CPSC 314
Computer Graphics

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Frames, Quiz Review

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Today

- Complete transformations wrt aux frame
- Review practice questions
- Additional tips

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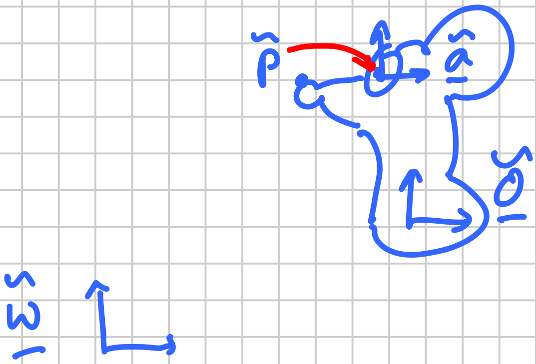
Note Title 2015-01-26

Very important case: How to transform a point w.r.t. an auxiliary frame.

Also in Sec 5.2



$\hat{\underline{a}} = \hat{\underline{w}}\,\underline{\bar{A}}$

$\hat{\underline{o}} = \hat{\underline{w}}\,\underline{\bar{O}}$ ← "model matrix"

Want to apply a transformation $\underline{\bar{M}}$ in frame $\hat{\underline{a}}$

Recipe: transform everything into a common frame. Then it's a matrix problem.

$\hat{p} = \hat{\underline{o}}\,\bar{p}$ "p is defined in object frame"

$= \hat{\underline{w}}\,\underline{\bar{O}}\,\bar{p}$

convert to $\hat{\underline{w}}$: $= \hat{\underline{w}}\, O\, p$

convert to $\hat{\underline{a}}$: $= \hat{\underline{a}}\, A^{-1}\, \underbrace{O\, p}_{p_w}$

transform by M → $\hat{\underline{a}}\, M A^{-1} O p$

transform to wherever I want, e.g. $\hat{\underline{w}}$: $= \hat{\underline{w}}\, \boxed{A M A^{-1}}\, O p$

$p_w \longrightarrow p_w' = A M A^{-1} p_w$

[Simplified notation for matrices and column matrix

(n×n)	
O	p
A	n
(caps)	(lowercase)

[For frames & points still $\hat{\underline{w}}$ $\hat{p}$

§ Interpreting sequences of transformations

$$\underline{\hat{w}}\ A\,B\,C\,(D\,(E\,(F\,p)))$$

Better if you think about changing shape of object

$\longleftarrow$ apply in this order for view ①

$\longrightarrow$ " " for view ②

$$((((\underline{\hat{w}}\,A)\,B)\,C)\,D\,E\,F)\,p$$

a new frame

Better for changing from model to world to camera

Both are correct! Just a question of convenience.

Frames in Graphics, continued

- Section 5.2 is very important, since it uses transformations in the most common ways in computer graphics, e.g., different versions of doMtoOwrtA (see p. 46 of book). Make sure you understand this section.

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Practice Homework

Last class you were told:

Come prepared with your answers for the next class

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C³: Vector Spaces

2014-01-13

Suppose $\vec{e}_1$ & $\vec{e}_2$ form an orthonormal basis, and $\vec{a}$ & $\vec{b}$ are two orthogonal vectors with coordinates $\bar{a}$ & $\bar{b}$ in basis $\underline{\vec{e}}$.

Consider these statements:

1. $\vec{e}_1 \times \vec{e}_2 = 0 \qquad \vec{e}_1 \times \vec{e}_1 = 1$
2. $\vec{e}_1 \cdot \vec{e}_2 = 0 \qquad \vec{e}_1 \cdot \vec{e}_1 = 1$
3. $(\bar{a})^T \bar{b} = 0$
4. $(\bar{a})^T \bar{b} = 1$

Choose the best:

A : ① and ③

B : ① and ④

C : ② and ③

D : ② and ④

E : ②, ③ & ④

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Practice: GLSL

- What is the mandatory output in a vertex shader?
  - a) gl_Position
  - b) The color of each vertex (e.g. fragColor in the textbook example)
  - c) The texture coordinates
  - d) All of the above

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Practice: Rotation

What kind of rotation is described by the matrix $\begin{bmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{bmatrix}$?

(a) A rotation about the y axis

(b) A rotation about the x axis

(c) A rotation about the line $x + y + z = 1$

(d) A rotation about the z axis

(e) Not a rotation

Y axis unchanged.

Tips:

Remember Rotation about x, Y, Z

$Rot(Y, 90°) = \begin{bmatrix} 0 & & 1 \\ & 1 & \\ -1 & & 0 \end{bmatrix}$ Esp, rotations by 90°, 180°, .

Practice : 3D Rotation

- Which of the following is equivalent to a 90 degree rotation about the z axis $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$?

a) 90 degree about z, then -90 about y:
$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) -90 degree about x, then -90 degree about y:
$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

c) -90 degree about y, then -90 degree about x:
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

d) None of the above

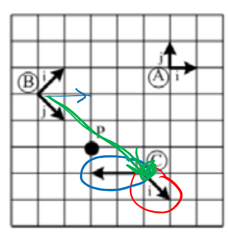
Practice: Homogeneous Coordinate

- Which of the following is a vector in 3D homogeneous coordinate?

a) $\begin{bmatrix} 0 \\ 3 \\ 3 \\ 3 \end{bmatrix}$ b) $\begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix}$ c) $\begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

Practice: Basis and Transformation

- What is the matrix that transforms a point from frame C to frame B?



a) $\begin{bmatrix} -1 & 0 & 0.5 \\ -1 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -1 & 3.5 \\ 0 & -1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} -1 & 1 & -3 \\ -1 & 0 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & -1 & 0.5 \\ 1 & -1 & 3.5 \\ 0 & 0 & 1 \end{bmatrix}$

$$\tilde{p} = \underline{\tilde{c}}\, p_c = \underline{\tilde{b}}\, p_b$$

$$\underline{\tilde{b}}\, C\, p_c$$

$$\boxed{\begin{array}{c|c|c} 0 & -1 & 0.5 \\ 1 & -1 & 3.5 \\ \hline 0 & 0 & 1 \end{array}}$$

Additional practice questions

Practice : Transformation

- Compute the transformation matrix that creates the following motion, all wrt the World frame. Rotate a point around the z axis by 90 degrees, and then scale the coordinates by ½ in all directions, and then translate by (2, 1, 3).

a) $$\begin{bmatrix} 0 & -0.5 & 0 & 2 \\ 0.5 & 0 & 0 & 1 \\ 0 & 0 & 0.5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $$\begin{bmatrix} 0 & -0.5 & 0 & 1 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) $$\begin{bmatrix} 0 & -0.5 & 0 & -0.5 \\ 0.5 & 0 & 0 & 1 \\ 0 & 0 & 0.5 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

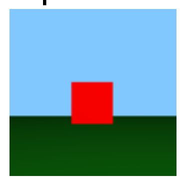
d) $$\begin{bmatrix} 0.5 & 0 & 0 & 1 \\ 0 & -0.5 & 0 & 0.5 \\ 0 & 0 & 0.5 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

e) None of the above

Practice: Moving an Object

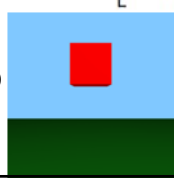
- The output on the screen corresponds to

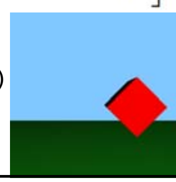
$$O = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \vec{o}^t = \vec{w}^t O$$

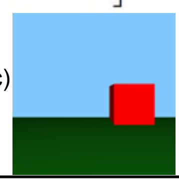


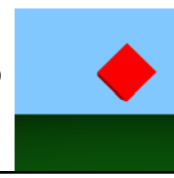
- Which of the following outputs corresponds to

$$O = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 0 & 0 \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) 

b) 

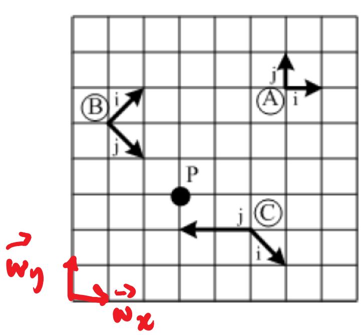
c) 

d) 

Practice: Interpreting Frames

- What is the matrix $\underline{C}$, in $\vec{c} = \vec{a}\, \underline{C}$

Note: $i \equiv x$, $j \equiv y$



(a) $\begin{bmatrix} 0 & -1 & -4 \\ -1/2 & -1/2 & -2.5 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1/\sqrt{2} & -1 & -1 \\ -1/\sqrt{2} & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -2 & -1 \\ -1 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & -1/2 & 1 \\ -1 & -1/2 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

(e) None of the above

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