Affine Transforms

Note Title 2015-01-19

Last class: Using homogeneous coordinates $(n+1)$ ↑dimension

can represent and manipulate both points and vectors.

All transforms we have seen so far can be represented as 4×4 matrices.

This is why graphics hardware (and software) is based on vec4's (ie 4 floats)

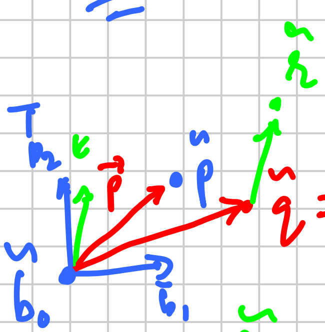
§ Affine transform ≃

linear transform of displacement vectors followed by a translation

$$\underline{\bar{A}} = \left[\begin{array}{c|c} I & \bar{t} \\ \hline & 1 \end{array}\right] \left[\begin{array}{c|c} \underline{\bar{l}}_{3\times3} & \\ \hline & \underline{1} \end{array}\right]$$

Eg

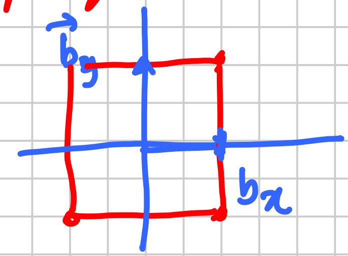
$$\left[\begin{array}{c|c} I & \bar{t} \\ \hline & 1 \end{array}\right] \underbrace{\left[\begin{array}{c|c} \underline{\bar{l}}_{3\times3} & \\ \hline & \underline{1} \end{array}\right] \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ 1 \end{pmatrix}}_{\text{first transform}}$$



$$\tilde{q} = f(\tilde{p})$$

$$\tilde{r} = \begin{pmatrix} q_1 + t_1 \\ q_2 + t_2 \\ 1 \end{pmatrix} = \left[\begin{array}{c|c} I & \bar{t} \\ \hline & \boxed{} \end{array}\right] \begin{pmatrix} q_1 \\ q_2 \\ 1 \end{pmatrix} = \underline{\bar{l}}_{3\times3} \begin{pmatrix} p_1 \\ p_2 \\ 1 \end{pmatrix}$$

Eg I A square in 2D



Scale along X (or 1) by a factor of 2

$$\overline{\underline{S}} = \begin{array}{c} \\ \\ 1 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} \quad & n & \quad 1 \end{array} \\ \left[\begin{array}{cc|c} 2 & & \\ & 1 & \\ \hline & & 1 \end{array}\right] \end{array}$$

Translation along X by 2 units

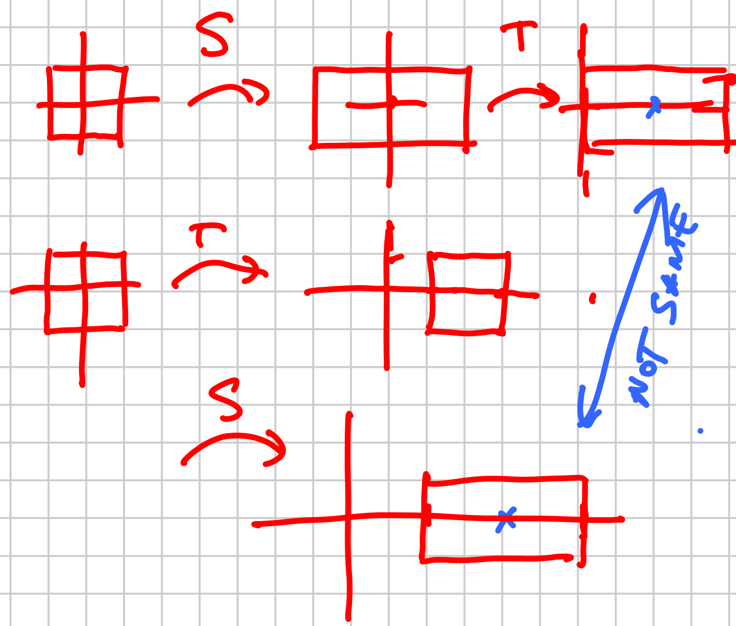
$$\overline{\underline{T}} = \left[\begin{array}{cc|c} 1 & & 2 \\ & 1 & 0 \\ \hline & & 1 \end{array}\right]$$

Does the order matter?

Note it's all in the same direction . .

$$\overline{\underline{T}}\ \overline{\underline{S}} = \left[\begin{array}{cc|c} 2 & & 2 \\ & 1 & \\ \hline & & 1 \end{array}\right]$$

$$\overline{\underline{S}}\ \overline{\underline{T}} = \left[\begin{array}{cc|c} 2 & & 4 \\ & 1 & 0 \\ \hline & & 1 \end{array}\right]$$



ORDER MATTERS !!

A note about notation in writing

- Zeros will be left blank

$$\left[\begin{array}{cc|c} 1 & & \\ & 1 & \\ \hline & & 1 \end{array}\right] \text{ instead of } \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right]$$

- Always "block" your matrices

$$\begin{array}{c} n \\ 1 \end{array}\!\!\begin{array}{c} \begin{array}{cc} n & \quad 1 \end{array} \\ \left[\begin{array}{c|c} \quad & \\ \hline & \end{array}\right] \end{array}$$

where n is usually 2 or 3

§ Structure of an affine transform matrix

$$\bar{A} = \left[\begin{array}{c|c} L & t \\ \hline & 1 \end{array}\right]$$

n x n linear transform

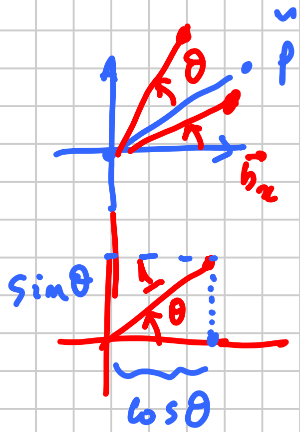
translation that follows L

last row is always $[0 \; 0 \; 0 \,|\, 1]$

§ Rotation

in 2D

what happens to basis vectors?



$$\bar{b}_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$\vec{b}_y = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

$$\bar{R} = \left(\begin{array}{cc|c} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

Check by plugging in unit vectors