Global Illumination

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Course News

Assignment 3 (project)
- Due Friday!!
- Demos in labs starting this Friday
- Demos are MANDATORY(!)

Reading
- Chapter 10 (ray tracing), except 10.8-10.10
- Chapter 14 (global illumination)

Area Light Sources

So far:
- All lights were either point-shaped or directional
  - Both for ray-tracing and the rendering pipeline
- Thus, at every point, we only need to compute lighting formula and shadowing for **one** light direction

In reality:
- All lights have a finite area
- Instead of just dealing with one direction, we now have to integrate over all directions that go to the light source

Area Light Sources

Point lights:
- Only one light direction:
  \[ I_{\text{reflected}} = \rho \cdot V \cdot I_{\text{light}} \]
- \( V \) is visibility of light (0 or 1)
- \( \rho \) is lighting model (e.g. diffuse or Phong)

Area Lights:
- Infinitely many light rays
- Need to integrate over all of them:
  \[ I_{\text{reflected}} = \int_{\text{rays}} \rho(\omega) \cdot V(\omega) \cdot I_{\text{light}}(\omega) \cdot d\omega \]
- Lighting model visibility and light intensity can now be different for every ray!
Integrating over Light Source

Rewrite the integration

Instead of integrating over directions

\[ I_{\text{reflected}} = \int_{\text{light}} \rho(\omega) \cdot V(\omega) \cdot I_{\text{light}}(\omega) \cdot d\omega \]

we can integrate over points on the light source

\[ I_{\text{reflected}}(q) = \int_{\omega} \frac{\rho(p-q) \cdot V(p-q)}{|p-q|^2} \cdot I_{\text{light}}(p) \cdot ds \cdot dt \]

where \( q \) point on reflecting surface, \( p = F(s,t) \) is a point on the area light

- We are integrating over \( p \)
- Denominator: quadratic falloff

Integration

Problem:
- Except for the simplest of scenes, either integral is not solvable analytically
- This is mostly due to the visibility term, which could be arbitrarily complex depending on the scene

So:
- Use numerical integration
- Effectively: approximate the light with a whole number of point lights

Numerical Integration

Regular grid of point lights

Problem:
- will see 4 hard shadows rather than as soft shadows
- Need LOTS of points to avoid this problem

Monte Carlo Integration

Batter:
- Randomly choose the points
- Use different points on light for computing the lighting in different points on reflecting surface

This produces random noise
- Visually preferable to structured artifacts

Monte Carlo Integration

Formally:
- Approximate integral with finite sum

\[ I_{\text{reflected}}(q) = \int_{\omega} \frac{\rho(p-q) \cdot V(p-q)}{|p-q|^2} \cdot I_{\text{light}}(p) \cdot ds \cdot dt \]

\[ \approx \frac{A}{N} \sum_{i=1}^{N} \frac{\rho(p_i-q) \cdot V(p_i-q)}{|p_i-q|^2} \cdot I_{\text{light}}(p_i) \]

where
- \( p_i \) are randomly chosen on the light source
- \( A \) is the total area of the light
- \( N \) is the number of samples (rays)
Sampling

Sample directions vs. sample light source
Most directions do not correspond to points on the light source
Thus, variance will be higher than sampling light directly

Images by Matt Pharr

Monte Carlo Integration

Note:
This approach of approximating lighting integrals with sums over randomly chosen points is much more flexible than this!
In particular, it can be used for global illumination
Light bouncing off multiple surfaces before hitting the eye

Global Illumination

So far:
Have considered only light directly coming form the light sources
As well as mirror reflections, refraction

In reality:
Light bouncing off diffuse and/or glossy surfaces also illuminates other surfaces
This is called global illumination

Images by Henrik Wann Jensen

Direct Illumination

Rendering Equation

Equation guiding global illumination:
\[ L_n(x, \omega_i) = L_e(x, \omega_i) + \int p(x, \omega_i, \omega_o) L_o(\omega_o) d\omega_o \]
\[ = L_e(x, \omega_i) + \int p(x, \omega_i, \omega_o) L_o(R(x, \omega_i), -\omega_i) d\omega_o \]

Where
\( p \) is the reflectance from \( \omega_i \) to \( \omega_o \) at point \( x \)
\( L_n \) is the outgoing (i.e., reflected) radiance at point \( x \) in direction \( \omega_i \)
Radiance is a specific physical quantity describing the amount of light along a ray
Radiance is constant along a ray
\( L_e \) is the emitted radiance (\( = 0 \) unless point \( x \) is on a light source)
\( R \) is the “ray-tracing function”. It describes what point is visible from \( x \) in direction \( \omega_i \)
### Rendering Equation

**Equation guiding global illumination:**
\[ L_e(x, \omega_e) = L_r(x, \omega_e) + \int p(x, \omega_i, \omega_e) L_e(x, \omega_i) d\omega_i \]

**Note:**
- The equation is an integral equation.
- This equation cannot be solved directly.
  - Ray-tracing function is complicated!
  - Similar to the problem we had computing illumination from area light sources!

### Ray Casting
- Cast a ray from the eye through each pixel.
- The following few slides are from Fred Durand (MIT)

### Ray Tracing
- Cast a ray from the eye through each pixel.
- Trace secondary rays (light, reflection, refraction).

### Monte Carlo Ray Tracing
- Cast a ray from the eye through each pixel.
- Cast random rays from the visible point.
  - Accumulate radiance contribution.

### Monte Carlo
- Cast a ray from the eye through each pixel.
- Cast random rays from the visible point.
  - Recurse.
Monte Carlo
- Systematically sample primary light

Monte Carlo Path Tracing
**In practice:**
- Do not branch at every intersection point
  - This would have exponential complexity in the ray depth!
- Instead:
  - Shoot some number of primary rays through the pixel (10s-1000s, depending on scene)
  - For each pixel and each intersection point, make a single, random decision in which direction to go next

Monte Carlo Path Tracing
- Trace only one secondary ray per recursion
- But send many primary rays per pixel
  - (performs antialiasing as well)

How to Sample?
**Simple sampling strategy:**
- At every point, choose between all possible reflection directions with equal probability
- This will produce very high variance/noise if the materials are specular or glossy
- Lots of rays are required to reduce noise!

**Better strategy: importance sampling**
- Focus rays in areas where most of the reflected light contribution will be found
- For example: if the surface is a mirror, then only light from the mirror direction will contribute!
- Glossy materials: prefer rays near the mirror direction

How to Sample?
Images by Veach & Guibas

- Naive sampling strategy
- Multiple importance sampling

How to Sample?
**Sampling strategies are still an active research area!**
- Recent years have seen drastic advances in performance
- Lots of excellent sampling strategies have been developed in statistics and machine learning
  - Many are useful for graphics
How to Sample?

**Objective:**
- Compute light transport in scenes using stochastic ray tracing
  - Monte Carlo, Sequential Monte Carlo
  - Metropolis

[Burke, Ghosh, Heldrich '05]
[Ghosh, Heldrich '06]

How to Sample?

- E.g. importance sampling (left) vs. Sequential Monte Carlo (right)

More on Global Illumination

**This was a (very) quick overview**
- More details in CPSC 514 (Computer Graphics: Rendering)
- Not offered this year, but in 20011/12