Perspective Projection

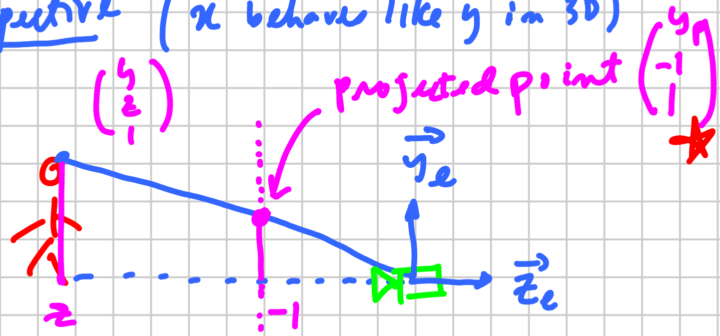
Perspective Cameras and Projective Transformation

- What do you get when you call
  var camera = new
  THREE.PerspectiveCamera(30, 1, 0.1, 1000);
  // view angle, aspect ratio, near, far

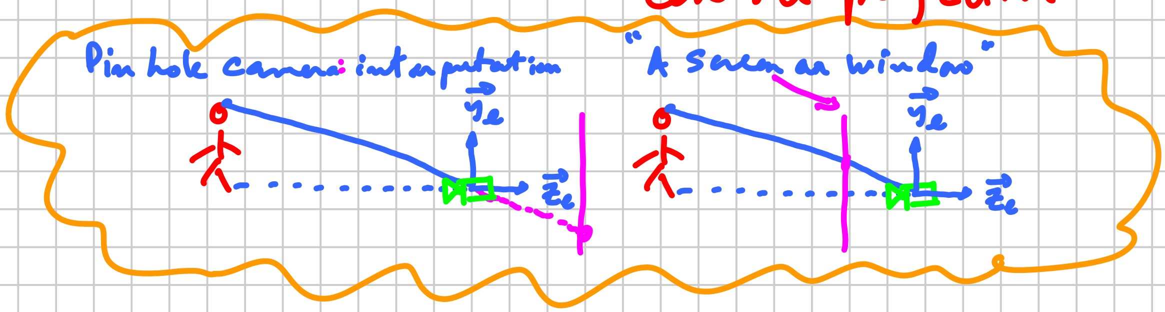
Perspective

Note Title 2015-02-11

2D perspective (x behave like y in 3D)



★ Want to represent the light ray through the origin "central projection"



To find y_p, use similar triangles

$$\frac{y_p}{y} = \frac{-1}{z} \quad \text{or} \quad y_p = -\frac{y}{z}$$

This is a <u>non-linear</u> transformation of 2D/3D space!

§ Representing this using a homogeneous transformation (i.e. $n+1 \times n+1$ matrix, where n is dimension)

$$\begin{pmatrix} y \\ z \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} -y/z \\ -1 \\ 1 \end{pmatrix} = -\tfrac{1}{z}\begin{pmatrix} y \\ z \\ -z \end{pmatrix} \overset{\text{equivalent}}{\simeq} \begin{pmatrix} y \\ z \\ -z \end{pmatrix}$$

e.g. gl-position.

From now on, identify all non-zero multiples of a point with itself. That is, we ignore the "scale"

Dividing by the last entry w in $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$ is called

homogenization.

Using this can write projection as a matrix!

$$P_b = \left[\begin{array}{cc|c} 1 & & \\ & 1 & \\ \hline & -1 & \end{array}\right]$$

achieves the projection above.

Note big change

But all depth info is lost. The matrix is singular

§ A better projection matrix that retains some depth info.

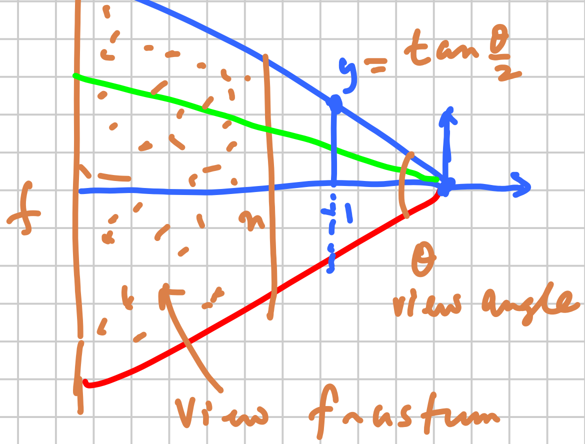
$$P_u = \left[\begin{array}{cc|c} 1 & & \\ & 0 & 1 \\ \hline & -1 & 0 \end{array}\right]$$

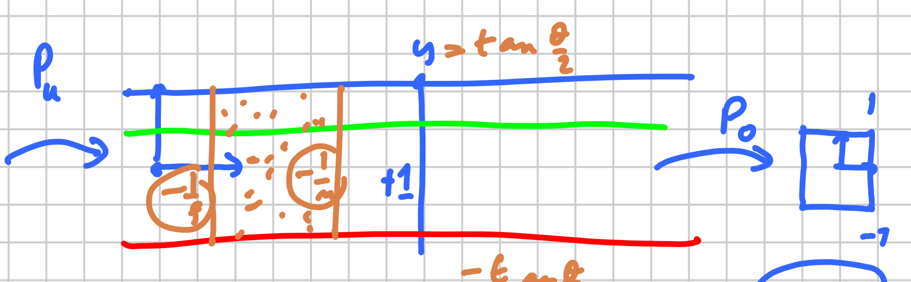
sometimes called an "unhinging" transform

What does this do to

$$\begin{pmatrix} y \\ z \\ 1 \end{pmatrix} \xrightarrow{P_u} \begin{pmatrix} y \\ 1 \\ -z \end{pmatrix} \xrightarrow[H]{\text{homogenize}} \begin{pmatrix} -y/z \\ -1/z \\ 1 \end{pmatrix}$$

Converts lines through the origin to parallel lines



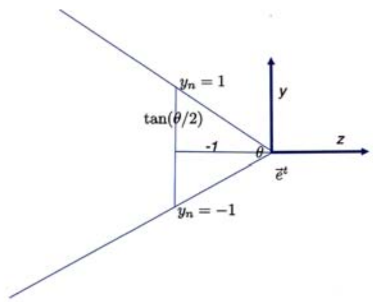


see L14

★ So this is now an Orthographic Projection

So the total perspective matrix is $P = P_o P_u$

PerspectiveCamera
Eye coords → Clip coords



$$\begin{bmatrix} \dfrac{1}{\alpha \tan\left(\dfrac{\theta}{2}\right)} & 0 & 0 & 0 \\ 0 & \dfrac{1}{\tan\left(\dfrac{\theta}{2}\right)} & 0 & 0 \\ 0 & 0 & \dfrac{f+n}{f-n} & -\dfrac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

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