L7. Affine Spaces

Note Title 2014-01-20

How to represent a point?

If we know one point "origin" $\mathring{o}$

Every other point is a "displacement vector" $\vec{v}$

Already know how to represent vectors in a basis.

Want to unify points and vectors in a single homogeneous system.

Extend the '+' operator. Allow:

$$\mathring{o} + \vec{v} = \mathring{p} \quad \text{and} \quad \mathring{p} - \mathring{o} = \vec{v}$$

$$1\,\mathring{o} = \mathring{o}$$

But not $\mathring{p} + \mathring{q}$ or $5\,\mathring{p}$

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§ A "basis" for this space

$$\mathring{p} = \mathring{b}_0 + \vec{v}$$

$$= 1\,\mathring{b}_0 + \sum_i v_i \vec{b}_i$$

$$= \begin{pmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 & \mathring{b}_0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 1 \end{pmatrix}$$

In our matrix notation

$$\boxed{\tilde{p} = \underline{\tilde{b}}\ \bar{v}}$$

Affine frame

Homogeneous coordinates in this frame

4×1 column matrix

★ Note! in book $\vec{b}^t$

Homogeneous coordinates of a point in a frame

$$\begin{pmatrix} x \\ x \\ x \\ 1 \end{pmatrix}$$

Homogeneous coords. of a vector?

$$\begin{pmatrix} x \\ x \\ x \\ 0 \end{pmatrix} \qquad \text{So } \underline{\tilde{b}} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 0 \end{pmatrix} = 0 + \sum_{i=1}^{3} v_i \vec{b}_i$$

Homogeneous coords of a linear transform

$$\underline{\bar{L}} = \begin{array}{c|c} & \begin{array}{cc} 3 & 1 \end{array} \\ \hline \begin{array}{c} 3 \\ 1 \end{array} & \left(\begin{array}{c|c} l_{3\times3} & \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \\ \hline 0\ 0\ 0 & 1 \end{array}\right) \end{array}$$

Check what this does to coordinates

$$\underline{\bar{L}}\,\bar{p} = \left(\begin{array}{c|c} l_{3\times3} & \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \\ \hline 0\ 0\ 0 & 1 \end{array}\right) \begin{pmatrix} v \\ \hline 1 \end{pmatrix} = \begin{pmatrix} l_{3\times3}\, v \\ 1 \end{pmatrix}$$

So $l_{3\times3}$ is linear transform of the displacement $\vec{v}$ from the origin of the frame $\tilde{b}_0$.