Interpolation and Approximation of functions

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Partly from
Textbook Chapter 9

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Today

- Reminders:
  - Assignment 4 is out
  - Midterm 2 coming soon (March 21)
  - Final exam: APR 26, 03:30 PM
- Wrap up texture mapping for now
- New topic: Interpolation and approximation
  - Foundation of a lot of topics (Chapter 9), including geometric modeling (Chapter 22), animation (Chapter 23) and dealing with images (Chapters 16-18)

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Assignment 4

- Available now, due March 25 (by popular demand)
- However: please make sure you do at least the first 3 parts before March 21 (midterm)
  - These parts will help you understand the topics covered in class and textbook Chapter 15, which will be on the midterm
- Use the extra time after midterm for “Creative Licence”. Texture mapping is particularly great for this… many of you will find this a lot of fun!

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C³ Review: Texture mapping

- Which texture mapping technique would you use to put the smiley face in the scene?
  a) Basic texturing
  b) Projector texture mapping
  c) Environment cube maps
  d) All of the above
  e) None of the above

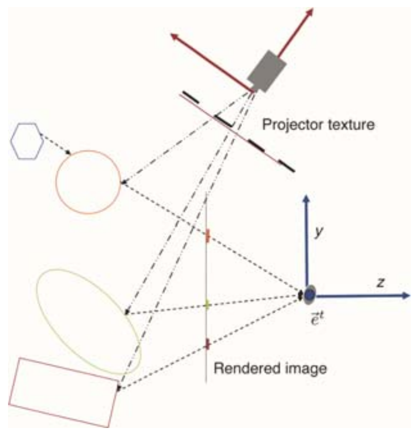
C^3 Review: Texture mapping

- In which of the following will the resulting surface color appear to be static when the eye (camera) is moving?
  a) Environment cube mapping
  b) Projector texture mapping
  c) Basic texture mapping
  d) B & C
  e) All of the above

C^3 Review: Texture mapping

- Can you use texture mapping along with other lighting/shading techniques?
  a) Yes. Texture mapping should be done in the vertex shader, and lighting/shading should be done in the fragment shader so they don't overlap.
  b) Yes. The lighting/shading effect can be added to the texture map on each fragment.
  c) No. The normal from the texture is in conflict with the normal from the model
  d) No. The final colour of each pixel can only be determined by one technique.

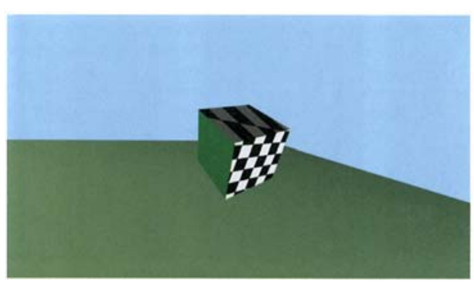
Geometry of Projector Textures



Projector texture mapping

- The slide projector is modeled using 4 by 4, modelview and projection matrices, M_s and P_s

$$\begin{bmatrix} x_t w_t \\ y_t w_t \\ - \\ w_t \end{bmatrix} = P_s M_s \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$



Projector texture mapping

- With the texture coordinates defined as

$$x_t = \frac{x_t w_t}{w_t} \text{ and } y_t = \frac{y_t w_t}{w_t}$$

- To color a point on a triangle with object coordinates $[x_o, y_o, z_o, 1]^t$, we fetch the texture data stored at location $[x_t, y_t]^t$

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Projector texture mapping

- Projector vertex shader

```
#version 330

uniform mat4 uModelViewMatrix;
uniform mat4 uProjMatrix;

uniform mat4 uSProjMatrix;
uniform mat4 uSModelViewMatrix;

in vec4 aVertex;
out vec4 vTexCoord;

void main(){
  vTexCoord = uSProjMatrix * uSModelViewMatrix * aVertex;
  gl_Position = uProjMatrix * uModelViewMatrix * aVertex;
}
```

Vertex shader generates texture coordinates! But not normalized

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Projector texture mapping

- Projector fragment shader

```
#version 330

uniform sampler2D vTexUnit0;

in vec4 aTexCoord;
out vec4 fragColor;

void main(){
  vec2 tex2;
  tex2.x = vTexCoord.x/vTexCoord.w;
  tex2.y = vTexCoord.y/vTexCoord.w;
  vec4 texColor0 = texture2D(vTexUnit0, tex2);
  fragCoor = texColor0;
}
```

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Interpolation & Approximation

Note Title 2014-03-12

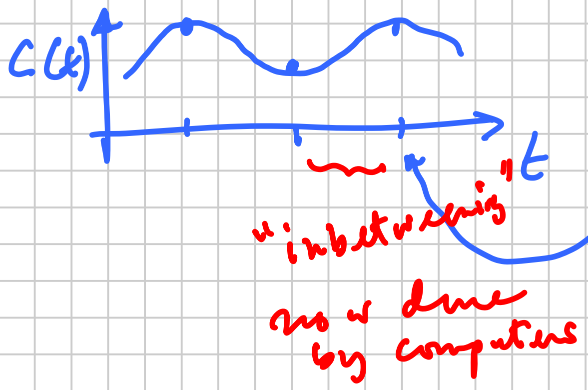
Motivation:

Digital / Discrete representation of Continuous (smooth) functions (in any dimension)

We will focus on 1D

- easier to explain
- Generalization to higher dim are conceptually straightforward
- lots of applications

eg. functions of time



Computer animation

"Keyframe animation"

Audio

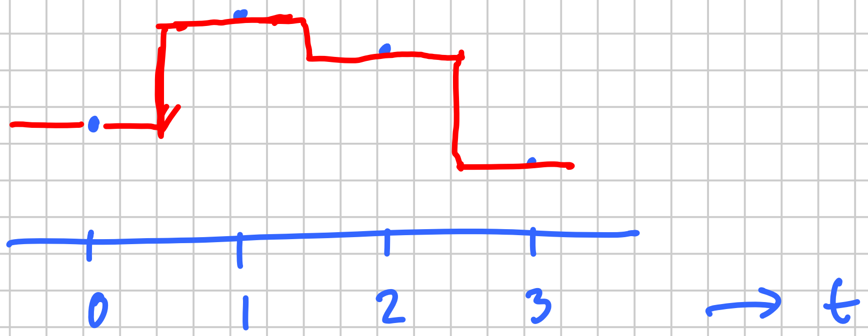
Key Questions

$$\text{Discrete} \xrightarrow{\text{reconstruction}} \text{Continuous}$$

Interpolation is one way to do this

$$\text{Discrete} \xleftarrow{\text{sampling}} \text{Continuous}$$

§ Constant (interpolation)



Obvious limitations

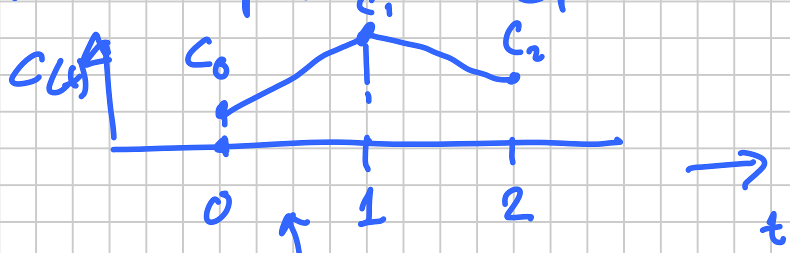
not continuous

may want smoother solutions

Note: this is what your monitor does!

when you display an image.

§ Linear Interpolation (piecewise)



Focus on this interval

$$c(t) = at + b$$

Fix boundaries

$$c_0 = a \cdot 0 + b \Rightarrow b = c_0$$

$$c_1 = a \cdot 1 + c_0 \Rightarrow a = c_1 - c_0$$

$$c(t) = (c_1 - c_0)\, t + c_0$$

☆ Key step. rewrite this

$$\boxed{C(t) = \underbrace{c_0}\,\underbrace{(1-t)} + \underbrace{c_1}\,\underbrace{t}}$$

Nicely separates data from the "blending weights" that are independent of data. Just depend on the type of interpolation.

c_i are called "control values"

In general c_i can be any dimensional vector

Eg: c_i could be the coordinates of a 3D point

In common usage c_i are called "control points"

Next class: Generalize to higher smoothness

Bezier curves, etc.