Ray-Tracing

Wolfgang Heidrich

Course News

Assignment 3 (project)

- Due April 1

Reading

- Chapter 10 (ray tracing), except 10.8-10.10
- Chapter 14 (global illumination)

Friday Lecture

- Out of town for program committee meeting
- Anika will continue discussion of ray-tracing

Overview

So far

- Real-time/HW rendering w/ Rendering Pipeline
- Rendering algorithms using the Rendering Pipeline

Now

- Ray-Tracing
  - *Simple algorithm for software rendering*
    - Usually offline (e.g. movies etc.)
    - But: research on making this method real-time
  - *Extremely flexible (new effects can easily be incorporated)*

Ray-Tracing

Basic Algorithm (Whithead):

```
for every pixel $p_i$ {
    Generate ray r from camera position through pixel $p_i$
      $p_i$= background color
    for every object o in scene {
        if( r intersects o && intersection is closer than previously
          found intersections )
            Compute lighting at intersection point, using local
          normal and material properties; store result in $p_i$
    }
}
```

Ray-Tracing

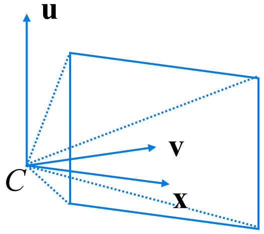
Issues:

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Efficient data structures so we don't have to test intersection with *every* object

Ray-Tracing – Generation of Rays

Camera Coordinate System

- Origin: C (camera position)
- Viewing direction: $\mathbf{v}$
- Up vector: u
- x direction: $\mathbf{x}= \mathbf{v}\times\mathbf{u}$



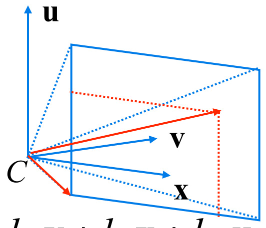
Note:

- Corresponds to viewing transformation in rendering pipeline!
- See gluLookAt…

Ray-Tracing – Generation of Rays

Other parameters:

- Distance of Camera from image plane: d
- Image resolution (in pixels): w, h
- Left, right, top, bottom boundaries in image plane: l, r, t, b



Then:

- Lower left corner of image: $O = C + d \cdot \mathbf{v} + l \cdot \mathbf{x} + b \cdot \mathbf{u}$
- Pixel at position i, j ($i=0..w-1, j=0..h-1$):

$$P_{i,j} = O + i \cdot \frac{r-l}{w-1} \cdot \mathbf{x} - j \cdot \frac{t-b}{h-1} \cdot \mathbf{u}$$
$$= O + i \cdot \Delta x \cdot \mathbf{x} - j \cdot \Delta y \cdot \mathbf{y}$$

Ray-Tracing – Generation of Rays

Ray in 3D Space:

$$\mathrm{R}_{i,j}(t) = C + t \cdot (P_{i,j} - C) = C + t \cdot \mathbf{v}_{i,j}$$

where $t = 0 \ldots \infty$

Ray-Tracing

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Ray Intersections

Task:

- Given an object o, find ray parameter t, such that $\mathbf{R}_{i,j}(t)$ is a point on the object
  - *Such a value for t may not exist*
- Intersection test depends on geometric primitive

Ray Intersections

Spheres at origin:

- Implicit function:

$$S(x,y,z): x^2 + y^2 + z^2 = r^2$$

- Ray equation:

$$\mathrm{R}_{i,j}(t) = C + t \cdot \mathbf{v}_{i,j} = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix} + t \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} c_x + t \cdot v_x \\ c_y + t \cdot v_y \\ c_z + t \cdot v_z \end{pmatrix}$$

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Ray Intersections

To determine intersection:

- Insert ray $\mathbf{R}_{i,j}(t)$ into $S(x,y,z)$:

$$(c_x + t \cdot v_x)^2 + (c_y + t \cdot v_y)^2 + (c_z + t \cdot v_z)^2 = r^2$$

- Solve for t (find roots)
  - *Simple quadratic equation*

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Ray Intersections

Other Primitives:

- Implicit functions:
  - *Spheres at arbitrary positions*
    - Same thing
  - *Conic sections (hyperboloids, ellipsoids, paraboloids, cones, cylinders)*
    - Same thing (all are quadratic functions!)
  - *Higher order functions (e.g. tori and other quartic functions)*
    - In principle the same
    - But root-finding difficult
    - Net to resolve to numerical methods

Ray Intersections

Other Primitives (cont)

- Polygons:
  - *First intersect ray with plane*
    - linear implicit function
  - *Then test whether point is inside or outside of polygon (2D test)*
  - *For convex polygons*
    - Suffices to test whether point in on the right side of every boundary edge
    - Similar to computation of outcodes in line clipping

Ray-Tracing

Issues:

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- Geometric transformations
- Lighting and shading
- Efficient data structures so we don't have to test intersection with *every* object

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Ray-Tracing – Geometric Transformations

Geometric Transformations:

- Similar goal as in rendering pipeline:
  - *Modeling scenes convenient using different coordinate systems for individual objects*
- Problem:
  - *Not all object representations are easy to transform*
    - This problem is fixed in rendering pipeline by restriction to polygons (affine invariance!)

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Ray-Tracing – Geometric Transformations

Geometric Transformations:

- Similar goal as in rendering pipeline:
  - *Modeling scenes convenient using different coordinate systems for individual objects*
- Problem:
  - *Not all object representations are easy to transform*
    - This problem is fixed in rendering pipeline by restriction to polygons (affine invariance!)
  - *Ray-Tracing has different solution:*
    - The ray itself is always affine invariant!
    - Thus: transform ray into object coordinates!

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Ray-Tracing – Geometric Transformations

Ray Transformation:

- For intersection test, it is only important that ray is in same coordinate system as object representation
- Transform all rays into object coordinates
  - *Transform camera point and ray direction by inverse of model/view matrix*
- Shading has to be done in world coordinates (where light sources are given)
  - *Transform object space intersection point to world coordinates*
  - *Thus have to keep both world and object-space ray*

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Ray-Tracing

Issues:

- Generation of rays
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Ray-Tracing
Lighting and Shading

Local Effects:

- Local Lighting
  - *Any reflection model possible*
  - *Have to talk about light sources, normals…*
- Texture mapping
  - *Color textures*
  - *Bump maps*
  - *Environment maps*
  - *Shadow maps*

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Ray-Tracing
Local Lighting

Light sources:

- For the moment: point and directional lights
- Later: are light sources
- More complex lights are possible
  - *Area lights*
  - *Global illumination*
    - Other objects in the scene reflect light
    - Everything is a light source!
    - Talk about this on Monday

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Ray-Tracing
Local Lighting

Local surface information (normal…)

- For implicit surfaces $F(x,y,z)$=0: normal $\mathbf{n}(x,y,z)$ can be easily computed at every intersection point using the gradient

$$\mathbf{n}(x,y,z)=\begin{pmatrix}\partial F(x,y,z)/\partial x\\ \partial F(x,y,z)/\partial y\\ \partial F(x,y,z)/\partial z\end{pmatrix}$$

- Example: $F(x,y,z)=x^2+y^2+z^2-r^2$

$$\mathbf{n}(x,y,z)=\begin{pmatrix}2x\\ 2y\\ 2z\end{pmatrix}$$

Needs to be normalized!

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Ray-Tracing
Local Lighting

Local surface information

- Alternatively: can interpolate per-vertex information for triangles/meshes as in rendering pipeline
  - *Phong shading!*
  - *Same as discussed for rendering pipeline*
- Difference to rendering pipeline:
  - *Interpolation cannot be done incrementally*
  - *Have to compute Barycentric coordinates for every intersection point (e.g plane equation for triangles)*

Ray-Tracing
Texture Mapping

Approach:

- Works in principle like in rendering pipeline
  - *Given s, t parameter values, perform texture lookup*
  - *Magnification, minification just as discussed*
- Problem: how to get s, t
  - *Implicit surfaces often don't have parameterization*
  - *For special cases (spheres, other conic sections), can use parametric representation*
  - *Triangles/meshes: use interpolation from vertices*

Ray-Tracing
Lighting and Shading

Global Effects

- Shadows
- Reflections/refractions

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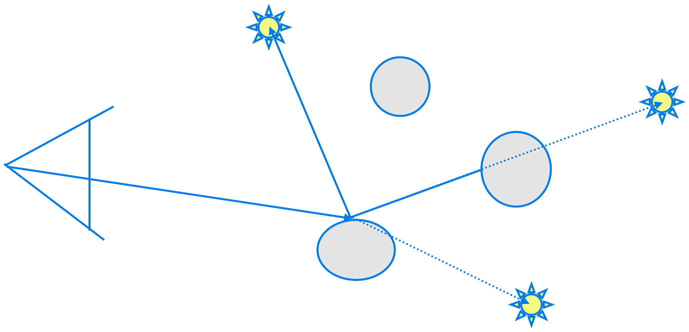
Ray-Tracing
Shadows

Approach:

- To test whether point is in shadow, send out *shadow rays* to all light sources
  - *If ray hits another object, the point lies in shadow*



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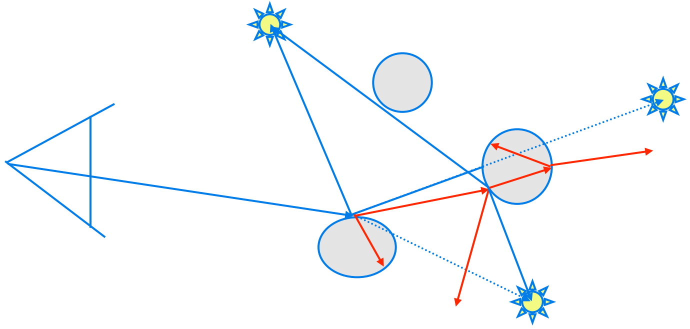
Ray-Tracing Reflections/Refractions

Approach:

- Send rays out in reflected and refracted direction to gather incoming light
- That light is multiplied by local surface color and Fresnel term, and added to result of local shading



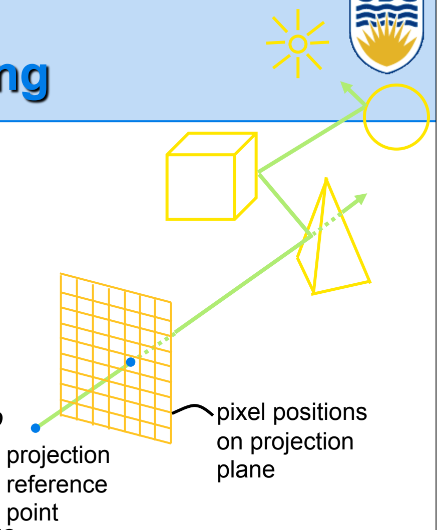
Recursive Ray Tracing

Ray tracing can handle

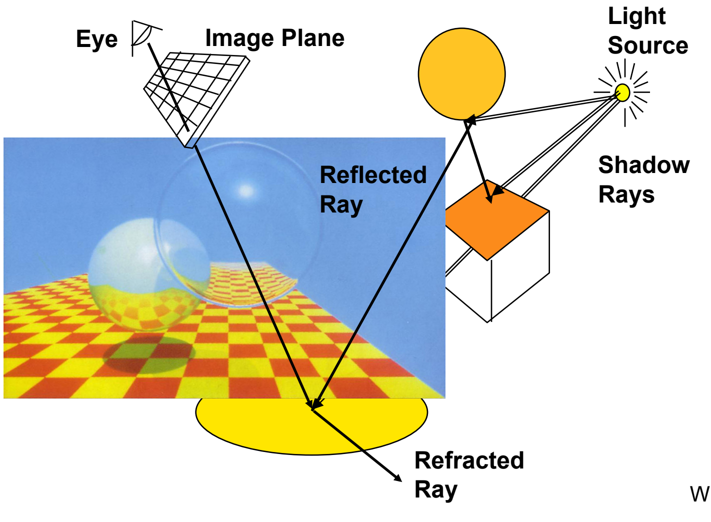
- Reflection (chrome)
- Refraction (glass)
- Shadows

Spawn secondary rays

- Reflection, refraction
  - *If another object is hit, recurse to find its color*
- Shadow
  - *Cast ray from intersection point to light source, check if intersects another object*



Recursive Ray-Tracing



Whitted, 1980

Recursive Ray-Tracing Algorithm

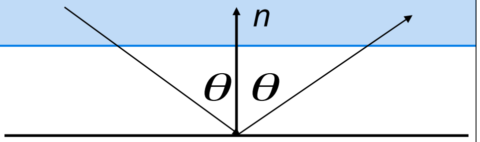
```
RayTrace(r,scene)
obj := FirstIntersection(r,scene)
if (no obj)  return BackgroundColor;
else begin
   if ( Reflect(obj) ) then
      reflect_color := RayTrace(ReflectRay(r,obj));
   else
      reflect_color := Black;
   if ( Transparent(obj) ) then
      refract_color := RayTrace(RefractRay(r,obj));
   else
      refract_color := Black;
   return Shade(reflect_color,refract_color,obj);
end;
```

Algorithm Termination Criteria

Termination criteria

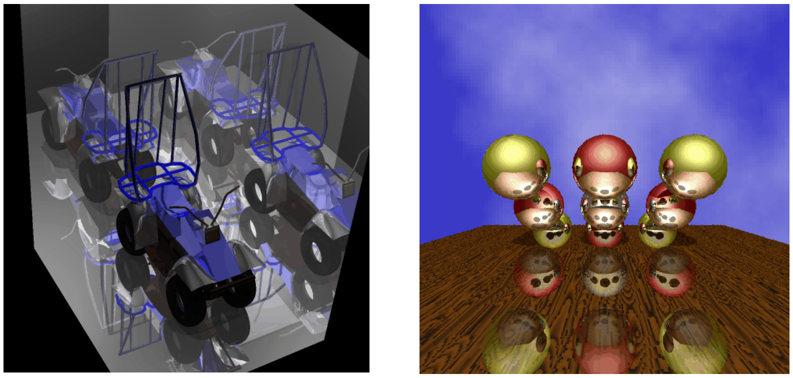
- No intersection
- Reach maximal depth
  - *Number of bounces*
- Contribution of secondary ray attenuated below threshold
  - *Each reflection/refraction attenuates ray*

Reflection

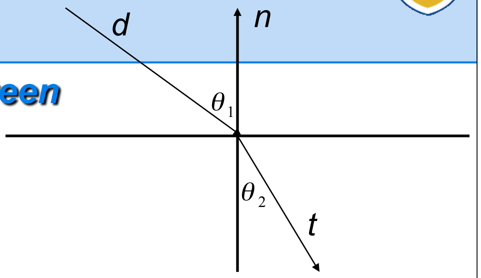


Mirror effects

- Perfect specular reflection



Refraction

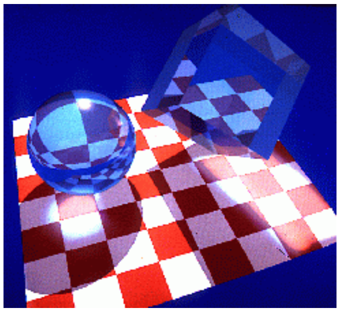


Happens at interface between transparent object and surrounding medium

- E.g. glass/air boundary

Snell's Law

- $$c_1 \sin\theta_1 = c_2 \sin\theta_2$$
- Light ray bends based on refractive indices c_1, c_2



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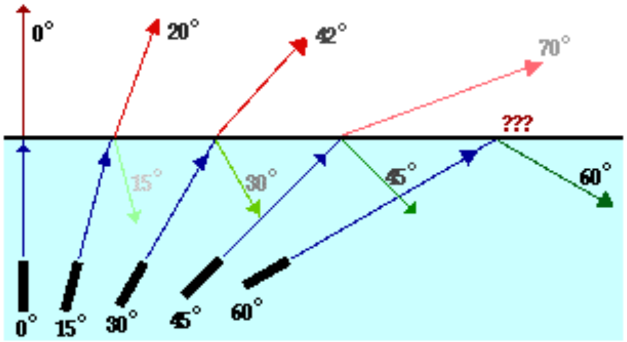
Total Internal Reflection

As the angle of incidence increases from 0 to greater angles ...



...the refracted ray becomes dimmer (there is less refraction)
...the reflected ray becomes brighter (there is more reflection)
...the angle of refraction approaches 90 degrees until finally
a refracted ray can no longer be seen.

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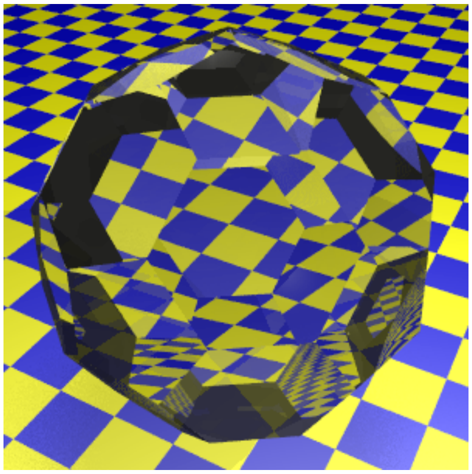
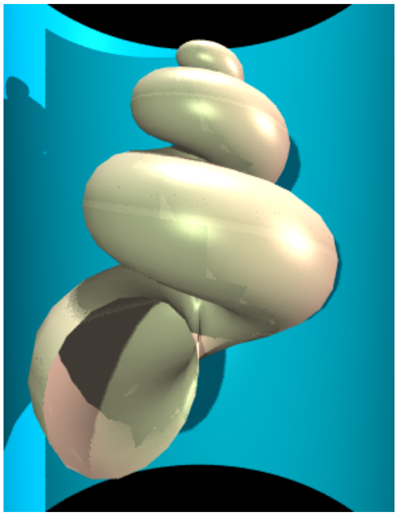
Ray-Tracing Example Images





Ray-Tracing Terminology

Terminology:

- Primary ray: ray starting at camera
- Shadow ray
- Reflected/refracted ray
- Ray tree: all rays directly or indirectly spawned off by a single primary ray

Note:

- Need to limit maximum depth of ray tree to ensure termination of ray-tracing process!

Ray-Tracing

Issues:

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Efficient data structures so we don't have to test intersection with *every* object

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Ray Tracing

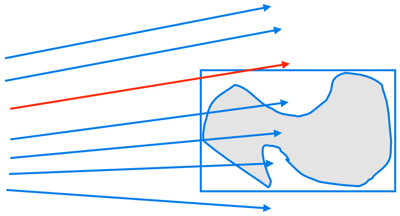
Data Structures

- Goal: reduce number of intersection tests per ray
- Lots of different approaches:
  - *(Hierarchical) bounding volumes*
  - *Hierarchical space subdivision*
    - Oct-tree, k-D tree, BSP tree

Bounding Volumes

Idea:

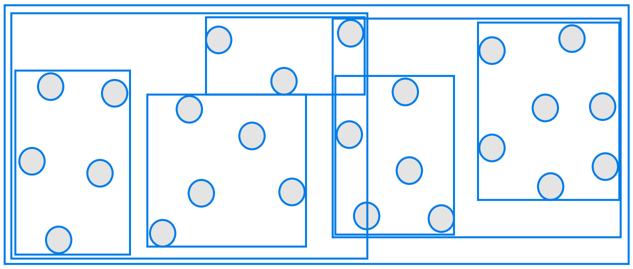
- Rather than testing every ray against a potentially very complex object (e.g. triangle mesh), do a quick *conservative* test first which eliminates most rays
  - *Surround complex object by simple, easy to test geometry (typically sphere or axis-aligned box)*
    - Want to make bounding volume as tight as possible!



Hierarchical Bounding Volumes

Extension of previous idea:

- Use bounding volumes for groups of objects



Spatial Subdivision Data Structures

Bounding Volumes:

- Find simple object completely enclosing complicated objects
  - *Boxes, spheres*
- Hierarchically combine into larger bounding volumes

Spatial subdivision data structure:

- Partition the whole space into cells
  - *Grids, oct-trees, (BSP trees)*
- Simplifies and accelerates traversal
- Performance less dependent on order in which objects are inserted

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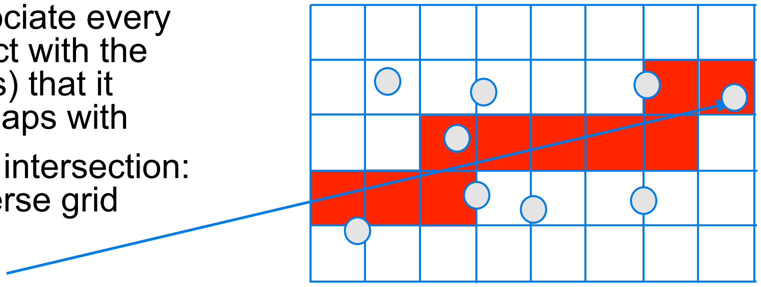
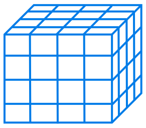
Regular Grid

Subdivide space into rectangular grid:

- Associate every object with the cell(s) that it overlaps with
- Find intersection: traverse grid



In 3D: regular grid of cubes (voxels):



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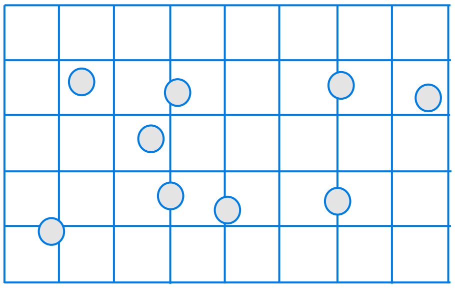
Creating a Regular Grid

Steps:

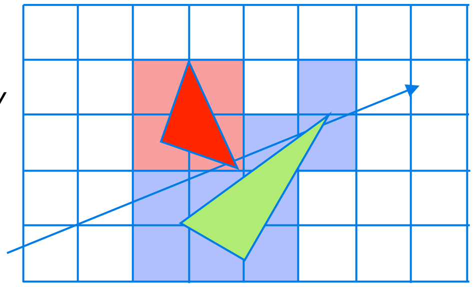
- Find bounding box of scene
- Choose grid resolution in x, y, z
- Insert objects
- Objects that overlap multiple cells get referenced by all cells they overlap



Grid Traversal

Traversal:

- Start at ray origin
- While no intersection found
  - *Go to next grid cell along ray*
  - *Compute intersection of ray with all objects in the cell*
  - *Determine closest such intersection*
  - ***Check if that intersection is inside the cell***
  - *If so, terminate search*



Traversal

Note:

- This algorithm calls for computing the intersection points multiple times (once per grid cell)
- In practice: store intersections for a (ray, object) pair once computed, reuse for future cells

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Regular Grid Discussion

Advantages?

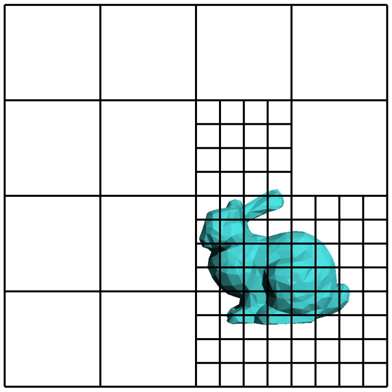
- Easy to construct
- Easy to traverse

Disadvantages?

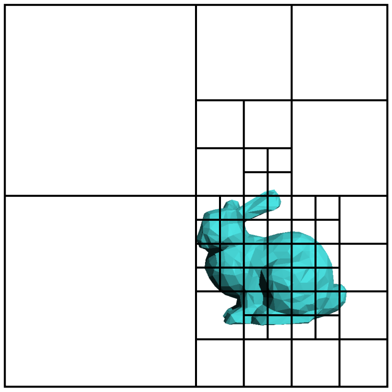
- May be only sparsely filled
- Geometry may still be clumped

Adaptive Grids

- Subdivide until each cell contains no more than *n* elements, or maximum depth *d* is reached



Nested Grids

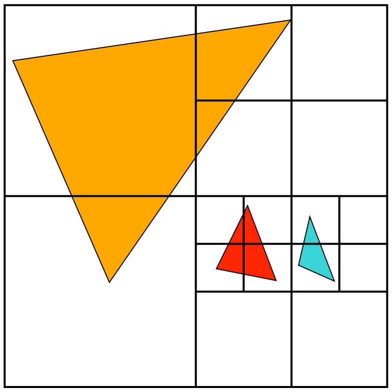


Octree/(Quadtree)

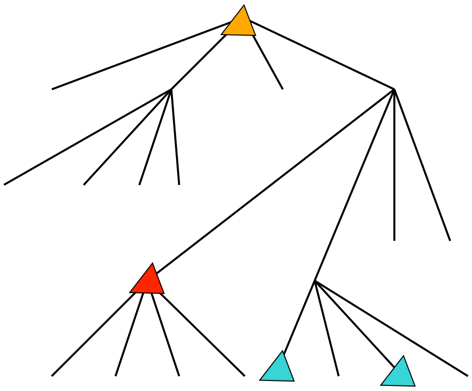
- This slide and the next are curtsey of Fredo Durand at MIT

Primitives in an Adaptive Grid

- Can live at intermediate levels, or be pushed to lowest level of grid



Octree/(Quadtree)



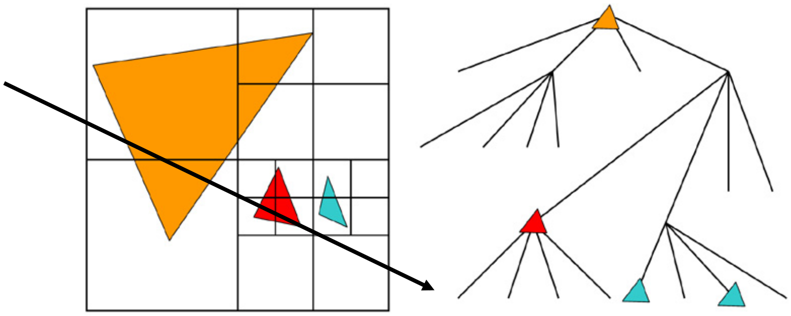
Adaptive Grid Discussion

Advantages

- Grid complexity matches geometric density

Disadvantages

- More expensive to traverse than regular grid



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Soft Shadows in Ray Tracing

CPSC 314

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Area Light Sources

So far:

- All lights were either point-shaped or directional
  - *Both for ray-tracing and the rendering pipeline*
- Thus, at every point, we only need to compute lighting formula and shadowing for ONE light direction

In reality:

- All lights have a finite area
- Instead of just dealing with one direction, we now have to integrate over all directions that go to the light source

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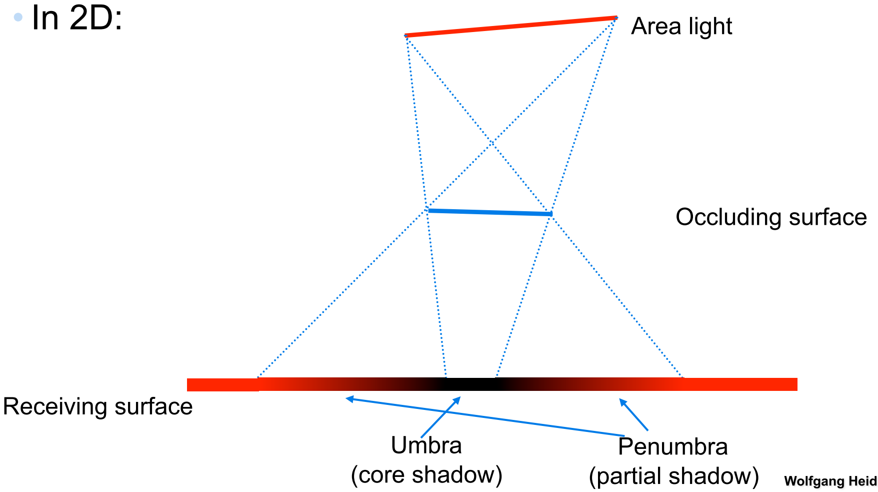
Area Light Sources

Area lights produce soft shadows:

- In 2D:



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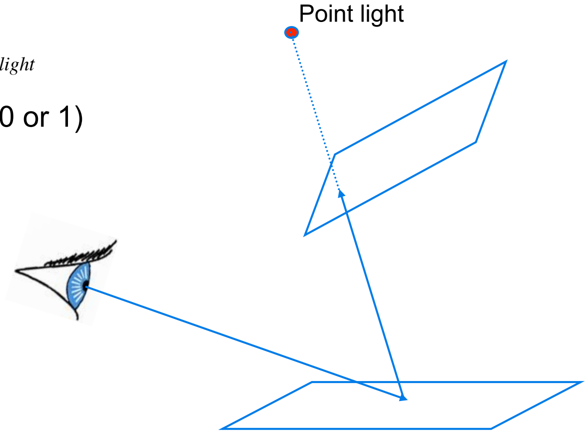
Area Light Sources

Point lights:

- Only one light direction:

$$I_{reflected} = \rho \cdot V \cdot I_{light}$$

- V is visibility of light (0 or 1)
- ρ is lighting model (e.g. diffuse or Phong)



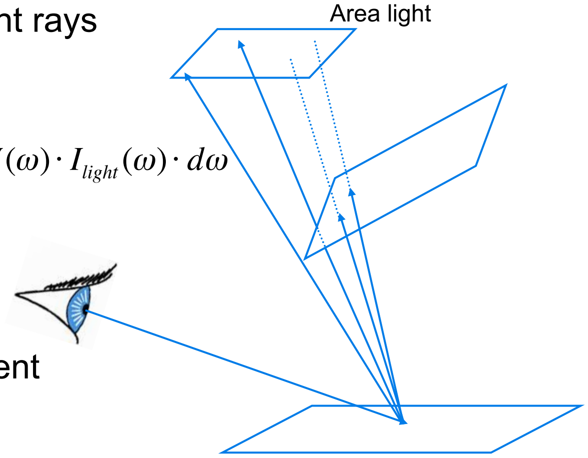
Are Light Sources

Area Lights:

- Infinitely many light rays
- Need to integrate over all of them:

$$I_{reflected} = \int_{\substack{light \\ directions}} \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

- Lighting model visibility and light intensity can now be different for every ray!



Integrating over Light Source

Rewrite the integration

- Instead of integrating over directions

$$I_{reflected} = \int_{\substack{light\\directions}} \rho(\omega)\cdot V(\omega)\cdot I_{light}(\omega)\cdot d\omega$$

we can integrate over points on the light source

$$I_{reflected}(q) = \int_{s,t} \frac{\rho(p-q)\cdot V(p-q)}{|p-q|^2}\cdot I_{light}(p)\cdot ds\cdot dt$$

where q: point on reflecting surface, p= F(s,t) is a point on the area light

  - *We are integrating over p*
  - *Denominator: quadratic falloff!*

Integration

Problem:

- Except for the simplest of scenes, either integral is **not solvable analytically**!
- This is mostly due to the visibility term, which could be arbitrarily complex depending on the scene

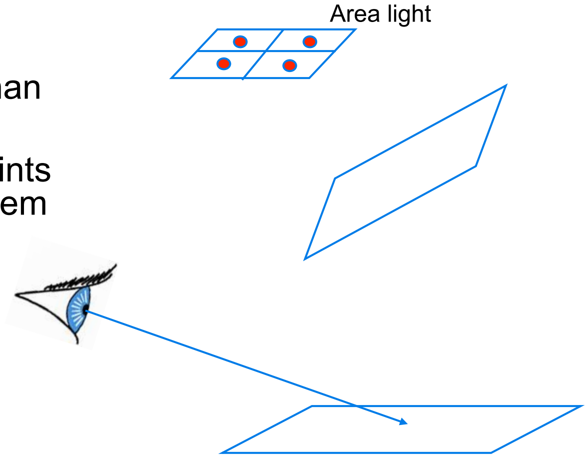
So:

- Use numerical integration
- Effectively: approximate the light with a whole number of point lights

Numerical Integration

Regular grid of point lights

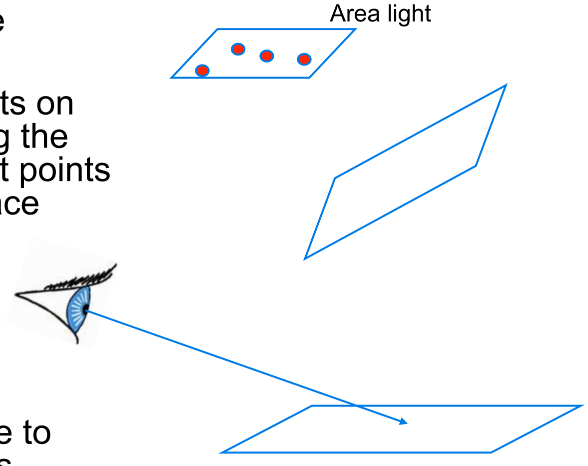
- Problem:
  will see 4 hard
  shadows rather than
  as soft shadow
- Need LOTS of points
  to avoid this problem



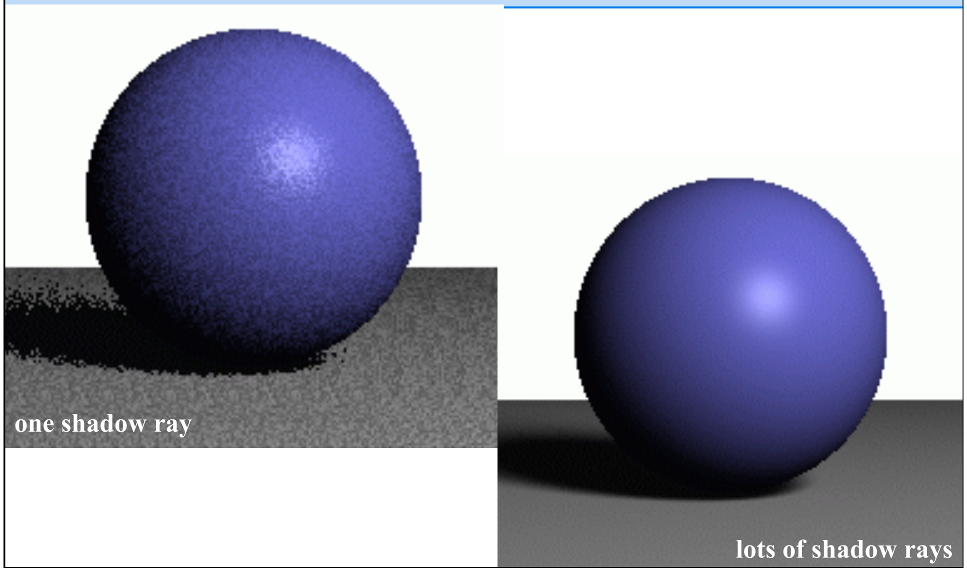
Monte Carlo Integration

Better:

- Randomly choose
  the points
- Use different points on
  light for computing the
  lighting in different points
  on reflecting surface
- This produces
  random noise
- Visually preferable to
  structured artifacts



Monte Carlo Integration



Monte Carlo Integration

Formally:

- Approximate integral with finite sum

$$I_{reflected}(q) = \int_{s,t} \frac{\rho(p-q)\cdot V(p-q)}{|p-q|^2}\cdot I_{light}(p)\cdot ds\cdot dt$$

$$\approx \frac{A}{N}\sum_{i=1}^{N} \frac{\rho(p_i-q)\cdot V(p_i-q)}{|p_i-q|^2}\cdot I_{light}(p_i)$$

where

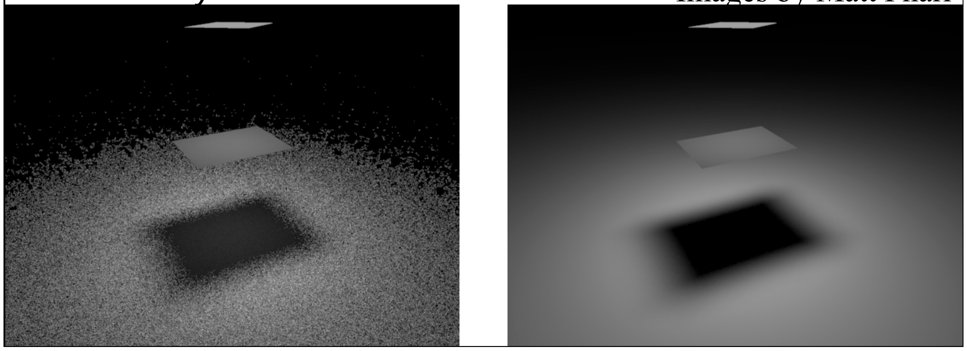
- *The p_i are randomly chosen on the light source*
  - With equal probability!
- *A is the total area of the light*
- *N is the number of samples (rays)*

Sampling

Sample directions vs. sample light source

- Most directions do not correspond to points on the light source
  - *Thus, variance will be higher than sampling light directly*

Images by Matt Pharr



Monte Carlo Integration

Note:

- This approach of approximating lighting integrals with sums over randomly chosen points is much more flexible than this!
- In particular, it can be used for global illumination
  - *Light bouncing off multiple surfaces before hitting the eye*

Coming Up…

Next Week:

- Global illumination

Wolfgang Heidrich