CPSC 314
Computer Graphics

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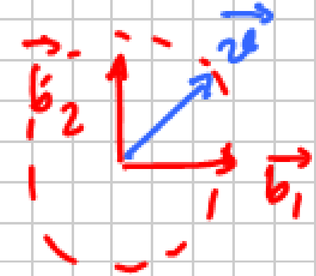
Linear transforms, scaling, rotation

Type of Linear Transformations

- Scaling

- Rotation

§ Scaling along coordinate axes



$$\vec{v} \text{ or } \vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \longrightarrow \begin{pmatrix} \alpha v_1 \\ \beta v_2 \\ \gamma v_3 \end{pmatrix}$$

Equivalently

$$\vec{v} \longrightarrow \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

§ Rotation

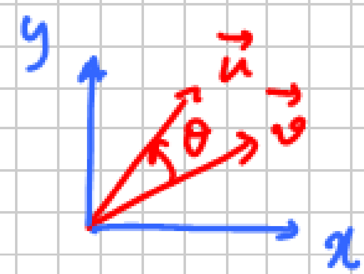
- A linear transformation that preserves dot product

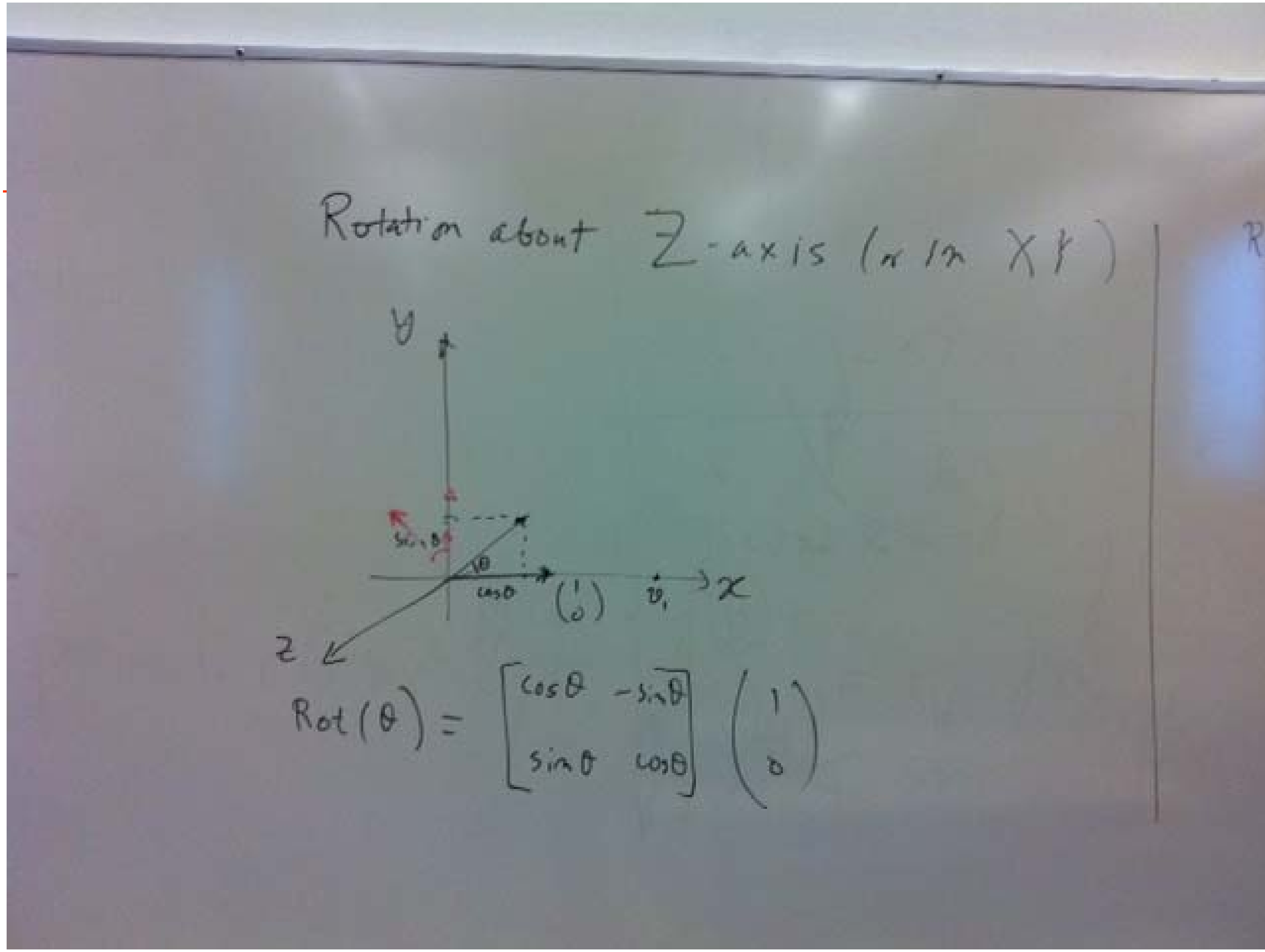
$\vec{v}_1 \cdot \vec{v}_2$ remain same

In particular $\|v\| = (v \cdot v)^{1/2}$ = length is preserved

AND preserves orientation (handedness)

Example rotations:




Rotation about Z-axis (or in XY)
y
x
Z
sin θ
θ
cos θ
Rot(θ) = [cosθ −sinθ ; sinθ cosθ] (1 ; 0)

Rotation in 3D about Z?

$$\mathrm{Rot}_z(\theta) = \left[\begin{array}{cc|c} c & -s & 0 \\ s & c & 0 \\ \hline 0 & 0 & 1 \end{array}\right]$$

$$\mathrm{Rot}_y(\theta) = \begin{bmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{bmatrix}$$

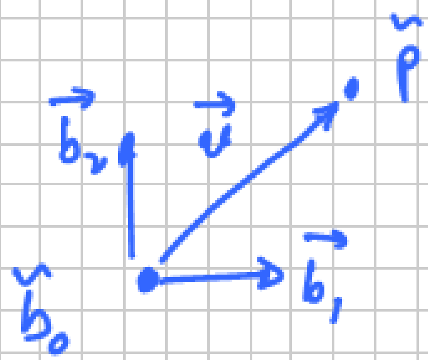
$$\mathrm{Rot}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ & c & -s \\ & s & c \end{bmatrix}$$

What happens if you multiply two rotations

P & Q

- PQ is also rotation
- Note: Matrix multiplication is <u>not</u> commutative
- It turns out that, with 3 rotations you express any possible rotation
- In an orthonormal basis, the columns/rows of a Rotation matrix are orthonormal too

§ Points



Can represent <u>any</u> point this way?

Include origin in the basis, extend algebra

$$\tilde{p} = \tilde{b}_0 + \vec{v}$$