Interpolation

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Partly from
Textbook Chapter 9

Midterm 2 update

- Textbook. Read ALL of these, except as noted
  - Ch 14 Materials (shading and lighting)
  - Ch 15 Texture Mapping
  - Ch 3.6 (transformation of normals)
  - Ch 9 Interpolation. Skip 9.2 and 9.3
  - Ch 10 Projection
  - Ch 12 From Vertex to Pixel
  - ~~Ch 11:~~ We'll cover this AFTER midterm, so Wed. will be review

§ Review Bernstein Polynomial

$$b_{\nu,n}(t) = \binom{n}{\nu} t^{\nu} (1-t)^{n-\nu}$$

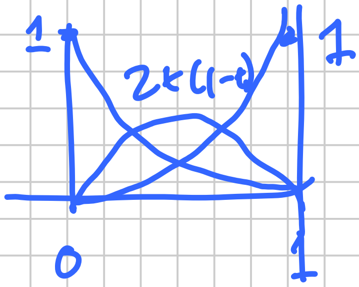
enumeration ↗ ↑ degree

"Binomial Coefficient"

Partition of unity

$$\sum_{\nu=0}^{n} b_{\nu,n}(t) = 1$$

$b_{\nu,2}$



§ Most important Case: degree 3 (cubic) & Bezier Curves

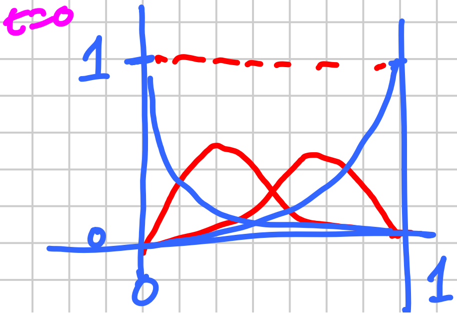
$\nu =$	0	1	2	3
$b_{\nu,3}$	$(1-t)^3$	$3t(1-t)^2$	$3t^2(1-t)$	t^3
$\left.\frac{db}{dt}\right\|_{t=0}$	-3	3	0	0

$-3(1-t)^2$

$3(1-t)^2 +$ $\cancel{3t}$ [illegible]

$\downarrow$

0



§ Bezier curve of degree n

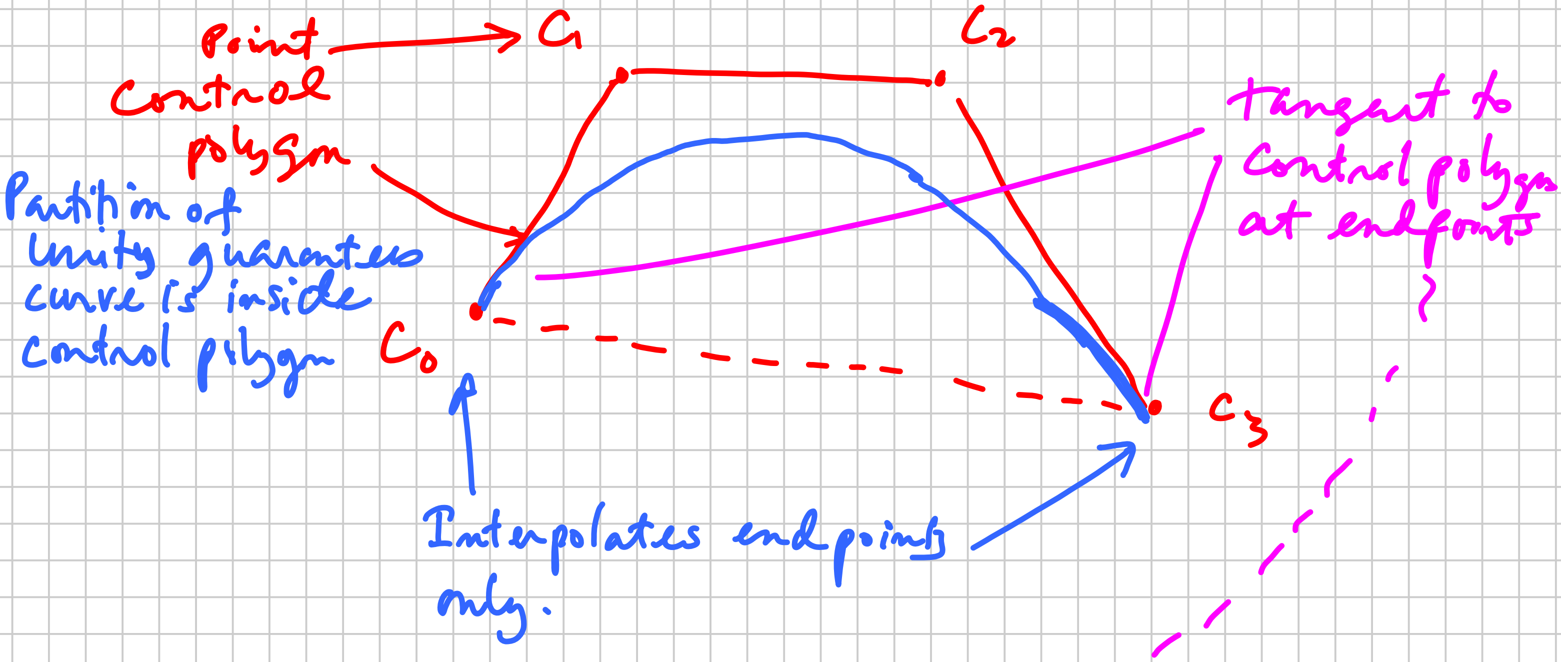
$$c(t) = \sum_{\nu=0}^{n} c_\nu \, b_{\nu,n}(t)$$

control points

blending functions/weights

Nice geometric intuition, easy to manipulate (so good user interface)

Introduced in Automotive CAD


Point
control polygon
c_0
c_1
c_2
c_3
Partition of unity guarantees curve is inside control polygon
tangent to control polygon at endpoints ?
Interpolates endpoints only.

[Sketch of proof of this (see purple entry of table above)

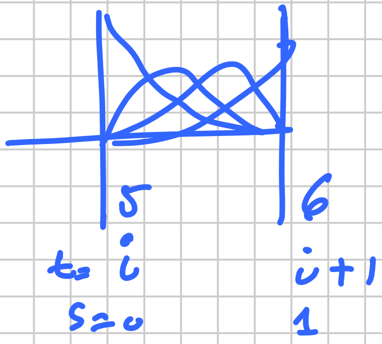
$$\frac{dc}{dt} = -3c_0 + 3c_1$$

$$= 3(c_1 - c_0)$$

§ Demo with Adobe Illustrator

§ Translate and scale the parameter t

So far $t \in [0, 1]$



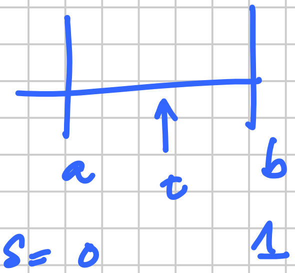
eg. translation

define $s = t - i$

$$b_{\nu,n}(t) = b_{\nu,n}(s)$$

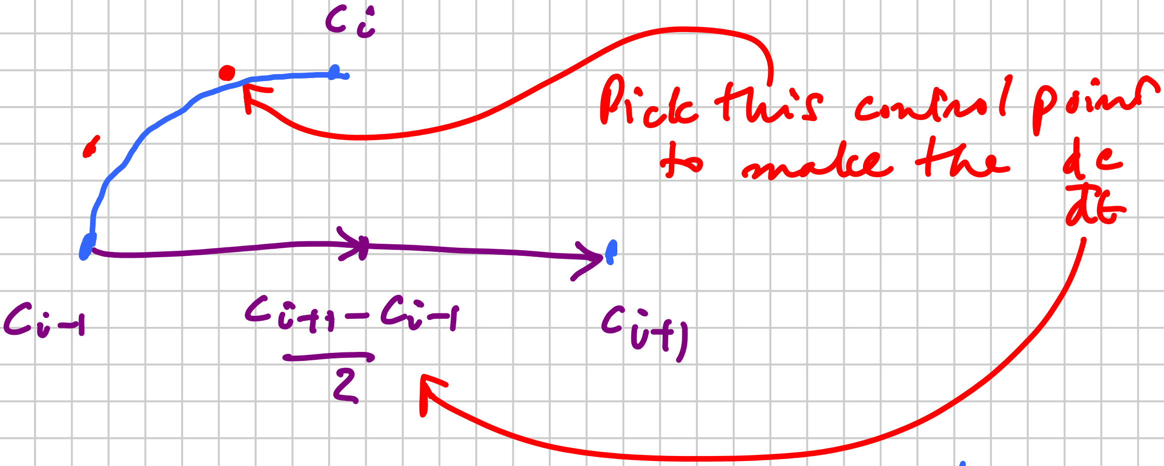
Scale

define $s = \frac{t-a}{b-a}$



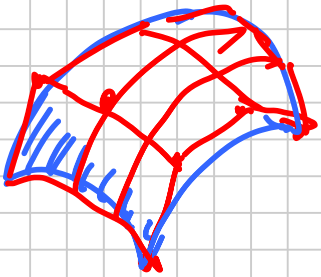
as long as you have a (redefined) param. $[0, 1]$, everything works.

§ Catmull-Rom splines


c_i
Pick this control point to make the $\frac{dc}{dt}$
c_{i-1}
$\frac{c_{i+1} - c_{i-1}}{2}$
c_{i+1}

widely used in animation. See textbook §9.2

§ 2D Surface Patch



Product of
Bezier curves

Bezier patch
$4 \times 4 = 16$ control points

★ Tesselation Shaders can evaluate these efficiently in OpenGL

The famous "Utah Teapot" is made using Bezier Patches.

§ B-Spline

see textbook 9.4

NURBs