Shadow Volumes
Ray-Tracing

Wolfgang Heidrich

Course News

Assignment 3 (project)

- Due April 1

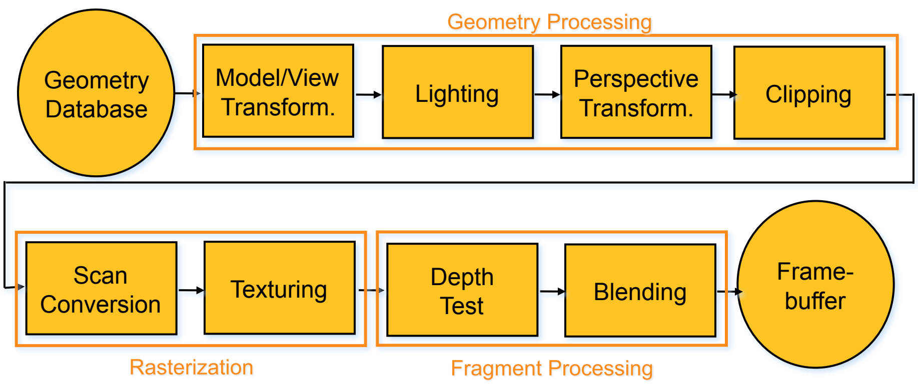
Reading

- Chapter 10 (ray tracing), except 10.8-10.10
- Chapter 14 (global illumination)

Homework 8

- Out today
- Last homework…

The Rendering Pipeline

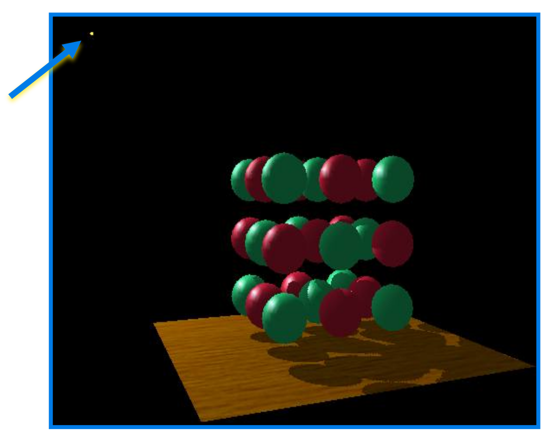


Wolfgang Heidrich

Visualizing the Shadow Mapping Technique (1)

A scene with fairly complex shadows

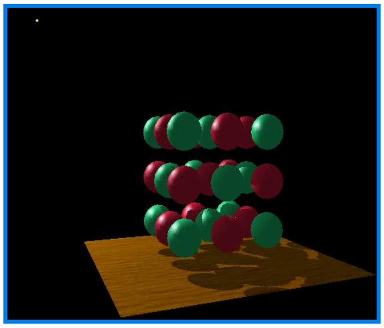
the point light source



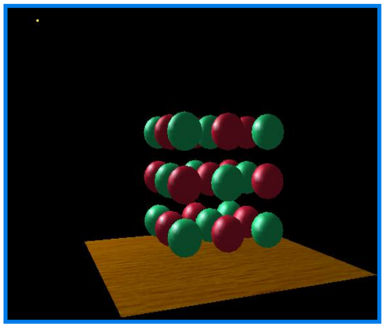
Wolfgang Heidrich

Visualizing the Shadow Mapping Technique (2)

Compare with and without shadows



with shadows

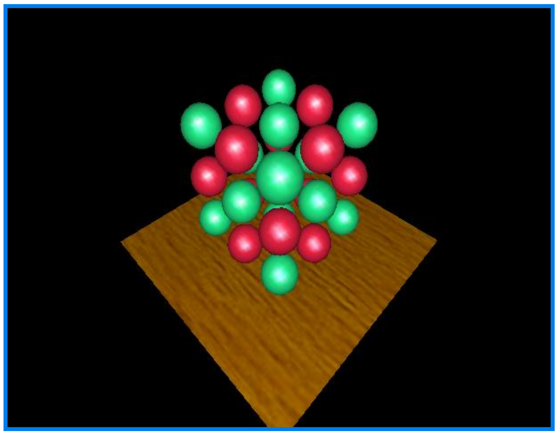


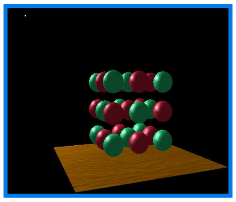
without shadows

Wolfgang Heidrich

Visualizing the Shadow Mapping Technique (3)

The scene from the light's point-of-view



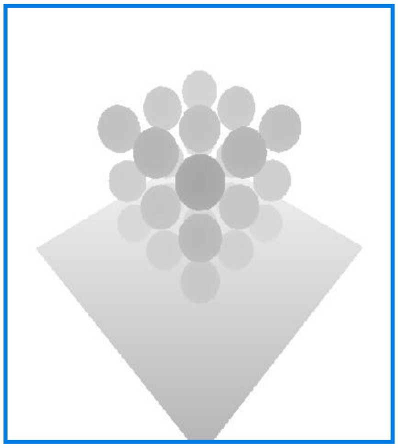


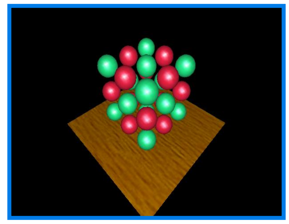
FYI: from the eye's point-of-view again

Wolfgang Heidrich

Visualizing the Shadow Mapping Technique (4)

The depth buffer from the light's point-of-view



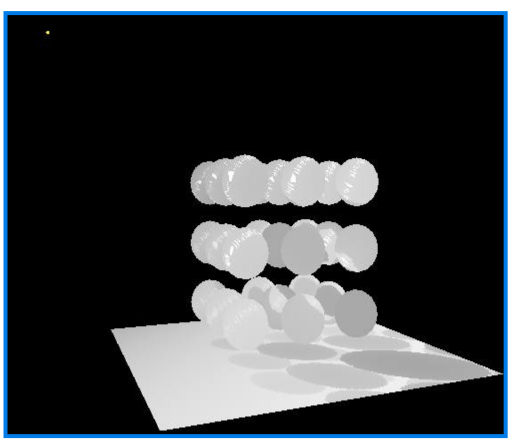


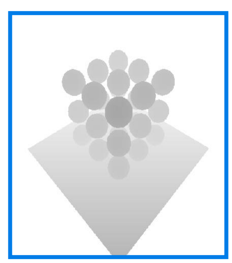
FYI: from the light's point-of-view again

Wolfgang Heidrich

Visualizing the Shadow Mapping Technique (5)

Projecting the depth map onto the eye's view



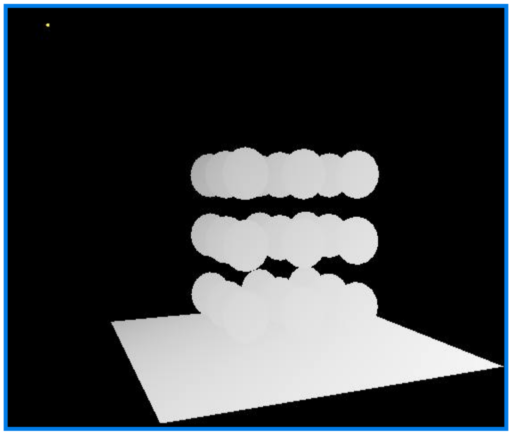


FYI: depth map for light's point-of-view again

Wolfgang Heidrich

Visualizing the Shadow Mapping Technique (6)

Projecting light's planar distance onto eye's view

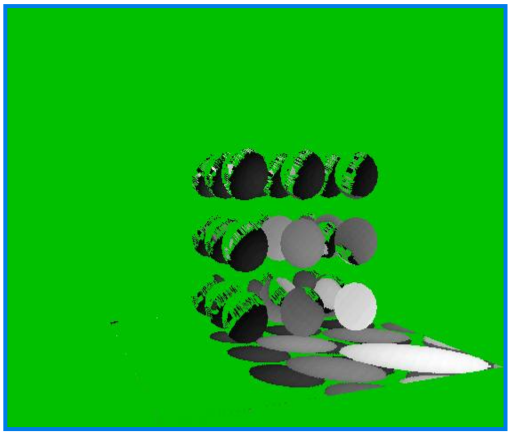


Wolfgang Heidrich

Visualizing the Shadow Mapping Technique (6)

Comparing light distance to light depth map

Green is where the light planar distance and the light depth map are approximately equal



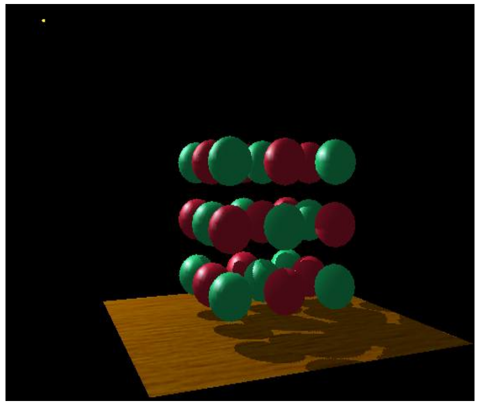
Non-green is where shadows should be

Wolfgang Heidrich

Visualizing the Shadow Mapping Technique (7)

Complete scene with shadows

Notice how specular highlights never appear in shadows



Notice how curved surfaces cast shadows on each other

Wolfgang Heidrich

Shadow Maps

Approach for shadows from point light sources

- Surface point is in shadow if it is not visible from the light source
- Use depth buffer to test visibility:
  - *Render scene from the point light source*
  - *Store resulting depth buffer as texture map*
  - *For every fragment generated while rendering from the camera position, project the fragment into the depth texture taken from the camera, and check if it passes the depth test.*

Wolfgang Heidrich

Shadow Volumes

Use new buffer: stencil buffer

- Just another channel of the framebuffer
- Can count how often a pixel is drawn

Algorithm (1):

- Generate silhouette polygons for all objects
  - *Polygons starting at silhouette edges of object*
  - *Extending away from light source towards infinity*
  - *These can be computed in vertex programs*

Wolfgang Heidrich

Shadow Volumes

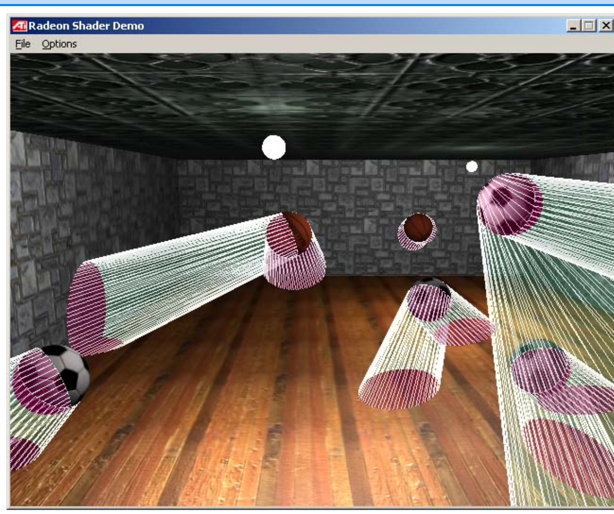


Image by ATI

Wolfgang Heidrich

Shadow Volumes

Algorithm (2):

- Render all original geometry into the depth buffer
  - *I.e. do not draw any colors (or only draw ambient illumination term)*
- Render front-facing silhouette polygons while incrementing the stencil buffer for every rendered fragment
- Render back-facing silhouette polygons while decrementing the stencil buffer for every rendered fragment
- Draw illuminated geometry where the stencil buffer is 0, shadow elsewhere

Wolfgang Heidrich

Shadow Volumes

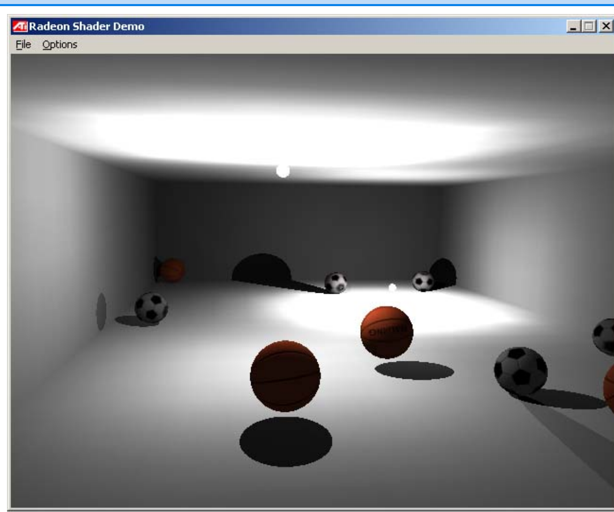


Image by ATI

Wolfgang Heidrich

Shadow Volumes

Discussion:

- Object space method therefore no precision issues
- Lots of large polygons: can be slow
  - *High geometry count*
  - *Large number of pixels rendered*

Ray Tracing

Wolfgang Heidrich

Course Topics for the Rest of the Term

Ray-tracing & Global Illumination

- Today, next week

Parametric Curves/Surfaces

Overview of current research

Wolfgang Heidrich

Overview

So far

- Real-time/HW rendering w/ Rendering Pipeline
- Rendering algorithms using the Rendering Pipeline

Now

- Ray-Tracing
  - *Simple algorithm for software rendering*
    - Usually offline (e.g. movies etc.)
    - But: research on making this method real-time
  - *Extremely flexible (new effects can easily be incorporated)*

Wolfgang Heidrich

Ray-Tracing

Basic Algorithm (Whithead):

for every pixel p_i {

Generate ray r from camera position through pixel p_i

p_i= background color

for every object o in scene {

if(r intersects o && intersection is closer than previously found intersections)

Compute lighting at intersection point, using local normal and material properties; store result in p_i

}

}

Wolfgang Heidrich

Ray-Tracing

Issues:

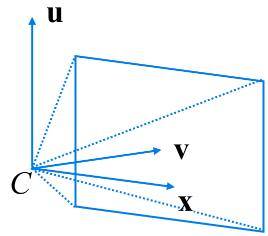
- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Efficient data structures so we don't have to test intersection with *every* object

Wolfgang Heidrich

Ray-Tracing – Generation of Rays

Camera Coordinate System

- Origin: C (camera position)
- Viewing direction: $\mathbf{v}$
- Up vector: $\mathbf{u}$
- x direction: $\mathbf{x} = \mathbf{v} \times \mathbf{u}$



Note:

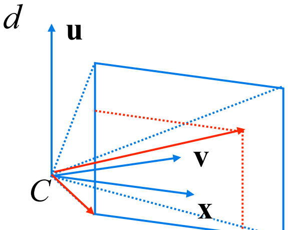
- Corresponds to viewing transformation in rendering pipeline!
- See gluLookAt…

Wolfgang Heidrich

Ray-Tracing – Generation of Rays

Other parameters:

- Distance of Camera from image plane: d
- Image resolution (in pixels): w, h
- Left, right, top, bottom boundaries in image plane: l, r, t, b



Then:

- Lower left corner of image: $O = C + d \cdot \mathbf{v} + l \cdot \mathbf{x} + b \cdot \mathbf{u}$
- Pixel at position i, j ($i=0..w-1, j=0..h-1$):

$$P_{i,j} = O + i \cdot \frac{r-l}{w-1} \cdot \mathbf{x} - j \cdot \frac{t-b}{h-1} \cdot \mathbf{u}$$

$$= O + i \cdot \Delta x \cdot \mathbf{x} - j \cdot \Delta y \cdot \mathbf{y}$$

Wolfgang Heidrich

Ray-Tracing – Generation of Rays

Ray in 3D Space:

$$R_{i,j}(t) = C + t \cdot (P_{i,j} - C) = C + t \cdot \mathbf{v}_{i,j}$$

where $t = 0 \ldots \infty$

Wolfgang Heidrich

Ray-Tracing

Issues:

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Efficient data structures so we don't have to test intersection with *every* object

Wolfgang Heidrich

Ray Intersections

Task:

- Given an object o, find ray parameter t, such that $\mathbf{R}_{i,j}(t)$ is a point on the object
  - *Such a value for t may not exist*
- Intersection test depends on geometric primitive

Wolfgang Heidrich

Ray Intersections

Spheres at origin:

- Implicit function:

$$S(x,y,z): x^2 + y^2 + z^2 = r^2$$

- Ray equation:

$$\mathrm{R}_{i,j}(t) = C + t \cdot \mathbf{v}_{i,j} = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix} + t \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} c_x + t \cdot v_x \\ c_y + t \cdot v_y \\ c_z + t \cdot v_z \end{pmatrix}$$

Wolfgang Heidrich

Ray Intersections

To determine intersection:

- Insert ray $\mathbf{R}_{i,j}(t)$ into $S(x,y,z)$:

$$(c_x + t \cdot v_x)^2 + (c_y + t \cdot v_y)^2 + (c_z + t \cdot v_z)^2 = r^2$$

- Solve for t (find roots)
  - *Simple quadratic equation*

Wolfgang Heidrich

Ray Intersections

Other Primitives:

- Implicit functions:
  - *Spheres at arbitrary positions*
    - Same thing
  - *Conic sections (hyperboloids, ellipsoids, paraboloids, cones, cylinders)*
    - Same thing (all are quadratic functions!)
  - *Higher order functions (e.g. tori and other quartic functions)*
    - In principle the same
    - But root-finding difficult
    - Net to resolve to numerical methods

Wolfgang Heidrich

Ray Intersections

Other Primitives (cont)

- Polygons:
  - *First intersect ray with plane*
    - linear implicit function
  - *Then test whether point is inside or outside of polygon (2D test)*
  - *For convex polygons*
    - Suffices to test whether point in on the right side of every boundary edge
    - Similar to computation of outcodes in line clipping

Wolfgang Heidrich

Ray-Tracing

Issues:

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Efficient data structures so we don't have to test intersection with *every* object

Wolfgang Heidrich

Ray-Tracing – Geometric Transformations

Geometric Transformations:

- Similar goal as in rendering pipeline:
  - *Modeling scenes convenient using different coordinate systems for individual objects*
- Problem:
  - *Not all object representations are easy to transform*
    - This problem is fixed in rendering pipeline by restriction to polygons (affine invariance!)

Ray-Tracing – Geometric Transformations

Geometric Transformations:

- Similar goal as in rendering pipeline:
  - *Modeling scenes convenient using different coordinate systems for individual objects*
- Problem:
  - *Not all object representations are easy to transform*
    - This problem is fixed in rendering pipeline by restriction to polygons (affine invariance!)
  - *Ray-Tracing has different solution:*
    - The ray itself is always affine invariant!
    - Thus: transform ray into object coordinates!

Ray-Tracing – Geometric Transformations

Ray Transformation:

- For intersection test, it is only important that ray is in same coordinate system as object representation
- Transform all rays into object coordinates
  - *Transform camera point and ray direction by inverse of model/view matrix*
- Shading has to be done in world coordinates (where light sources are given)
  - *Transform object space intersection point to world coordinates*
  - *Thus have to keep both world and object-space ray*

Wolfgang Heidrich

Ray-Tracing

Issues:

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Efficient data structures so we don't have to test intersection with *every* object

Wolfgang Heidrich

Ray-Tracing
Lighting and Shading

Local Effects:

- Local Lighting
  - *Any reflection model possible*
  - *Have to talk about light sources, normals…*
- Texture mapping
  - *Color textures*
  - *Bump maps*
  - *Environment maps*
  - *Shadow maps*

Wolfgang Heidrich

Ray-Tracing
Local Lighting

Light sources:

- For the moment: point and directional lights
- Later: are light sources
- More complex lights are possible
  - *Area lights*
  - *Global illumination*
    - Other objects in the scene reflect light
    - Everything is a light source!
    - Talk about this on Monday

Wolfgang Heidrich

Ray-Tracing Local Lighting

Local surface information (normal…)

- For implicit surfaces $F(x,y,z)$=0: normal $\mathbf{n}(x,y,z)$ can be easily computed at every intersection point using the gradient

$$\mathbf{n}(x,y,z) = \begin{pmatrix} \partial F(x,y,z)/\partial x \\ \partial F(x,y,z)/\partial y \\ \partial F(x,y,z)/\partial z \end{pmatrix}$$

- Example: $F(x,y,z) = x^2 + y^2 + z^2 - r^2$

$$\mathbf{n}(x,y,z) = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

Needs to be normalized!

Wolfgang Heidrich

Ray-Tracing Local Lighting

Local surface information

- Alternatively: can interpolate per-vertex information for triangles/meshes as in rendering pipeline
  - *Phong shading!*
  - *Same as discussed for rendering pipeline*
- Difference to rendering pipeline:
  - *Interpolation cannot be done incrementally*
  - *Have to compute Barycentric coordinates for every intersection point (e.g plane equation for triangles)*

Wolfgang Heidrich

Ray-Tracing Texture Mapping

Approach:

- Works in principle like in rendering pipeline
  - *Given s, t parameter values, perform texture lookup*
  - *Magnification, minification just as discussed*
- Problem: how to get s, t
  - *Implicit surfaces often don't have parameterization*
  - *For special cases (spheres, other conic sections), can use parametric representation*
  - *Triangles/meshes: use interpolation from vertices*

Ray-Tracing Lighting and Shading

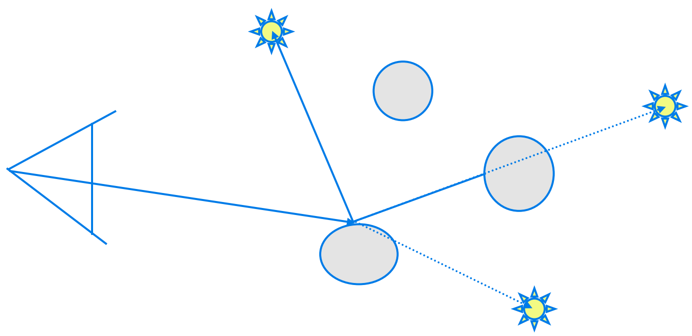
Global Effects

- Shadows
- Reflections/refractions

Ray-Tracing Shadows

Approach:

- To test whether point is in shadow, send out *shadow rays* to all light sources
  - *If ray hits another object, the point lies in shadow*

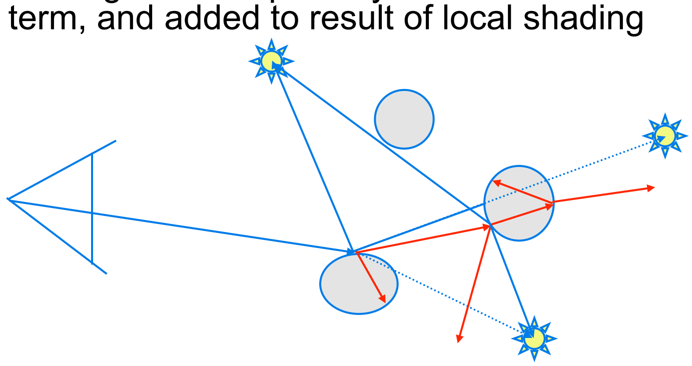


Wolfgang Heidrich

Ray-Tracing Reflections/Refractions

Approach:

- Send rays out in reflected and refracted direction to gather incoming light
- That light is multiplied by local surface color and Fresnel term, and added to result of local shading



Wolfgang Heidrich

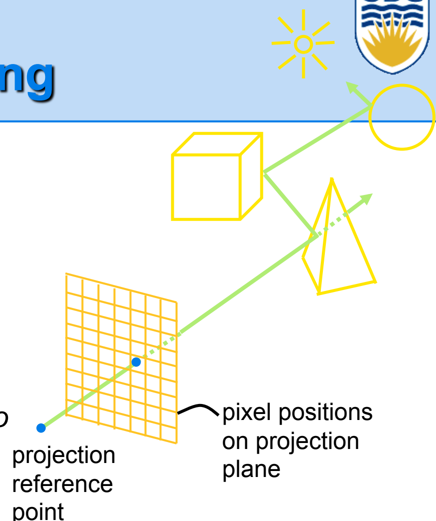
Recursive Ray Tracing

Ray tracing can handle

- Reflection (chrome)
- Refraction (glass)
- Shadows

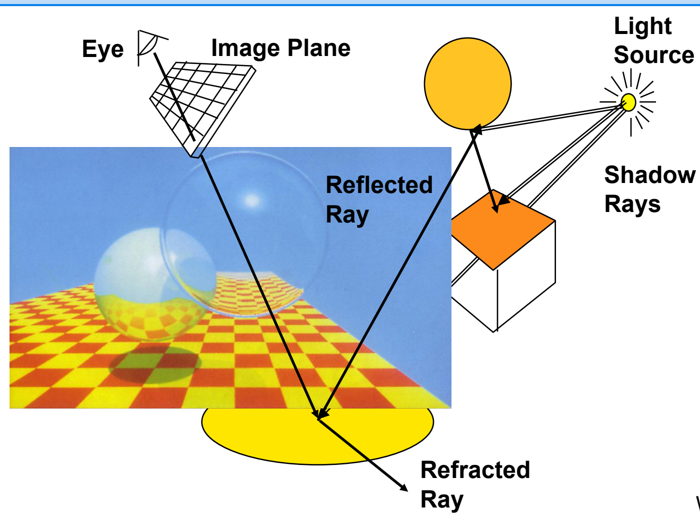
Spawn secondary rays

- Reflection, refraction
  - *If another object is hit, recurse to find its color*
- Shadow
  - *Cast ray from intersection point to light source, check if intersects another object*



Wolfgang Heidrich

Recursive Ray-Tracing



Whitted, 1980

Wolfgang Heidrich

Recursive Ray-Tracing Algorithm

```
RayTrace(r,scene)
obj := FirstIntersection(r,scene)
if (no obj)  return BackgroundColor;
else begin
   if ( Reflect(obj) ) then
     reflect_color := RayTrace(ReflectRay(r,obj));
   else
     reflect_color := Black;
  if ( Transparent(obj) ) then
     refract_color := RayTrace(RefractRay(r,obj));
   else
     refract_color := Black;
  return Shade(reflect_color,refract_color,obj);
end;
```

Wolfgang Heidrich

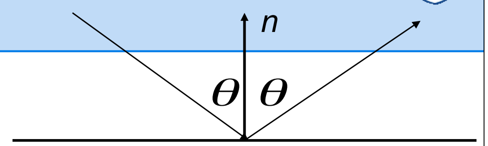
Algorithm Termination Criteria

Termination criteria

- No intersection
- Reach maximal depth
  - *Number of bounces*
- Contribution of secondary ray attenuated below threshold
  - *Each reflection/refraction attenuates ray*

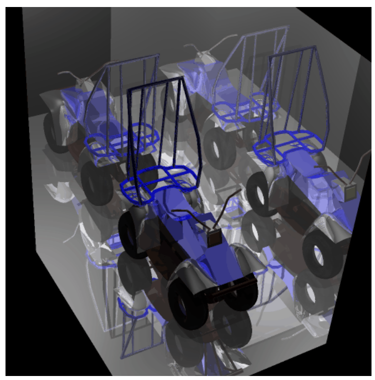
Wolfgang Heidrich

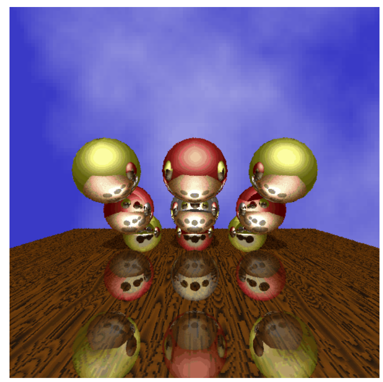
Reflection



Mirror effects

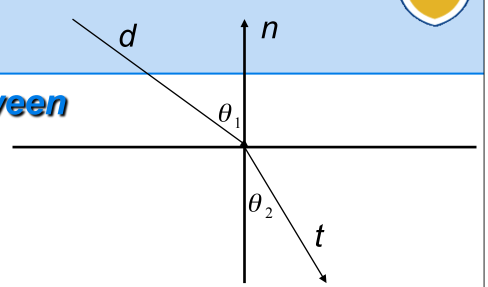
- Perfect specular reflection





Wolfgang Heidrich

Refraction

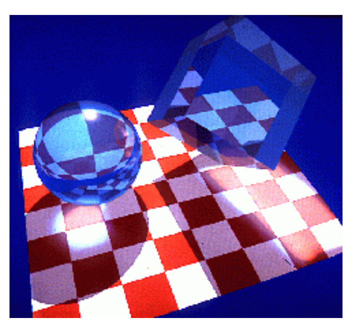


Happens at interface between transparent object and surrounding medium

- E.g. glass/air boundary

Snell's Law

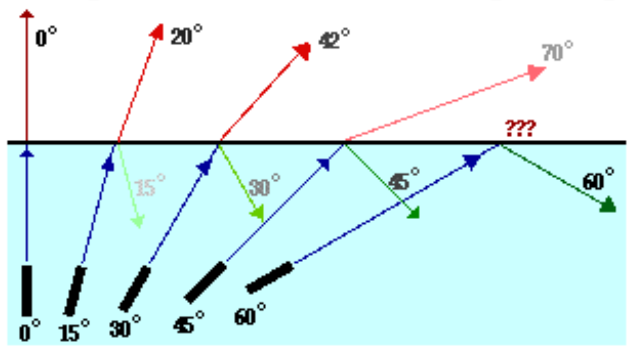
- $c_1 \sin\theta_1 = c_2 \sin\theta_2$
- Light ray bends based on refractive indices c_1, c_2



Wolfgang Heidrich

Total Internal Reflection

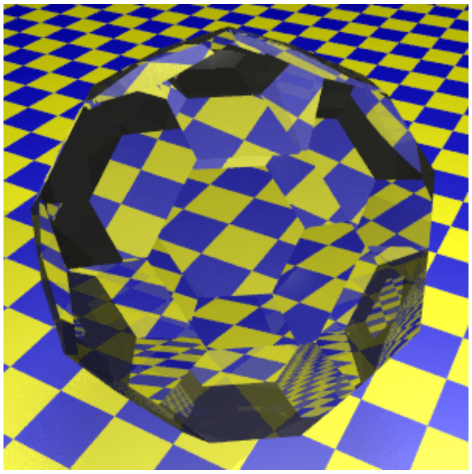
As the angle of incidence increases from 0 to greater angles ...

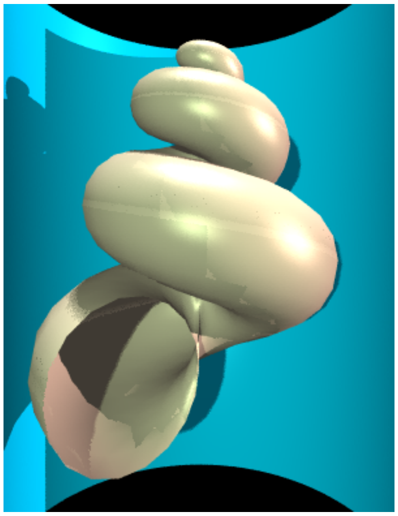


...the refracted ray becomes dimmer (there is less refraction)
...the reflected ray becomes brighter (there is more reflection)
...the angle of refraction approaches 90 degrees until finally a refracted ray can no longer be seen.

Wolfgang Heidrich

Ray-Tracing Example Images





Wolfgang Heidrich

Ray-Tracing Terminology

Terminology:

- Primary ray: ray starting at camera
- Shadow ray
- Reflected/refracted ray
- Ray tree: all rays directly or indirectly spawned off by a single primary ray

Note:

- Need to limit maximum depth of ray tree to ensure termination of ray-tracing process!

Wolfgang Heidrich

Ray-Tracing

Issues:

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Efficient data structures so we don't have to test intersection with *every* object

Wolfgang Heidrich

Ray Tracing

Data Structures

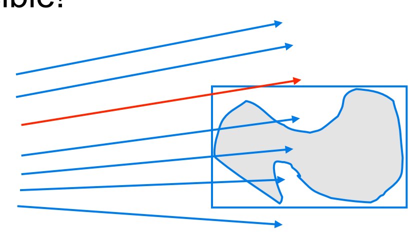
- Goal: reduce number of intersection tests per ray
- Lots of different approaches:
  - *(Hierarchical) bounding volumes*
  - *Hierarchical space subdivision*
    - Oct-tree, k-D tree, BSP tree

Wolfgang Heidrich

Bounding Volumes

Idea:

- Rather than testing every ray against a potentially very complex object (e.g. triangle mesh), do a quick *conservative* test first which eliminates most rays
  - *Surround complex object by simple, easy to test geometry (typically sphere or axis-aligned box)*
    - Want to make bounding volume as tight as possible!

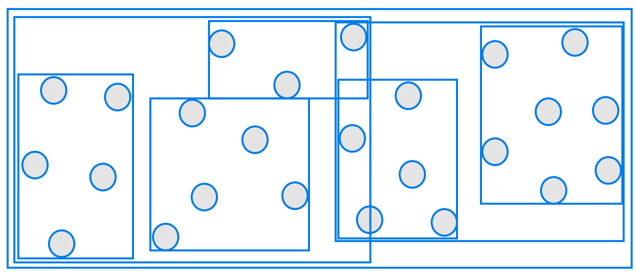


Wolfgang Heidrich

Hierarchical Bounding Volumes

Extension of previous idea:

- Use bounding volumes for groups of objects



Wolfgang Heidrich

Spatial Subdivision Data Structures

Bounding Volumes:

- Find simple object completely enclosing complicated objects
  - *Boxes, spheres*
- Hierarchically combine into larger bounding volumes

Spatial subdivision data structure:

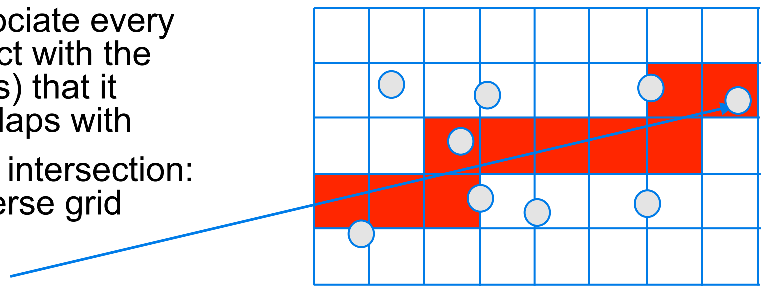
- Partition the whole space into cells
  - *Grids, oct-trees, (BSP trees)*
- Simplifies and accelerates traversal
- Performance less dependent on order in which objects are inserted

Wolfgang Heidrich

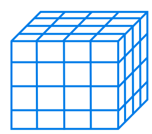
Regular Grid

Subdivide space into rectangular grid:

- Associate every object with the cell(s) that it overlaps with
- Find intersection: traverse grid



In 3D: regular grid of cubes (voxels):

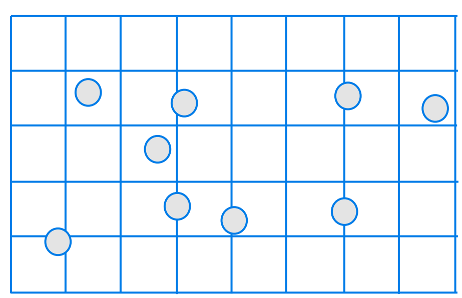


Wolfgang Heidrich

Creating a Regular Grid

Steps:

- Find bounding box of scene
- Choose grid resolution in x, y, z
- Insert objects
- Objects that overlap multiple cells get referenced by all cells they overlap

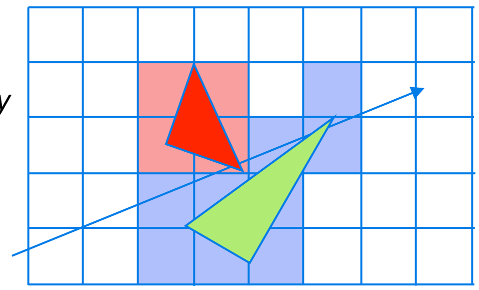


Wolfgang Heidrich

Grid Traversal

Traversal:

- Start at ray origin
- While no intersection found
  - *Go to next grid cell along ray*
  - *Compute intersection of ray with all objects in the cell*
  - *Determine closest such intersection*
  - ***Check if that intersection is inside the cell***
  - *If so, terminate search*



Wolfgang Heidrich

Traversal

Note:

- This algorithm calls for computing the intersection points multiple times (once per grid cell)
- In practice: store intersections for a (ray, object) pair once computed, reuse for future cells

Wolfgang Heidrich

Regular Grid Discussion

Advantages?

- Easy to construct
- Easy to traverse

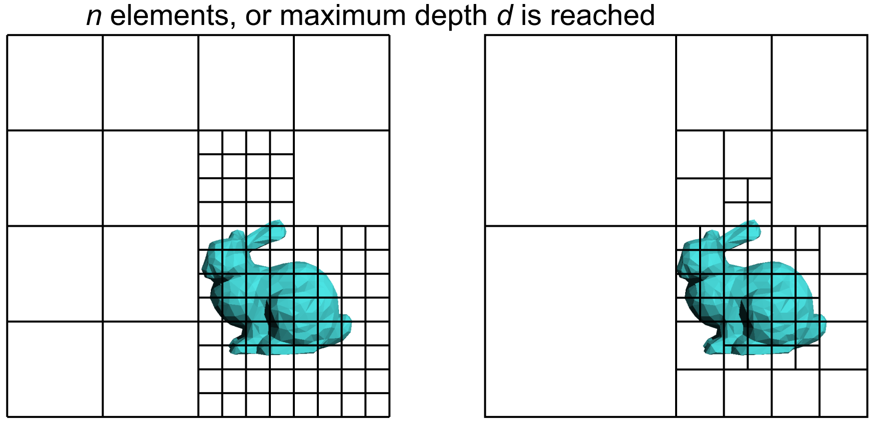
Disadvantages?

- May be only sparsely filled
- Geometry may still be clumped

Wolfgang Heidrich

Adaptive Grids

- Subdivide until each cell contains no more than *n* elements, or maximum depth *d* is reached



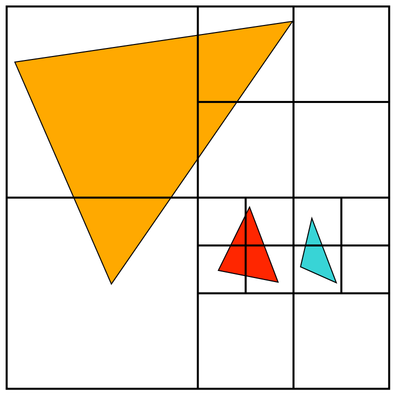
Nested Grids Octree/(Quadtree)

- This slide and the next are curtsey of Fredo Durand at MIT

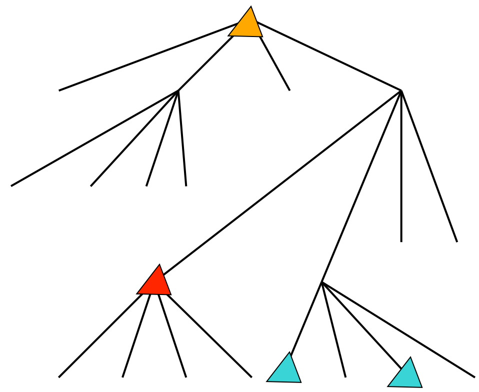
Wolfgang Heidrich

Primitives in an Adaptive Grid

- Can live at intermediate levels, or be pushed to lowest level of grid



Octree/(Quadtree)



Wolfgang Heidrich

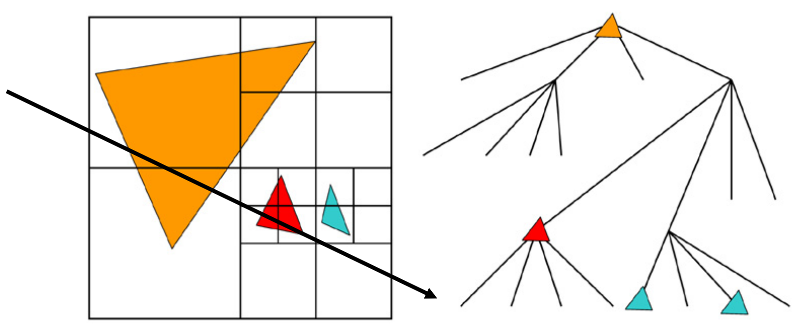
Adaptive Grid Discussion

Advantages

- Grid complexity matches geometric density

Disadvantages

- More expensive to traverse than regular grid



Wolfgang Heidrich

Coming Up…

Wednesday:

- More ray-tracing

Next Week:

- Global illumination

Wolfgang Heidrich