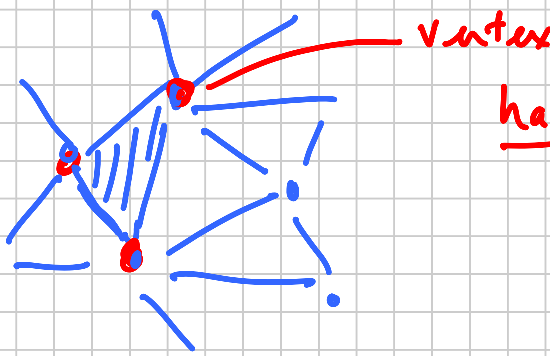
L3. Points and Vectors

Note Title 1/9/2015

Objectives: - Distinguish between points and vectors
- How to represent these
    - In a computer program (coordinates)
    - In mathematics (Notation)



has attributes:
point
normal
color
neighbours

NOT: vertex is a point

A point is a location in space, If you know a distinguished point, which we can call "origin"

$\vec{v}$ is a displacement vector

$$\hat{p} = \hat{o} + \vec{v}$$

There are other kinds of vectors: velocity, normals, . . .

§ Vector space

$$V = \{ \vec{v}, \vec{a}, \vec{b}, \ldots \}$$

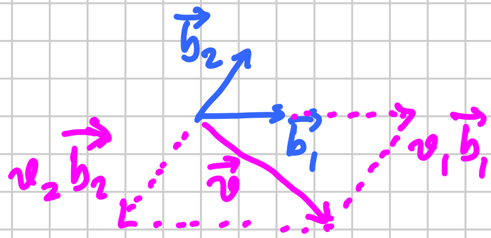
$$\vec{a} + \vec{b} \in V \quad \text{if} \quad \vec{a}, \vec{b} \in V$$

$$\alpha \vec{a} \in V \quad \text{if} \quad \vec{a} \in V$$

§ Basis $\vec{b}_1, \vec{b}_2, \ldots$

for any $\vec{v} = v_1 \vec{b}_1 + v_2 \vec{b}_2$

coordinates of $\vec{v}$ in the basis



A basis is a linearly independent set of vectors, which is complete for V.
The size of the basis is called the dimension of V.
The basis is not unique.

Why do we need this? Gives us coordinates

§ Orthonormal basis

Suppose we have a "dot" product

$$\vec{v}_1 \cdot \vec{v}_2 \longrightarrow \text{scalar}$$

Can define $\vec{a} \cdot \vec{a} \overset{\text{def}}{=} \|a\|^2$ where $\|a\|$ is a norm
i.e., a measure of size

Can "normalize" a vector, i.e. $\dfrac{\vec{a}}{\|a\|} = \dfrac{1}{\|a\|} \vec{a}$

Two vectors are <u>orthogonal</u> if $\vec{v}_1 \cdot \vec{v}_2 = 0$

A basis in which all vectors are mutually orthogonal, and have norm $= 1$ is called "<u>orthonormal</u>"

☆ Dot product of 2 vectors in an orthonormal basis has a simple form.

$$\vec{u} = u_1 \vec{b}_1 + u_2 \vec{b}_2$$
$$\vec{v} = v_1 \vec{b}_1 + v_2 \vec{b}_2$$

$$\vec{u} \cdot \vec{v} = u_1 v_1 \underbrace{\vec{b}_1 \cdot \vec{b}_1}_{=1} + u_1 v_2 \underbrace{\vec{b}_1 \cdot \vec{b}_2}_{=0} + \cdots$$

$$= u_1 v_1 + u_2 v_2$$ just involves <u>scalars!</u>

This is why orthonormal is great, we'll use whenever possible.

§ <u>Important No(ta)tion</u>

	<u>Pai</u>	<u>Book</u>
Point	$\hat{p}$	✓
vector	$\vec{v}$	✓
Column Matrix	$\bar{a}$	$\mathbf{a}$
Row Matrix	$\underline{a}$	$\mathbf{a}^t$

bold face

☆ <u>Change from Book.</u>

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$[a_1 \; a_2 \; a_3]$$