Global Illumination

Wolfgang Heidrich

Course News

Assignment 3 (project)
- Due Friday!!
- Demos in labs starting this Friday
- Demos are MANDATORY(!)

Reading
- Chapter 10 (ray tracing), except 10.8-10.10
- Chapter 14 (global illumination)

Area Light Sources

So far:
- All lights were either point-shaped or directional
  - *Both for ray-tracing and the rendering pipeline*
- Thus, at every point, we only need to compute lighting formula and shadowing for ONE light direction

In reality:
- All lights have a finite area
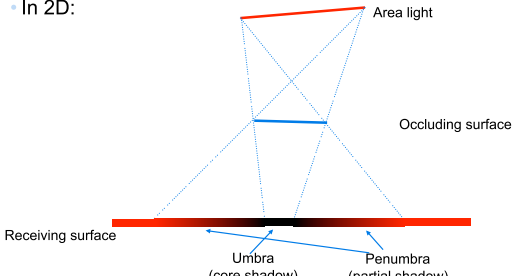- Instead of just dealing with one direction, we now have to integrate over all directions that go to the light source

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Area Light Sources

Area lights produce soft shadows:
- In 2D:

Area light

Occluding surface

Receiving surface

Umbra (core shadow)   Penumbra (partial shadow)

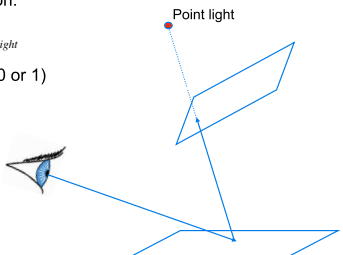## Area Light Sources

Point lights:
- Only one light direction:

$$I_{reflected} = \rho \cdot V \cdot I_{light}$$

- V is visibility of light (0 or 1)
- ρ is lighting model (e.g. diffuse or Phong)
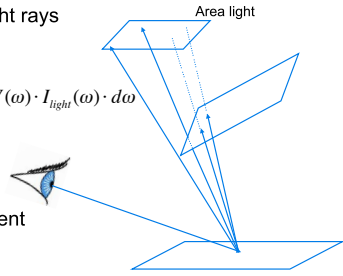
Point light

Are Light Sources

Area Lights:
- Infinitely many light rays
- Need to integrate over all of them:

$$I_{reflected} = \int_{\substack{light \\ directions}} \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

- Lighting model visibility and light intensity can now be different for every ray!

Area light

Integrating over Light Source

Rewrite the integration

- Instead of integrating over directions

$$I_{reflected} = \int_{\substack{light \\ directions}} \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

we can integrate over points on the light source

$$I_{reflected}(q) = \int_{s,t} \frac{\rho(p-q) \cdot V(p-q)}{|p-q|^2} \cdot I_{light}(p) \cdot ds \cdot dt$$

where q: point on reflecting surface, p= F(s,t) is a point on the area light

- We are integrating over p
- Denominator: quadratic falloff!

Integration

Problem:

- Except for the simplest of scenes, either integral is **not solvable analytically**!
- This is mostly due to the visibility term, which could be arbitrarily complex depending on the scene

So:

- Use numerical integration
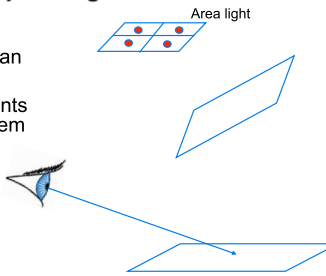- Effectively: approximate the light with a whole number of point lights

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Numerical Integration

Regular grid of point lights

Area light

- Problem: will see 4 hard shadows rather than as soft shadow
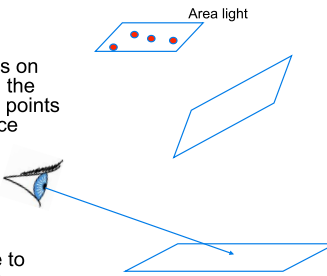- Need LOTS of points to avoid this problem

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Monte Carlo Integration

Better:

Area light

- **Randomly** choose the points
- Use different points on light for computing the lighting in different points on reflecting surface

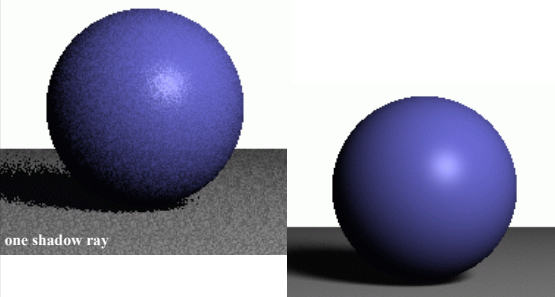- This produces random noise
- Visually preferable to structured artifacts

Monte Carlo Integration

one shadow ray

lots of shadow rays

Monte Carlo Integration

Formally:

- Approximate integral with finite sum

$$I_{reflected}(q) = \int_{s,t} \frac{\rho(p-q) \cdot V(p-q)}{|p-q|^2} \cdot I_{light}(p) \cdot ds \cdot dt$$

$$\approx \frac{A}{N} \sum_{i=1}^{N} \frac{\rho(p_i-q) \cdot V(p_i-q)}{|p_i-q|^2} \cdot I_{light}(p_i)$$

where

- The p_i are randomly chosen on the light source
  - With equal probability!
- A is the total area of the light
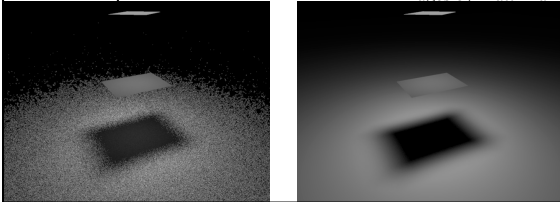- N is the number of samples (rays)

Sampling

Sample directions vs. sample light source

- Most directions do not correspond to points on the light source
  - *Thus, variance will be higher than sampling light directly*

Images by Matt Pharr

Monte Carlo Integration

Note:

- This approach of approximating lighting integrals with sums over randomly chosen points is much more flexible than this!
- In particular, it can be used for global illumination
  - *Light bouncing off multiple surfaces before hitting the eye*

Global Illumination

So far:

- Have considered only light directly coming form the light sources
  - *As well as mirror reflections, refraction*

In reality:

- Light bouncing off diffuse and/or glossy surfaces also illuminates other surfaces
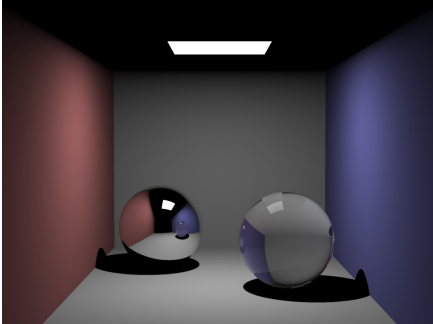 - *This is called global illumination*

Direct Illumination

Image by
Henrik Wann Jensen
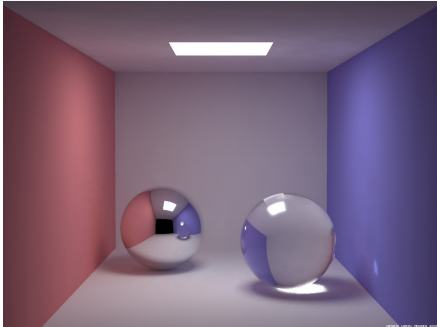

Global Illumination

Image by
Henrik Wann Jensen

Rendering Equation

Equation guiding global illumination:

$$L_o(x,\omega_o) = L_e(x,\omega_o) + \int_\Omega \rho(x,\omega_i,\omega_0)L_i(\omega_i)d\omega_i$$

$$= L_e(x,\omega_o) + \int_\Omega \rho(x,\omega_i,\omega_0)L_o(R(x,\omega_i),-\omega_i)d\omega_i$$

Where

- ρ is the reflectance from ω_i to ω_o at point x
- L_o is the outgoing (I.e. reflected) **radiance** at point x in direction ω_i
  - *Radiance is a specific physical quantity describing the amount of light along a ray*
  - *Radiance is constant along a ray*
- L_e is the emitted radiance (=0 unless point x is on a light source)
- R is the "ray-tracing function". It describes what point is visible from x in direction ω_i

Rendering Equation

Equation guiding global illumination:

$$L_o(x,\omega_o) = L_e(x,\omega_o) + \int_\Omega \rho(x,\omega_i,\omega_0)L_i(\omega_i)d\omega_i$$

$$= L_e(x,\omega_o) + \int_\Omega \rho(x,\omega_i,\omega_0)L_o(R(x,\omega_i),-\omega_i)d\omega_i$$
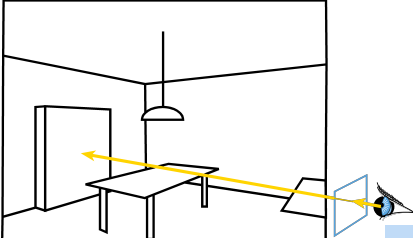
Note:

- The rendering equation is an **integral equation**
- This equation cannot be solved directly
  - *Ray-tracing function is complicated!*
  - *Similar to the problem we had computing illumination from area light sources!*

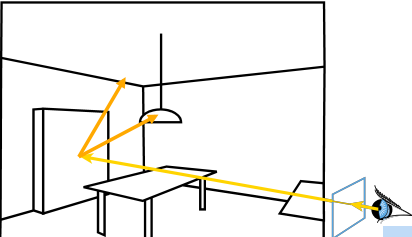Wolfgang Heidrich

Ray Casting

- Cast a ray from the eye through each pixel
- The following few slides are from Fred Durand (MIT)
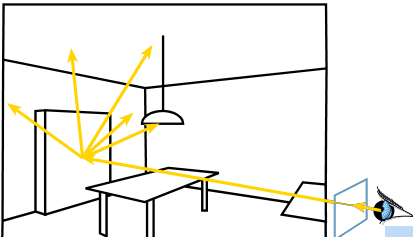

Ray Tracing

- Cast a ray from the eye through each pixel
- Trace secondary rays (light, reflection, refraction)

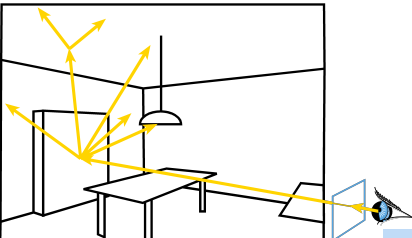## Monte Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
  - *Accumulate radiance contribution*
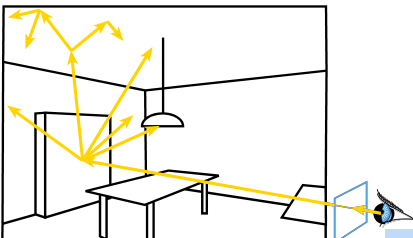

Monte Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse

Monte Carlo

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse

Monte Carlo

• Systematically sample primary light

Monte Carlo Path Tracing

In practice:

• Do not branch at every intersection point
  – *This would have exponential complexity in the ray depth!*
• Instead:
  – *Shoot some number of primary rays through the pixel (10s-1000s, depending on scene!)*
  – *For each pixel and each intersection point, make a single, random decision in which direction to go next*

Monte Carlo Path Tracing

• Trace only one secondary ray per recursion
• But send many primary rays per pixel
• (performs antialiasing as well)

How to Sample?

Simple sampling strategy:

• At every point, choose between all possible reflection directions with equal probability
• This will produce very high variance/noise if the materials are specular or glossy
• Lots of rays are required to reduce noise!

Better strategy: importance sampling

• Focus rays in areas where most of the reflected light contribution will be found
• For example: if the surface is a mirror, then only light from the mirror direction will contribute!
• Glossy materials: prefer rays near the mirror direction

How to Sample?

• Images by Veach & Guibas

Naive sampling strategy

Multiple importance sampling

How to Sample?

Sampling strategies are still an active research area!

• Recent years have seen drastic advances in performance
• Lots of excellent sampling strategies have been developed in statistics and machine learning
  – *Many are useful for graphics*

How to Sample?

UBC

Objective:

- Compute light transport in scenes using stochastic ray tracing
  - *Monte Carlo, Sequential Monte Carlo*
  - *Metropolis*

[Burke, Ghosh, Heidrich '05]
[Ghosh, Heidrich '06],
[Ghosh, Doucet, Heidrich '06]

How to Sample?

UBC

- E.g: importance sampling (left) vs. Sequential Monte Carlo (right)

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More on Global Illumination

UBC

This was a (very) quick overview

- More details in CPSC 514 (Computer Graphics: Rendering)
- Not offered this year, but in 20011/12

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