Interpolation and Approximation of functions

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Partly from
Textbook Chapter 9

1

Today

- Midterm 2 preparation
- Interpolation and approximation

2

Midterm 2 Preparation

- In class, F 1-1:50. Please be on time.
- Review lecture notes, and assignments.
- Everything covered in lecture could be on the exam
- Everything covered in listed textbook chapters could be on the exam
- I will provide some practice problems on Monday
- Office hours next week:
  - W 3-4 (usual time)
  - R 3:30-4:30 (extra office hour)

3

Midterm 2 Preparation

- Textbook. Read ALL of these, except as noted
  - Ch 14 Materials (shading and lighting)
  - Ch 15 Texture Mapping
  - Ch 3.6 (transformation of normals)
  - Ch 9 Interpolation
  - Ch 10 (note: only basic projection was covered in midterm 1)
  - Ch 11: skip 11.2.1. We'll cover this next week
  - Ch 12 From Vertex to Pixel
- Topics from Midterm 1 will be assumed as pre-requisites (e.g., it is assumed you now know coordinate frames and how to transform them)

4

C^3 Review: Interpolation

- Given 2 points P1 = (1,5), P2 = (5,3), what is the corresponding y value of an interpolated point with x = 3? Use linear interpolation.
  a) 3
  b) 4
  c) 5
  d) Not well defined

C^3 Review: Constant Interpolation

- Given 2 points P1 = (1,5), P2 = (5,3), what is the corresponding y value of an interpolated point with x = 3? Use constant interpolation.
  a) 3
  b) 4
  c) 5
  d) Not well defined

Interpolation

Note Title 2014-03-14

Last class:

Linear interpolation

Key conceptual step

$$c(t) = c_0 \underbrace{(1-t)} + c_1 \underbrace{t}$$

Blending functions
Polynomials

Today: generalize this idea.

(1) c can be of high dimension.

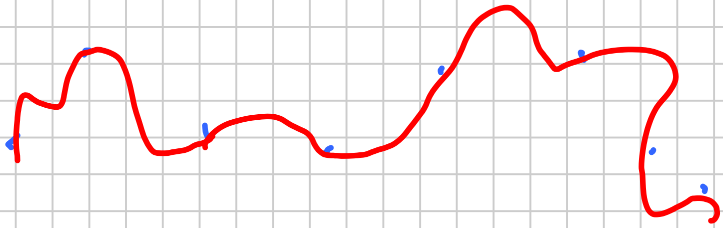
If $\bar{c} = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}$ $\quad \bar{c}_0 = \begin{pmatrix} c_{0x} \\ c_{0y} \\ c_{0z} \end{pmatrix}$ . . .

Interpolate one component at a time.

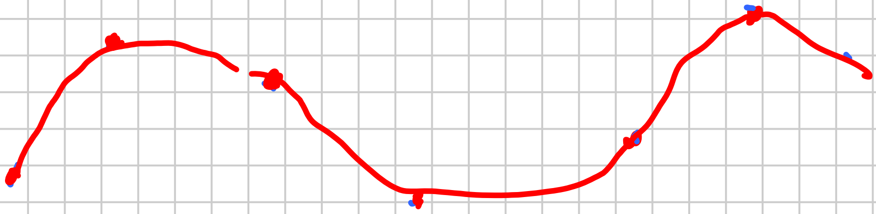
$$c_x(t) = c_{0x}(1-t) + c_{1x}\, t$$

$$\bar{c}(t) = \bar{c}_0 (1-t) + \bar{c}_1\, t$$

(2) Higher degree polynomial blending functions 2 (quadratic), $\boxed{3 \text{ (cubic)}}$, . . .
Higher order is possible but never used (too wiggly)



(3) Splines (piecewise polynomials, stitched together with desired continuity properties.



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§ Quadratic interpolation

c_1 •

c_0 • • c_2

Shortcut: Bernstein Polynomials as blending functions

deg 0	1		
deg 1	$(1-t)$	t	
deg 2	$(1-t)^2$	$2(1-t)t$	t^2

deg 3 $\quad (1-t)^3 \quad 3(1-t)^2 t \quad 3(1-t)t^2 \quad t^3$

deg n $\quad \boxed{b_{\nu,n}(t) = \binom{n}{\nu} t^{\nu} (1-t)^{n-\nu}}$

So quadratic interpolation

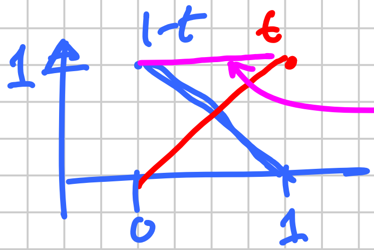
$$c(t) = c_0 (1-t)^2 + c_1\big(2(1-t)t\big) + c_2 t^2$$

Let's visualize this. All are function on [0,1]

deg 0

constants

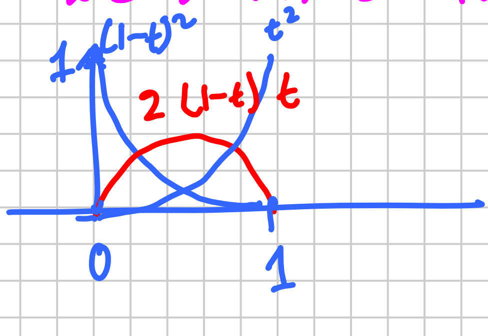
deg 1



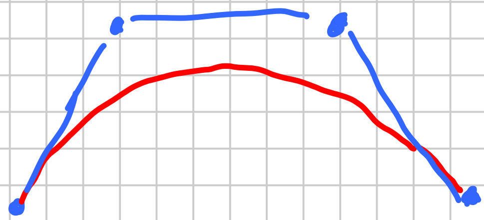
notice: the two blending function add to 1

Technical term:
"Partition of Unity"
(Book is terse on this)
Harder to see for higher degree but we will maintain this

deg 2



§ Bezier curves



deg 3 curves, specified by 4 control points.