CPSC 314
Computer Graphics

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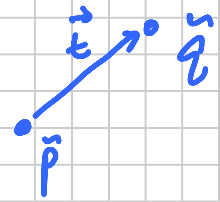
Homogeneous Coordinates and their transformations

Announcements

- Assignment 1: sign up for grading. Link posted on Piazza
  http://doodle.com/dx74v4k87mtgrsdp
- Lateness policy:
  up to **three days in the entire term**
  details will be posted on course web page
  Ensure you have submitted before your grading time slot
- Assignment 2 will be out this weekend

§ Translations of points

$$\tilde{p} \xRightarrow[\text{transforms to}]{} \tilde{p} + \vec{t}$$



$$\tilde{q} \underset{\text{equals}}{=} \tilde{p} + \vec{t}$$

$$\underline{\overline{T}} \overset{\text{define}}{=} \begin{array}{c c} & \begin{array}{cc} 3 & 1 \end{array} \\ \begin{array}{c} 3 \\ 1 \end{array} & \left[\begin{array}{ccc|c} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

So in a fixed basis

$$\underline{\overline{T}}\,\bar{p} = \left[\begin{array}{c|c} I & \begin{array}{c} t_1 \\ t_2 \\ t_3 \end{array} \\ \hline 0 \; 0 \; 0 & 1 \end{array}\right] \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} p_1 + t_1 \\ p_2 + t_2 \\ p_3 + t_3 \\ 1 \end{pmatrix}$$

$$= \bar{q}$$

So a translation is a <u>linear</u> transform in homogeneous coordinates!! So a 4×4 matrix.

Already know that rotations, scaling are also 4×4 matrices.

So all common manipulations are 4×4 matrices.

This is why these are in the DNA of OpenGL.

— x —

Recap:

in 3D orthogonal matrix

$R = \begin{bmatrix} \big| & \big|_{r} & \big| \end{bmatrix}$

each column

$r_1^T r_1 = 1$

$r_1^T r_2 = 0$

eg: $\begin{bmatrix} c & -s & \cdot \\ s & c & \cdot \\ \cdot & \cdot & 1 \end{bmatrix}$

r_1

$r_1^T r_1 = \cos^2 + \sin^2 = 1$

$r_1^T r_2 = -cs + sc + 0$
$= 0$

§ Special cases

Rotation $\left[\begin{array}{c|c} R & \vdots \\ \hline \cdots & 1 \end{array}\right]$

★ Rotates its "input" about the origin of the frame

Scaling $\begin{bmatrix} s_1 & & & \\ & s_2 & & \\ & & s_3 & \\ & & & 1 \end{bmatrix}$

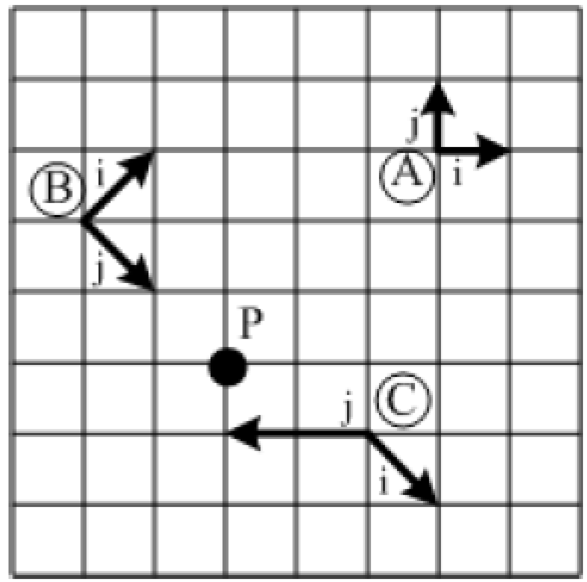
zeros elsewhere

Put it all together

$$\bar{T}\,\bar{L} = \left[\begin{array}{c|c} I & t \\ \hline & 1 \end{array}\right] \left[\begin{array}{c|c} l_{3\times3} & \\ \hline & 1 \end{array}\right] = \underbrace{\left[\begin{array}{c|c} l_{3\times3} & t \\ \hline \cdots & 1 \end{array}\right]}$$

This is the general form of any affine transformation.

C^3Homework: Basis and Transformation



- What are the coordinates of point P in frame A, B, and C?
- Which frame is orthonormal?
- How to transform a point from frame C to frame B?