Sampling & Reconstruction

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Course News

Assignment 3

- Due April 1

Homework 6

- Out today
- Questions on texture mapping

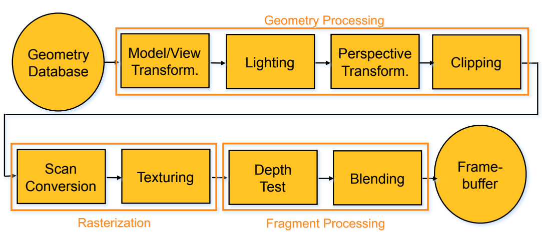
Reading

- Chapter 11 (Texture Mapping)

Quiz 2

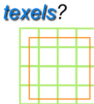
- On Wednesday, Mar 9
- Topics: full Rendering Pipeline, except transformations

The Rendering Pipeline

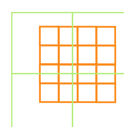


Texture Lookup – Sampling & Reconstruction

- How to deal with:
  - *Pixels that are much larger than texels?*
    - Apply filtering, "averaging"
    - "Minification"



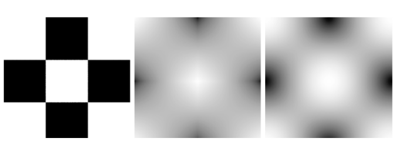
  - *Pixels that are much smaller than texels ?*
    - Interpolate
    - "Magnification"



Magnification: Interpolating Textures

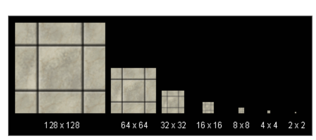
- Nearest neighbor
- Bilinear
- Hermite (cubic)

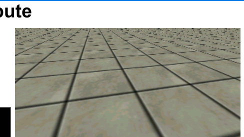
1	0	1
0	1	0
1	0	1



Minification: MIPmapping

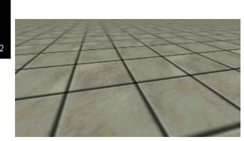
use "image pyramid" to precompute averaged versions of the texture





Without MIP-mapping

store whole pyramid in single block of memory



With MIP-mapping

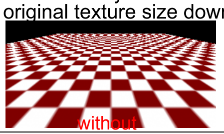
MIPmaps

Multum in parvo

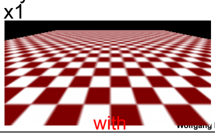
- “many things in a small place”
- Series of prefiltered texture maps of decreasing resolutions
- Avoid shimmering and flashing as objects move

gluBuild2DMipmaps

- Automatically constructs a family of textures from original texture size down to 1x1



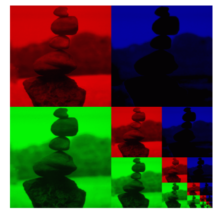
without



with

MIPmap storage

Only 1/3 more space required



Sampling & Reconstruction

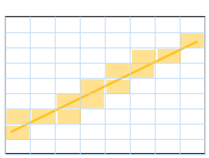
CPSC 314

Samples

- Most things in the real world are continuous
- Everything in a computer is discrete
- The process of mapping a continuous function to a discrete one is called sampling
- The process of mapping a discrete function to a continuous one is called reconstruction
- The process of mapping a continuous variable to a discrete one is called quantization
- Rendering an image requires both sampling and quantization
- Displaying an image involves reconstruction

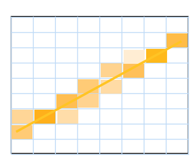
Line Segments

- We tried to sample a line segment so it would map to a 2D raster display
- We quantized the pixel values to 0 or 1
- We saw stair steps, or jaggies



Line Segments

- Instead, quantize to many shades
- But what sampling algorithm is used?



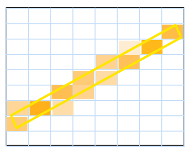
Unweighted Area Sampling

Shade pixels wrt area covered by thickened line

Equal areas cause equal intensity, regardless of distance from pixel center to area

- Rough approximation formulated by dividing each pixel into a finer grid of pixels

Primitive cannot affect intensity of pixel if it does not intersect the pixel



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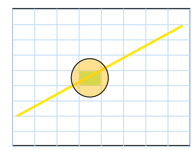
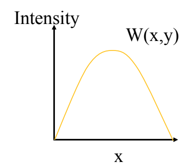
Weighted Area Sampling

Intuitively, pixel cut through the center should be more heavily weighted than one cut along corner

Weighting function, W(x,y)

- Specifies the contribution of primitive passing through the point (x, y) from pixel center





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Images

An image is a 2D function I(x, y)

- Specifies intensity for each point (x, y)
- (we consider each color channel independently)

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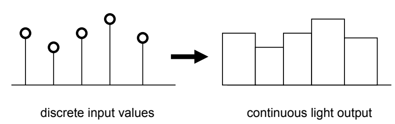
Image Sampling and Reconstruction

- Convert continuous image to discrete set of samples
- Display hardware reconstructs samples into continuous image
  - *Finite sized source of light for each pixel*



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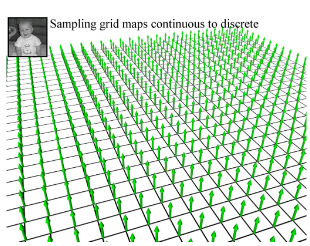
Point Sampling an Image

- Simplest sampling is on a grid
- Sample depends solely on value at grid points



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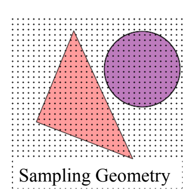
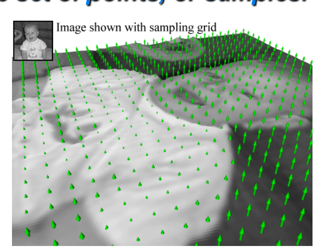
Point Sampling

Multiply sample grid by image intensity to obtain a discrete set of points, or samples.





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Sampling Errors

Some objects missed entirely, others poorly sampled

- Could try unweighted or weighted area sampling
- But how can we be sure we show everything?

Need to think about entire class of solutions!

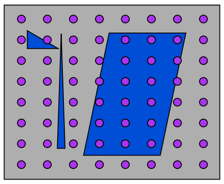

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Image As Signal

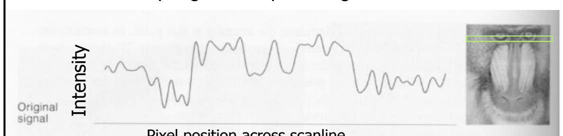
Image as spatial signal

2D raster image

- Discrete sampling of 2D spatial signal

1D slice of raster image

- Discrete sampling of 1D spatial signal



Examples from Foley, van Dam, Feiner, and Hughes

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Sampling Theory

How would we generate a signal like this out of simple building blocks?

Theorem

- Any signal can be represented as an (infinite) sum of sine waves at different frequencies

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Sampling Theory in a Nutshell

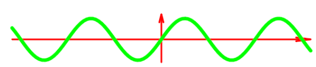
Terminology

- Wavelength – length of repeated sequence on infinite signal
- Frequency – 1/wavelength (number of repeated sequences in unit length)

Example – sine wave

- Wavelength = 2π
- Frequency = $1/ 2\pi$

$\sin(t)$



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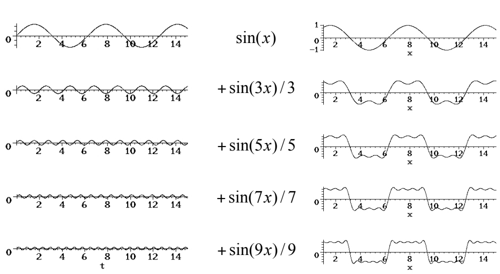
Fourier Transform – Summing Sinusoids



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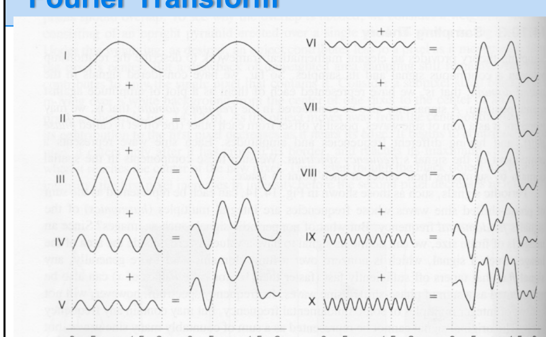
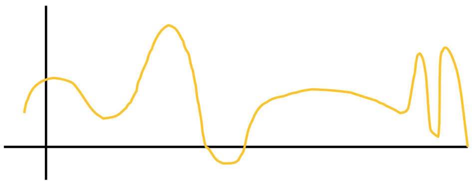
Fourier Transform



1D Sampling and Reconstruction



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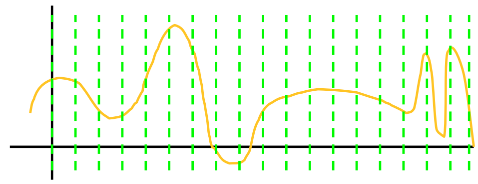
1D Sampling and Reconstruction



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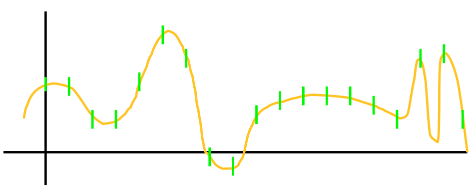
1D Sampling and Reconstruction



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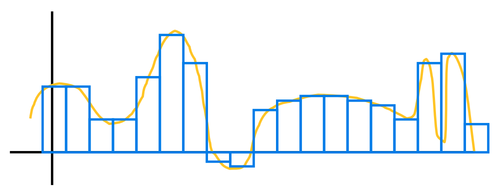
1D Sampling and Reconstruction



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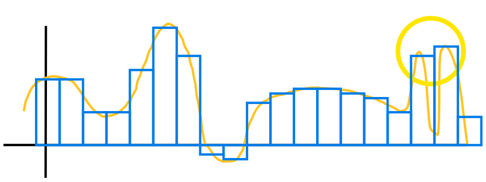
1D Sampling and Reconstruction

Problems

- Jaggies – abrupt changes
- Lose data



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1D Sampling and Reconstruction

Problems

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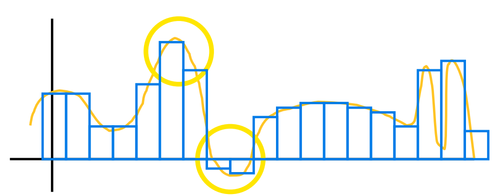

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Sampling Theorem

- Continuous signal can be completely recovered from its samples

Iff

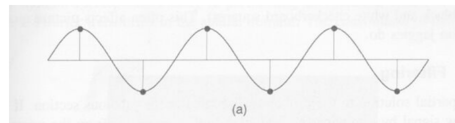
- Sampling rate greater than twice highest frequency present in signal

- Claude Shannon

Nyquist Rate

Lower bound on sampling rate

- Twice the highest frequency component in the image's spectrum



Falling Below Nyquist Rate

When sampling below Nyquist Rate, resulting signal looks like a lower-frequency one

- This is aliasing!

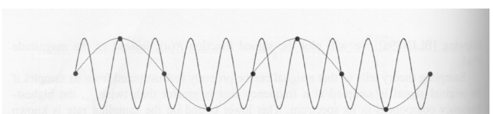
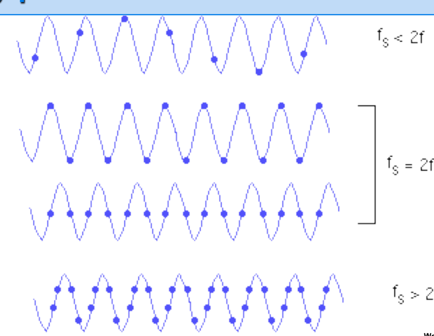


Fig. 14.17 Sampling below the Nyquist rate. (Courtesy of George Wolberg, Columbia University.)

Nyquist Rate



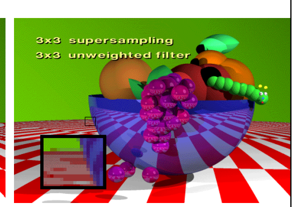
Aliasing

Incorrect appearance of high frequencies as low frequencies

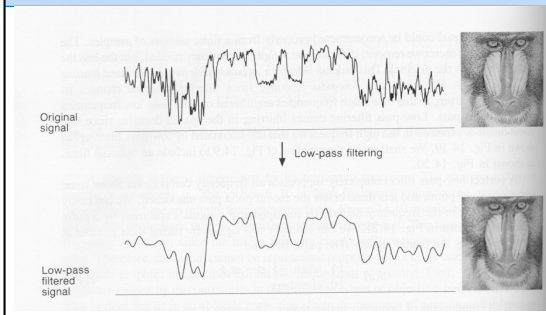
To avoid: anti-aliasing

- Supersample
  - *Sample at higher frequency*
- Low pass filtering
  - *Remove high frequency function parts*
  - *Aka prefiltering, band-limiting*

Supersampling



Low-Pass Filtering



Low-Pass Filtering

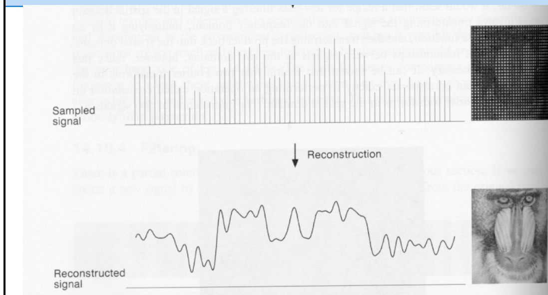
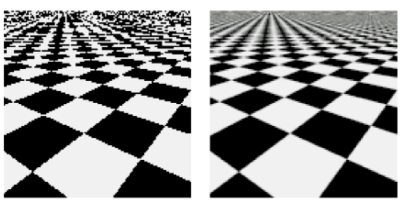


Fig. 14.20 The sampling pipeline with filtering. (Courtesy of George Wolberg, Columbia University.)

Previous Antialiasing Example

Texture mipmapping: low pass filter



(a) (b)

Discussion

Sampling & Reconstruction

- Fundamental issue in graphics, vision, and many other areas of computer science
  - *Whenever continuous signals need to be represented in a computer*
- Aliasing refers to the problem of reconstruction errors due to frequencies above the Nyquist limit
  - *These frequencies show up as erroneous low frequency content*

Discussion

Anti-Aliasing Approaches

- Low-pass filtering (before sampling!)
  - *Avoids aliasing*
  - *May not be practical in all settings*
  - *For images: artifacts around edges?!*
- Supersampling
  - *General algorithmic approach*
  - *Hoever: even the higher resolution image has a Nyquist limit!*
  - *Slow*

Coming Up:

Friday

- Sampling & reconstruction