Transformations

Note Title 2015-01-14

§ Basis Matrix / Coordinate System for a vector space

$$\vec{v} = v_1 \vec{b}_1 + v_2 \vec{b}_2 + v_3 \vec{b}_3$$

$$= \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$= \underline{\vec{b}} \; \bar{v}$$

Basis matrix   coordinate (vector)

§ Linear transformations of a vector space

$$\vec{v} \longrightarrow \mathcal{L}(\vec{v})$$

$$\mathcal{L}(\vec{u} + \vec{v}) = \mathcal{L}(\vec{u}) + \mathcal{L}(\vec{v})$$
$$\mathcal{L}(a\vec{v}) = a\,\mathcal{L}(\vec{v})$$

|| definition

Eg: scale / stretch uniformly ✓

translation? NO!

[check add $\vec{t}$ to all vectors

$\vec{u} \to \vec{u} + \vec{t}$ $\vec{v} \to \vec{v} + \vec{t}$

$\mathcal{L}(\vec{u} + \vec{v}) = \vec{u} + \vec{v} + \vec{t} \neq (\vec{u} + \vec{t}) + (\vec{v} + \vec{t})$]

Reflection ✓

Rotation ✓

§ Coordinates

Once you know what happens to basis vectors,
can reconstruct what happens to any vector

$$\vec v = \sum_i v_i \vec b_i = \underline{\vec b}\,\bar v$$

$$L(\vec v) = \sum v_i L(\vec b_i)$$

$$= \left[L(\vec b_1) \quad L(\vec b_2) \quad L(\vec b_3) \right] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Since these are vectors

$$= \left[\underline{\vec b}\,\bar L_1 \quad \underline{\vec b}\,\bar L_2 \quad \underline{\vec b}\,\bar L_3 \right] \bar v$$

$$= \underline{\vec b}\, \underbrace{\left[\bar L_1 \quad \bar L_2 \quad \bar L_3 \right]} \bar v$$

$$\underline{\vec b}\,\bar v = \vec v \longrightarrow \underline{\vec b}\; \underline{\bar L}\; \bar v$$

Linear transform in coordinates

A 2D array, written as a an array of column matrices

If we fix a basis

$$\bar v \longrightarrow \underline{\bar L}\,\bar v$$

Aside: GLSL provides built in functionality for this kind of operation.

$$\text{mat3 } L; \;\rightarrow\; \left[\begin{bmatrix} L_0 \end{bmatrix} \begin{bmatrix} L_1 \end{bmatrix} \begin{bmatrix} L_2 \end{bmatrix} \right]$$

(*) $L[0]$

Matrices stored in Column major form!

$$\text{vec3 } v;$$

$$v_{new} = L * v;$$

§ Scaling

$$\bar{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \rightarrow \begin{pmatrix} \alpha v_1 \\ \beta v_2 \\ \gamma v_3 \end{pmatrix}$$

$$L = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix}$$

$$Lv = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix}$$

§ Reflection about y-z plane

$$L = \begin{bmatrix} -1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$