



# Global Illumination

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## Course News

### ***Assignment 3 (project)***

- Due Friday!!
- Demos in labs starting this Friday
- Demos are MANDATORY(!)

### ***Reading***

- Chapter 10 (ray tracing), except 10.8-10.10
- Chapter 14 (global illumination)

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## Area Light Sources

### So far:

- All lights were either point-shaped or directional
  - Both for ray-tracing and the rendering pipeline
- Thus, at every point, we only need to compute lighting formula and shadowing for **ONE** light direction

### In reality:

- All lights have a finite area
- Instead of just dealing with one direction, we now have to **integrate** over all directions that go to the light source

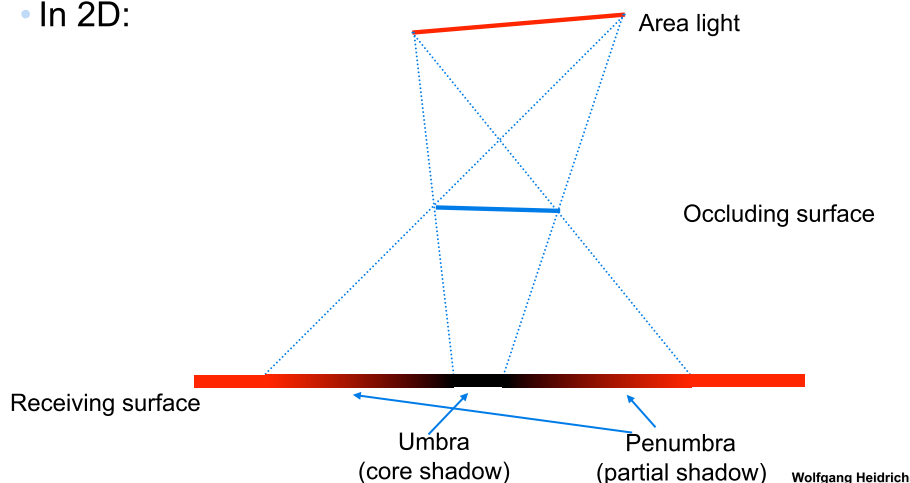
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## Area Light Sources

### Area lights produce soft shadows:

- In 2D:



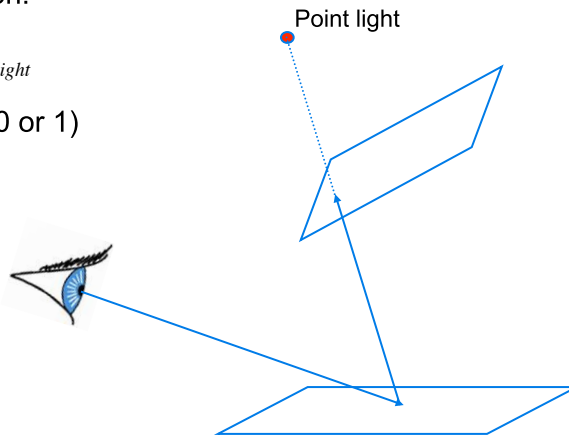
## Area Light Sources

### Point lights:

- Only one light direction:

$$I_{reflected} = \rho \cdot V \cdot I_{light}$$

- V is visibility of light (0 or 1)
- $\rho$  is lighting model (e.g. diffuse or Phong)



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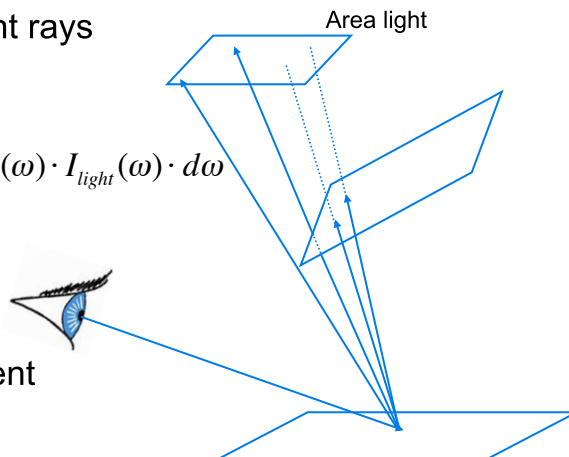
## Area Light Sources

### Area Lights:

- Infinitely many light rays
- Need to integrate over all of them:

$$I_{reflected} = \int_{\text{light directions}} \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

- Lighting model visibility and light intensity can now be different for every ray!



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## Integrating over Light Source

### Rewrite the integration

- Instead of integrating over directions

$$I_{\text{reflected}} = \int_{\text{light directions}} \rho(\omega) \cdot V(\omega) \cdot I_{\text{light}}(\omega) \cdot d\omega$$

we can integrate over points on the light source

$$I_{\text{reflected}}(q) = \int_{s,t} \frac{\rho(p-q) \cdot V(p-q)}{|p-q|^2} \cdot I_{\text{light}}(p) \cdot ds \cdot dt$$

where q: point on reflecting surface, p= F(s,t) is a point on the area light

- We are integrating over p
- Denominator: quadratic falloff!

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## Integration

### Problem:

- Except for the simplest of scenes, either integral is **not solvable analytically!**
- This is mostly due to the visibility term, which could be arbitrarily complex depending on the scene

### So:

- Use numerical integration
- Effectively: approximate the light with a whole number of point lights

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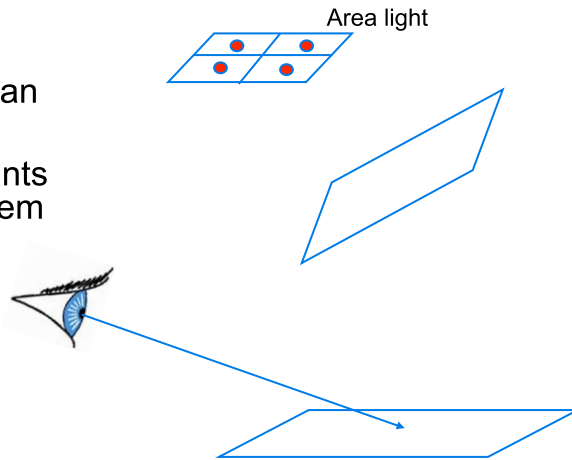




## Numerical Integration

### Regular grid of point lights

- Problem: will see 4 hard shadows rather than as soft shadow
- Need LOTS of points to avoid this problem



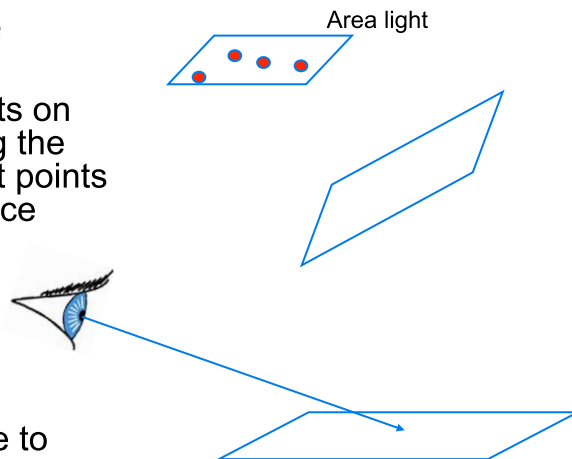
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## Monte Carlo Integration

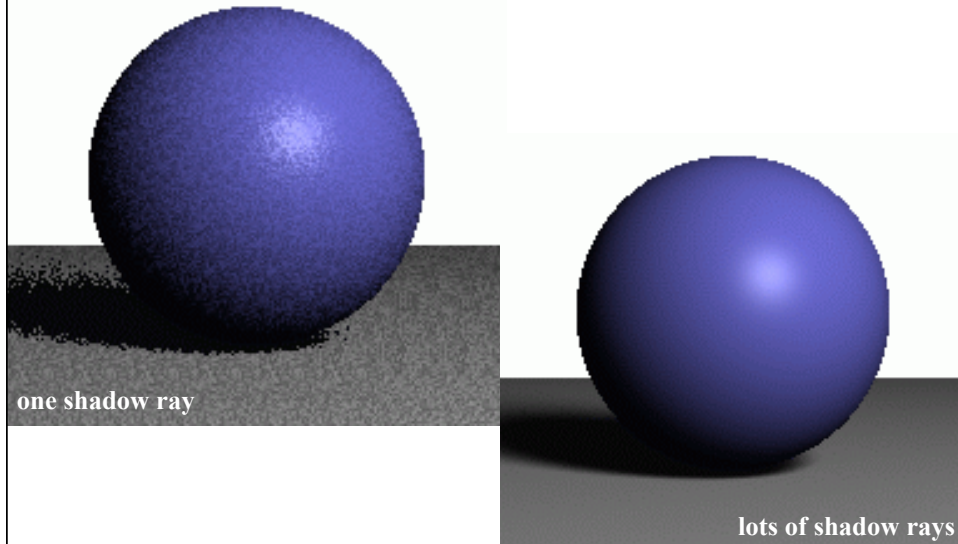
### Better:

- **Randomly** choose the points
- Use different points on light for computing the lighting in different points on reflecting surface
- This produces random noise
- Visually preferable to structured artifacts



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## Monte Carlo Integration



one shadow ray

lots of shadow rays

## Monte Carlo Integration

### Formally:

- Approximate integral with finite sum

$$I_{\text{reflected}}(q) = \int_{s,t} \frac{\rho(p-q) \cdot V(p-q)}{|p-q|^2} \cdot I_{\text{light}}(p) \cdot ds \cdot dt$$

$$\approx \frac{A}{N} \sum_{i=1}^N \frac{\rho(p_i-q) \cdot V(p_i-q)}{|p_i-q|^2} \cdot I_{\text{light}}(p_i)$$

where

- The  $p_i$  are randomly chosen on the light source
  - With equal probability!
- $A$  is the total area of the light
- $N$  is the number of samples (rays)

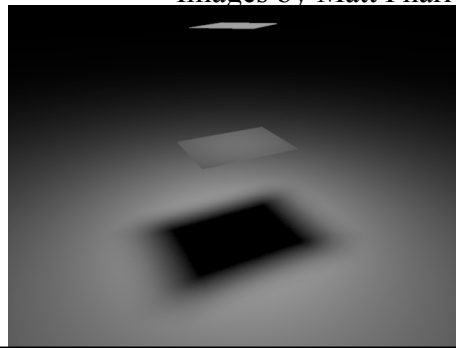
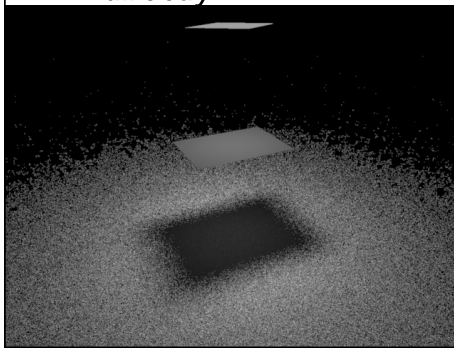


## Sampling

### **Sample directions vs. sample light source**

- Most directions do not correspond to points on the light source
  - Thus, variance will be higher than sampling light directly

Images by Matt Pharr



## Monte Carlo Integration

### **Note:**

- This approach of approximating lighting integrals with sums over randomly chosen points is much more flexible than this!
- In particular, it can be used for global illumination
  - Light bouncing off multiple surfaces before hitting the eye

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## Global Illumination

### **So far:**

- Have considered only light directly coming from the light sources
  - *As well as mirror reflections, refraction*

### **In reality:**

- Light bouncing off diffuse and/or glossy surfaces also illuminates other surfaces
  - *This is called global illumination*

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## Direct Illumination

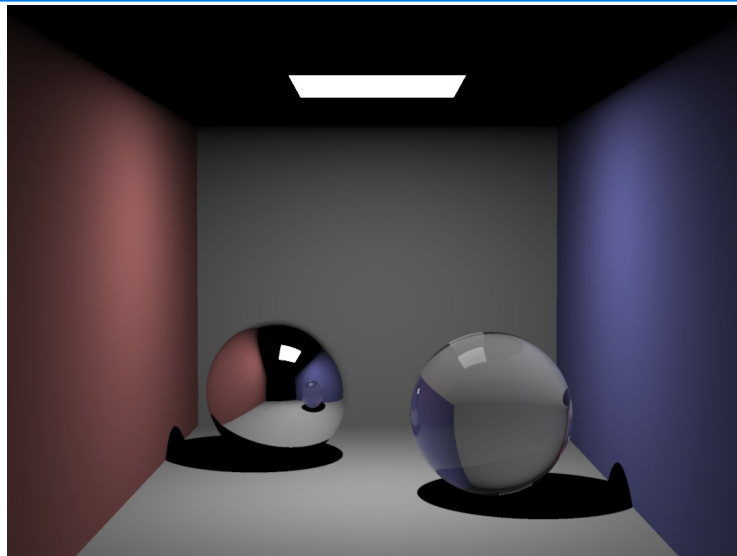
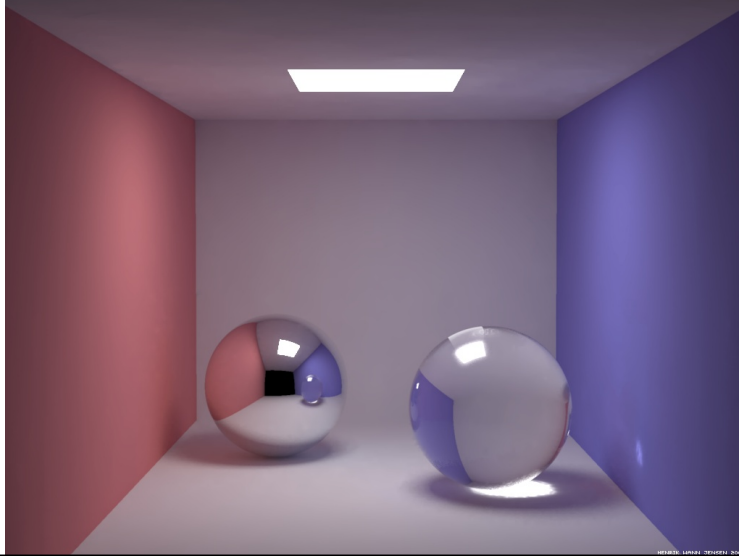


Image by  
Henrik Wann Jensen



## Global Illumination

Image by  
Henrik Wann Jensen



## Rendering Equation

### Equation guiding global illumination:

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega} \rho(x, \omega_i, \omega_o) L_i(\omega_i) d\omega_i$$

$$= L_e(x, \omega_o) + \int_{\Omega} \rho(x, \omega_i, \omega_o) L_o(R(x, \omega_i), -\omega_i) d\omega_i$$

### Where

- $\rho$  is the reflectance from  $\omega_i$  to  $\omega_o$  at point  $x$
- $L_o$  is the outgoing (i.e. reflected) **radiance** at point  $x$  in direction  $\omega_i$ 
  - Radiance is a specific physical quantity describing the amount of light along a ray
  - Radiance is constant along a ray
- $L_e$  is the emitted radiance (=0 unless point  $x$  is on a light source)
- $R$  is the “ray-tracing function”. It describes what point is visible from  $x$  in direction  $\omega_i$

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## Rendering Equation

### Equation guiding global illumination:

$$\begin{aligned}L_o(x, \omega_o) &= L_e(x, \omega_o) + \int_{\Omega} \rho(x, \omega_i, \omega_o) L_i(\omega_i) d\omega_i \\ &= L_e(x, \omega_o) + \int_{\Omega} \rho(x, \omega_i, \omega_o) L_o(R(x, \omega_i), -\omega_i) d\omega_i\end{aligned}$$

### Note:

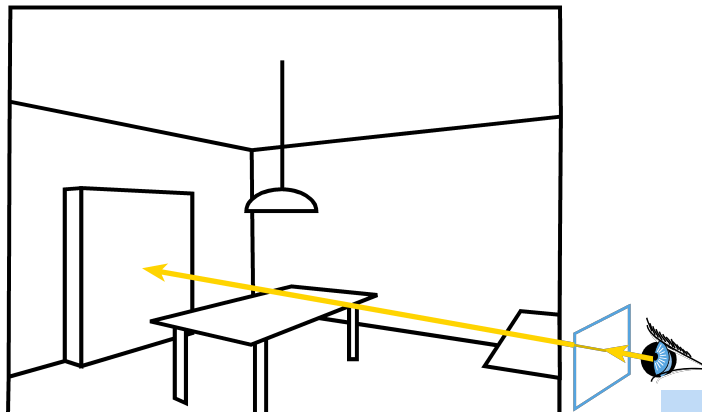
- The rendering equation is an **integral equation**
- This equation cannot be solved directly
  - *Ray-tracing function is complicated!*
  - *Similar to the problem we had computing illumination from area light sources!*

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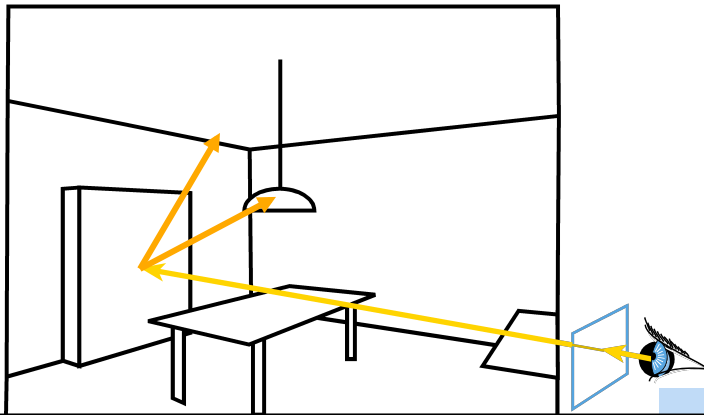
## Ray Casting

- Cast a ray from the eye through each pixel
- The following few slides are from Fred Durand (MIT)



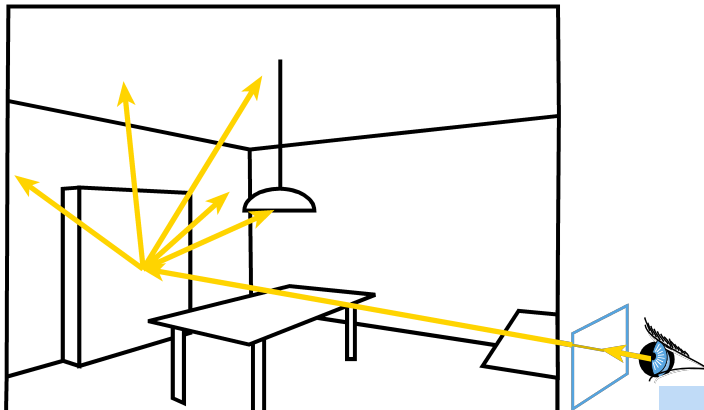
## Ray Tracing

- Cast a ray from the eye through each pixel
- Trace secondary rays (light, reflection, refraction)



## Monte Carlo Ray Tracing

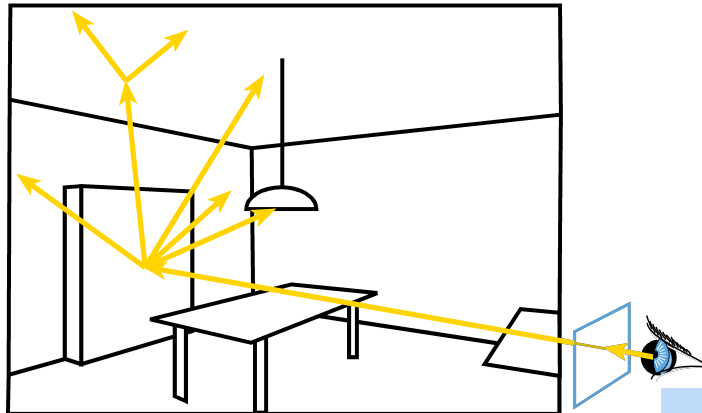
- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
  - Accumulate radiance contribution





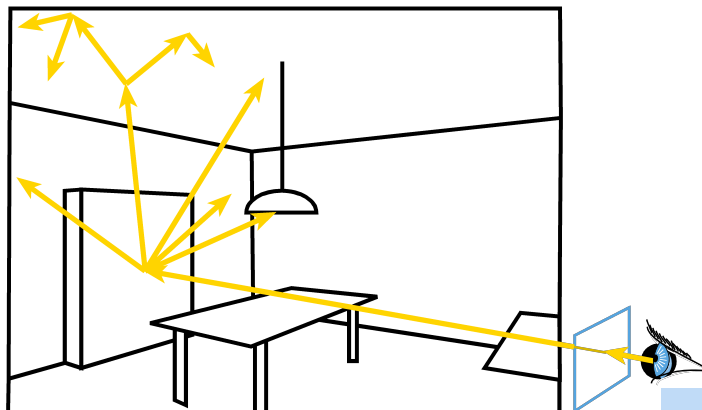
## Monte Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



## Monte Carlo

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse

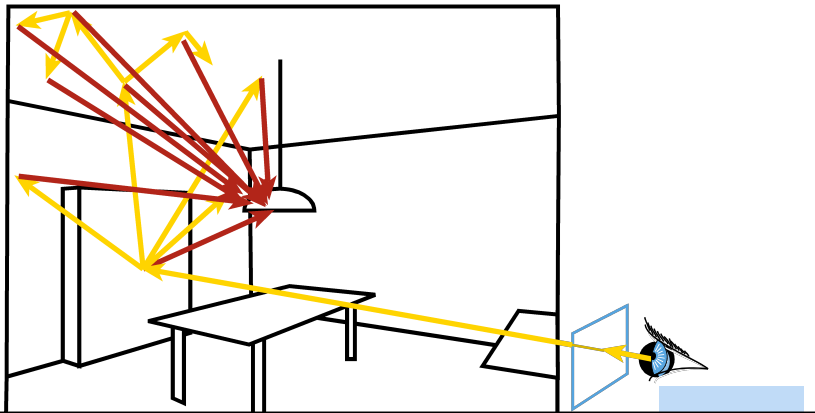






## Monte Carlo

- Systematically sample primary light



## Monte Carlo Path Tracing

### *In practice:*

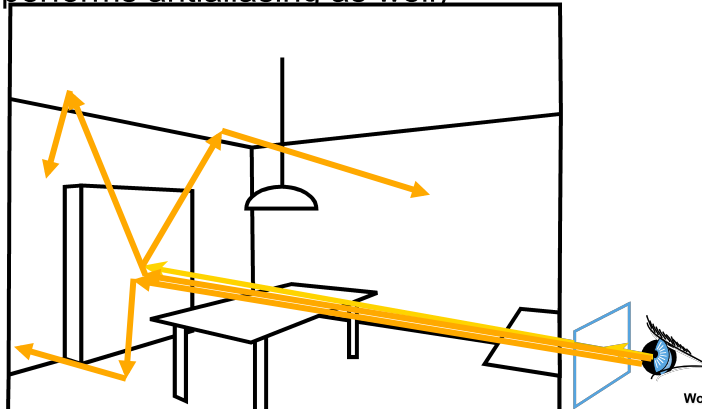
- Do not branch at every intersection point
  - *This would have exponential complexity in the ray depth!*
- Instead:
  - *Shoot some number of primary rays through the pixel (10s-1000s, depending on scene!)*
  - *For each pixel and each intersection point, make a **single, random** decision in which direction to go next*

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## Monte Carlo Path Tracing

- Trace only one secondary ray per recursion
- But send many primary rays per pixel
- (performs antialiasing as well)



## How to Sample?

### **Simple sampling strategy:**

- At every point, choose between all possible reflection directions with equal probability
- This will produce very high variance/noise if the materials are specular or glossy
- Lots of rays are required to reduce noise!

### **Better strategy: importance sampling**

- Focus rays in areas where most of the reflected light contribution will be found
- For example: if the surface is a mirror, then only light from the mirror direction will contribute!
- Glossy materials: prefer rays near the mirror direction

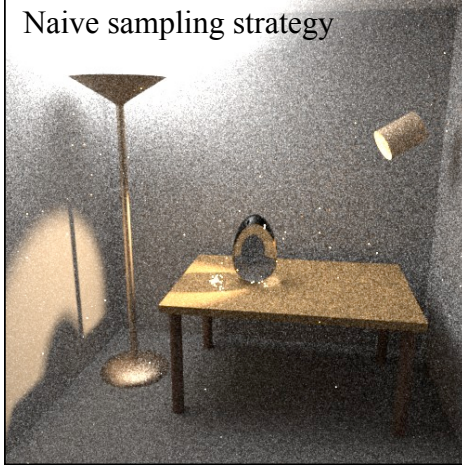
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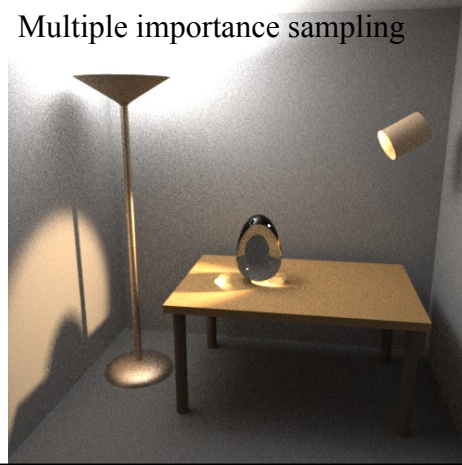
## How to Sample?

- Images by Veach & Guibas

Naive sampling strategy



Multiple importance sampling



## How to Sample?

### ***Sampling strategies are still an active research area!***

- Recent years have seen drastic advances in performance
- Lots of excellent sampling strategies have been developed in statistics and machine learning
  - *Many are useful for graphics*



## How to Sample?

### Objective:

- Compute light transport in scenes using stochastic ray tracing
  - Monte Carlo, Sequential Monte Carlo
  - Metropolis

[Burke, Ghosh, Heidrich '05]  
[Ghosh, Heidrich '06]  
[Ghosh, Doucet, Heidrich '06]



## How to Sample?

- E.g: importance sampling (left) vs. Sequential Monte Carlo (right)



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## More on Global Illumination

### ***This was a (very) quick overview***

- More details in CPSC 514 (Computer Graphics: Rendering)
- Not offered this year, but in 20011/12