Scan Conversion

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Course News

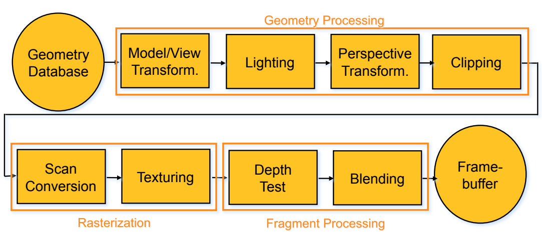
Assignment 2
- Due Monday, Feb 28

Homework 3
- Discussed in labs this week

Homework 4
- Hidden surface removal, out today

Reading
- Chapters 8, 9
- Hidden surface removal, shading

The Rendering Pipeline

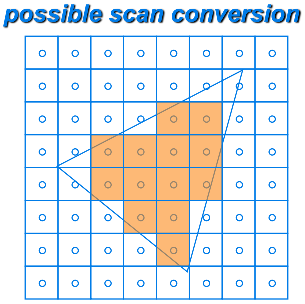


Scan Conversion - Rasterization

Convert continuous rendering primitives into discrete fragments/pixels
- Lines
  - *Midpoint/Bresenham*
- Triangles
  - *Flood fill*
  - *Scanline*
  - *Implicit formulation*
- Interpolation

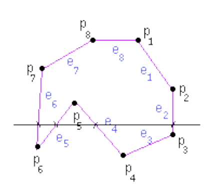
Scan Conversion of Polygons

One possible scan conversion



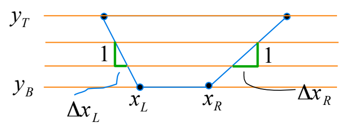
Scan Conversion of Polygons

A General Algorithm
- Intersect each scanline with all edges
- Sort intersections in x
- Calculate parity to determine in/out
- Fill the ‘in’ pixels



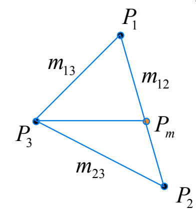
Edge Walking

```
for (y=yB; y<=yT; y++) {
 for (x=xL; x<=xR; x++)
     setPixel(x,y);
 xL += DxL;
 xR += DxR;
}
```



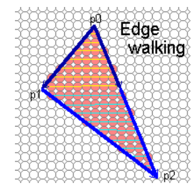
Edge Walking Triangles

- Split triangles into two regions with continuous left and right edges



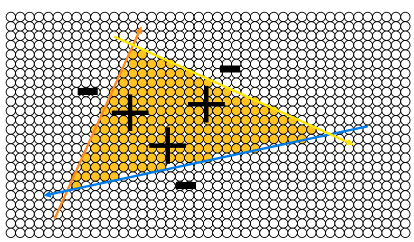
scanTrapezoid($x_3, x_m, y_3, y_1, \frac{1}{m_{13}}, \frac{1}{m_{12}}$)

scanTrapezoid($x_2, x_2, y_2, y_3, \frac{1}{m_{23}}, \frac{1}{m_{12}}$)



Modern Rasterization: Edge Equations

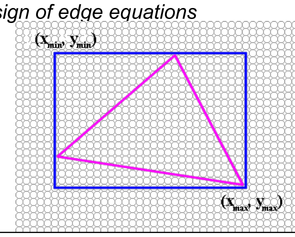
Define a triangle as follows:



Using Edge Equations

Usage:

- Go over each pixel in bounding rectangle
- Check if pixel is inside/outside of triangle
  - *Using sign of edge equations*



Computing Edge Equations

Implicit equation of a triangle edge:

$$L(x,y) = \frac{(y_e - y_s)}{(x_e - x_s)}(x - x_s) - (y - y_s) = 0$$

(see Bresenham algorithm)

- L(x,y) positive on one side of edge, negative on the other

Question:

- What happens for vertical lines?

Edge Equations

Multiply with denominator

$$L(x,y) = (y_e - y_s)(x - x_s) - (y - y_s)(x_e - x_s) = 0$$

- Avoids singularity
- Works with vertical lines

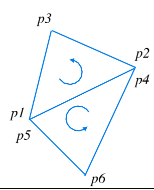
What about the sign?

- Which side is in, which is out?

Edge Equations

Determining the sign

- Which side is "in" and which is "out" depends on order of start/end vertices…
- Convention: specify vertices in counter-clockwise order



Edge Equations

Counter-Clockwise Triangles

- The equation L(x,y) as specified above is *negative inside, positive outside*
  - *Flip sign:*

$$L(x,y) = -(y_e - y_s)(x - x_s) + (y - y_s)(x_e - x_s) = 0$$

Clockwise triangles

- *Use original formula*

$$L(x,y) = (y_e - y_s)(x - x_s) - (y - y_s)(x_e - x_s) = 0$$

Discussion of Polygon Scan Conversion Algorithms

On old hardware:

- Use first scan-conversion algorithm
  - *Scan-convert edges, then fill in scanlines*
  - *Compute interpolated values by interpolating along edges, then scanlines*
- Requires clipping of polygons against viewing volume
- Faster if you have a few, large polygons
- Possibly faster in software

Discussion of Polygon Scan Conversion Algorithms

Modern GPUs:

- Use edge equations
  - *And plane equations for attribute interpolation*
  - *No clipping of primitives required*
- Faster with many small triangles

Additional advantage:

- Can control the order in which pixels are processed
- Allows for more memory-coherent traversal orders
  - *E.g. tiles or space-filling curve rather than scanlines*

Triangle Rasterization Issues (Independent of Algorithm)

Exactly which pixels should be lit?

- A: Those pixels inside the triangle edge (of course)

But what about pixels exactly on the edge?

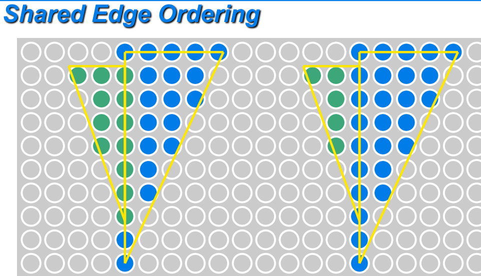
- Draw them: order of triangles matters (it shouldn't)
- Don't draw them: gaps possible between triangles

We need a consistent (if arbitrary) rule

- Example: draw pixels on left or top edge, but not on right or bottom edge

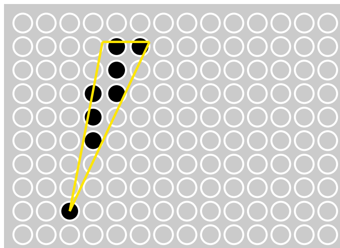
Triangle Rasterization Issues

Shared Edge Ordering



Triangle Rasterization Issues

Sliver



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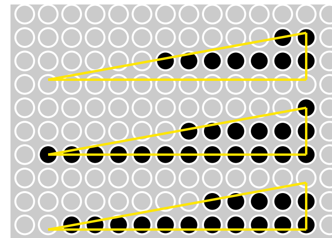
Triangle Rasterization Issues

Moving Slivers



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Triangle Rasterization Issues

These are ALIASING Problems

- Problems associated with representing continuous functions (triangles) with finite resolution (pixels)
- More on this problem when we talk about sampling…

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Shading

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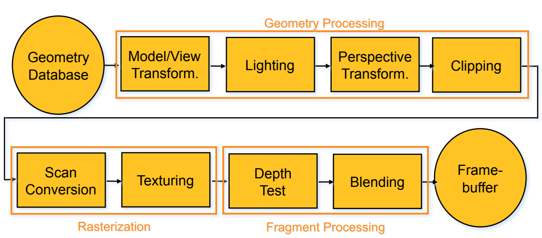
The Rendering Pipeline



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Shading

Input to Scan Conversion:

- Vertices of triangles (lines, quadrilaterals…)
- Color (per vertex)
  - *Specified with glColor*
  - *Or: computed with lighting*
- World-space normal (per vertex)
  - *Left over from lighting stage*

Shading Task:

- Determine color of every pixel in the triangle

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Shading

How can we assign pixel colors using this information?

- Easiest: flat shading
  - *Whole triangle gets one color (color of 1st vertex)*
- Better: Gouraud shading
  - *Linearly interpolate color across triangle*
- Even better:
  - *Linearly interpolate the normal vector*
  - *Compute lighting for every pixel*
  - *Note: not supported by rendering pipeline as discussed so far*

Flat Shading

- Simplest approach calculates illumination at a single point for each polygon

- Obviously inaccurate for smooth surfaces

Flat Shading Approximations

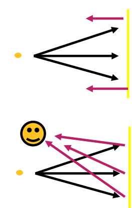
If an object really is faceted, is this accurate?

Flat Shading Approximations

If an object really is faceted, is this accurate?

no!

- For point sources, the direction to light varies across the facet
- For specular reflectance, direction to eye varies across the facet

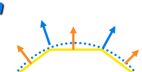


Improving Flat Shading

What if evaluate Phong lighting model at each pixel of the polygon?

- Better, but result still clearly faceted

For smoother-looking surfaces we introduce vertex normals at each vertex

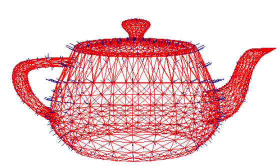


- Usually different from facet normal
- Used *only* for shading
- Think of as a better approximation of the *real* surface that the polygons approximate

Vertex Normals

Vertex normals may be

- Provided with the model
- Computed from first principles
- Approximated by averaging the normals of the facets that share the vertex

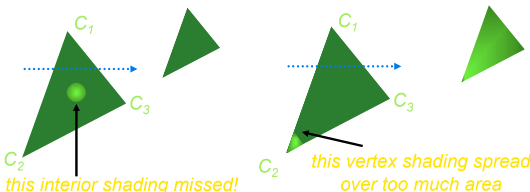


Gouraud Shading Artifacts

often appears dull, chalky

lacks accurate specular component

- if included, will be averaged over entire polygon



Gouraud Shading Artifacts

Mach bands

- Eye enhances discontinuity in first derivative
- Very disturbing, especially for highlights

Phong Shading

linearly interpolating surface normal across the facet, applying Phong lighting model at every pixel

- Same input as Gouraud shading
- Pro: much smoother results
- Con: considerably more expensive

Not the same as Phong lighting

- Common confusion
- Phong lighting: empirical model to calculate illumination at a point on a surface

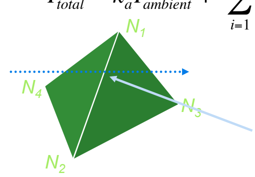
Phong Shading

Linearly interpolate the vertex normals

- Compute lighting equations at each pixel
- Can use specular component

$$I_{total} = k_a I_{ambient} + \sum_{i=1}^{\#\,lights} I_i \left(k_d \left(\mathbf{n} \cdot \mathbf{l_i} \right) + k_s \left(\mathbf{v} \cdot \mathbf{r_i} \right)^{n_{shiny}} \right)$$

remember: normals used in diffuse and specular terms



discontinuity in normal's rate of change harder to detect

Phong Shading Difficulties

Computationally expensive

- Per-pixel vector normalization and lighting computation!
- Floating point operations required

Lighting after perspective projection

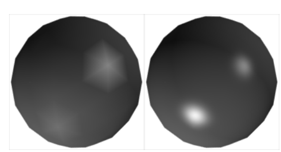
- Messes up the angles between vectors
- Have to keep eye-space vectors around

No direct support in standard rendering pipeline

- But can be simulated with texture mapping, procedural shading hardware (see later)

Shading Artifacts: Silhouettes

Polygonal silhouettes remain



Gouraud *Phong*

How to Interpolate?

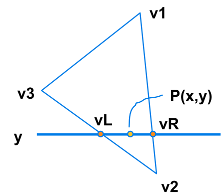
Need to propagate vertex attributes to pixels

- Interpolate between vertices:
  - *z (depth)*
  - *r,g,b   color components*
  - N_x, N_y, N_z *surface normals*
  - *u,v   texture coordinates (talk about these later)*
- Three equivalent ways of viewing this (for triangles)
  1. *Linear interpolation*
  2. *Barycentric coordinates*
  3. *Plane Equation*

1. Linear Interpolation

Interpolate quantity along L and R edges

- (as a function of y)
- Then interpolate quantity as a function of x

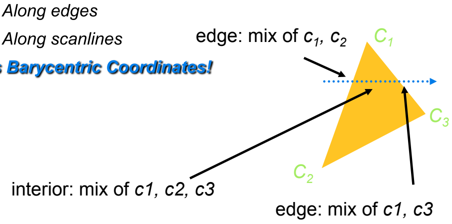


Linear Interpolation

Most common approach, and what OpenGL does

- Perform Phong lighting at the vertices
- Linearly interpolate the resulting colors over faces
  - *Along edges*
  - *Along scanlines*

Same as Barycentric Coordinates!



2. Barycentric Coordinates

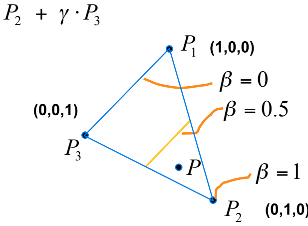
Have seen this before

- Barycentric Coordinates: weighted combination of vertices, with weights summing to 1

$$P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3$$

$$\alpha + \beta + \gamma = 1$$

$$0 \le \alpha, \beta, \gamma \le 1$$



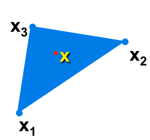
Barycentric Coordinates

- Convex combination of 3 points

$$\mathbf{x} = \alpha \cdot \mathbf{x}_1 + \beta \cdot \mathbf{x}_2 + \gamma \cdot \mathbf{x}_3$$

$$\text{with } \alpha + \beta + \gamma = 1,\ 0 \le \alpha, \beta, \gamma \le 1$$

- α, β, and γ are called *barycentric coordinates*



Barycentric Coordinates

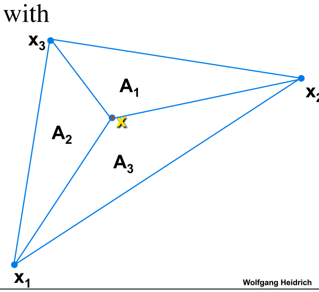
One way to compute them:

$$\mathbf{x} = \alpha \mathbf{x}_1 + \beta \mathbf{x}_2 + \gamma \mathbf{x}_3 \text{ with}$$

$$\alpha = A_1 / A$$

$$\beta = A_2 / A$$

$$\gamma = A_3 / A$$

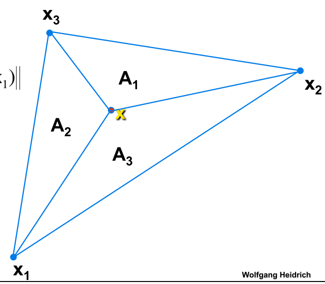


Barycentric Coordinates

How to compute areas?

- Cross products!
- e.g:

$$A_1 = \frac{1}{2}\left\|(\mathbf{x}_2 - \mathbf{x}_1) \times (\mathbf{x} - \mathbf{x}_1)\right\|$$



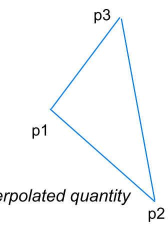
3. Plane Equation

Observation: Quantities vary linearly across image plane

- E.g.: r = Ax + By + C
  - *r= red channel of the color*
  - *Same for g, b, Nx, Ny, Nz, z…*
- From info at vertices we know:

$$r_1 = Ax_1 + By_1 + C$$
$$r_2 = Ax_2 + By_2 + C$$
$$r_3 = Ax_3 + By_3 + C$$

  - *Solve for A, B, C*
  - *One-time set-up cost per triangle and interpolated quantity*



Coming Up:

Wednesday/Friday

- Clipping, hidden surface removal