Transformations of Normal Vectors
Intro to Lighting

Wolfgang Heidrich

Course News

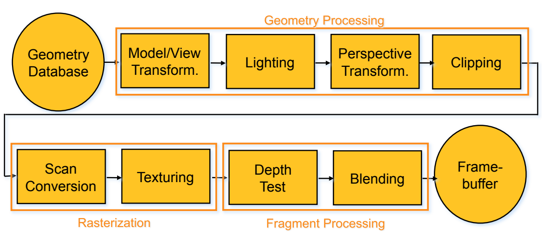
Assignment 1

- Due Monday!

Homework 2

- Discussed in labs this week

The Rendering Pipeline



Normals & Affine Transformations

Question:

- If we transform some geometry with an affine transformation, how does that affect the normal vector?

Consider

- Rotation
- Translation
- Scaling
- Shear

Normals & Affine Transformations

Want:

- Representation for normals that allows us to easily describe how they change under affine transformation

Why?

- Normal vectors will be of special interest when we talk about lighting (next week)

Homogeneous Planes And Normals

Planes in Cartesian Coordinates:

$$\{(x, y, z)^T \mid n_x x + n_y y + n_z z + d = 0\}$$

- n_x, n_y, n_z, and d are the parameters of the plane (normal and distance from origin)
- d is positive
- n point to half-space containing origin

Planes in Homogeneous Coordinates:

$$\{[x, y, z, 1]^T \mid n_x x + n_y y + n_z z + d \cdot 1 = 0\}$$

Homogeneous Planes And Normals

Planes in homogeneous coordinates are represented as row vectors

- E=$[n_x, n_y, n_z, d]$
- Condition that a point $[x,y,z,w]^T$ is located in E

$$\begin{bmatrix}x\\y\\z\\1\end{bmatrix} \in E = [n_x, n_y, n_z, d] \Leftrightarrow [n_x, n_y, n_z, d] \cdot \begin{bmatrix}x\\y\\z\\1\end{bmatrix} = 0$$

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Homogeneous Planes And Normals

Transformations of planes

$$[n_x, n_y, n_z, d] \cdot \begin{bmatrix}x\\y\\z\\1\end{bmatrix} = 0 \quad \Leftrightarrow T([n_x, n_y, n_z, d]) \cdot (\mathbf{A} \cdot \begin{bmatrix}x\\y\\z\\1\end{bmatrix}) = 0$$

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Homogeneous Planes And Normals

Transformations of planes

$$[n_x, n_y, n_z, d] \cdot \begin{bmatrix}x\\y\\z\\1\end{bmatrix} = 0 \quad \Leftrightarrow ([n_x, n_y, n_z, d] \cdot \mathbf{A}^{-1}) \cdot (\mathbf{A} \cdot \begin{bmatrix}x\\y\\z\\1\end{bmatrix}) = 0$$

- Works for $T([n_x, n_y, n_z, d]) = [n_x, n_y, n_z, d]\mathbf{A}^{-1}$
- Thus: planes have to be transformed by the *inverse* of the affine transformation (multiplied from left as a row vector)!

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Homogeneous Planes And Normals

Homogeneous Normals

- The plane definition also contains its normal
- Normal written as a vector $[n_x, n_y, n_z, 0]^T$

$$(\begin{bmatrix}n_x\\n_y\\n_z\\0\end{bmatrix} \cdot \begin{bmatrix}v_x\\v_y\\v_z\\0\end{bmatrix}) = 0 \Leftrightarrow ((\mathbf{A}^{-T} \cdot \begin{bmatrix}n_x\\n_y\\n_z\\0\end{bmatrix}) \cdot (\mathbf{A} \cdot \begin{bmatrix}v_x\\v_y\\v_z\\0\end{bmatrix})) = 0$$

- Thus: the normal to any surface has to be transformed by the inverse transpose of the affine transformation (multiplied from the right as a column vector)!

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Transforming Homogeneous Normals

Inverse Transpose of

- Rotation by α
  - *Rotation by α*
- Scale by s
  - *Scale by 1/s*
- Translation by t
  - *Identity matrix!*
- Shear by a along x axis
  - *Shear by -a along y axis*

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Intro to Lighting

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Lighting

Goal
- Model the interaction of light with surfaces to render realistic images

Contributing Factors
- Light sources
  - *Shape and color*
- Surface materials
  - *How surfaces reflect light*

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Materials

Analyzing surface reflectance
- Illuminate surface point with a ray of light from different directions
- Observe how much light is reflected in all possible directions

Does this tell us anything about general lighting conditions?

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Materials

Light is linear
- If two rays illuminate the surface point the result is just the sum of the individual reflections for each ray
- For N directions the reflection is the sum of the individual N reflections
- For light arriving from a *continuum* of directions, the reflection is the *integral* over the reflections caused by the individual directions
  - *More on this when we talk about global illumination at the end of the term*

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Experiment

Goal:
- Visualize reflected light distribution for a given illuminating ray

Physical setup:
- Laser illumination
- Water tank with fluorescent dye
  - *Makes laser light visible as it travels through "empty" space*

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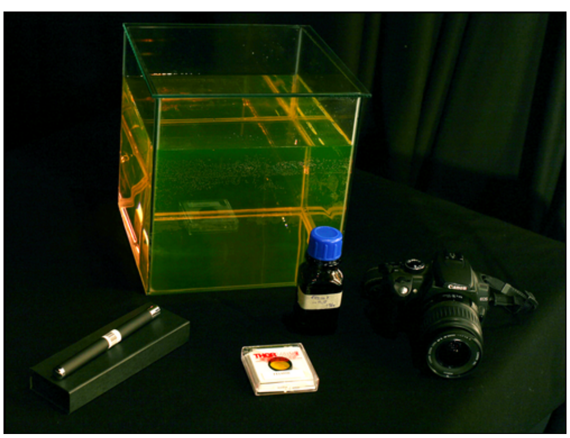

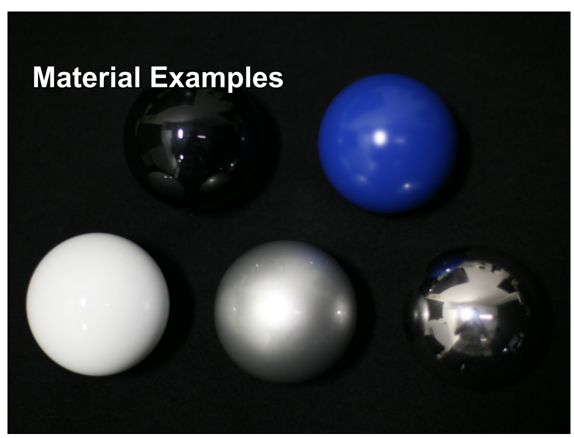

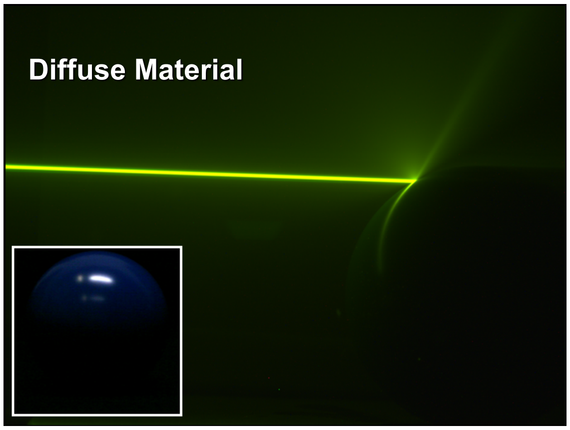

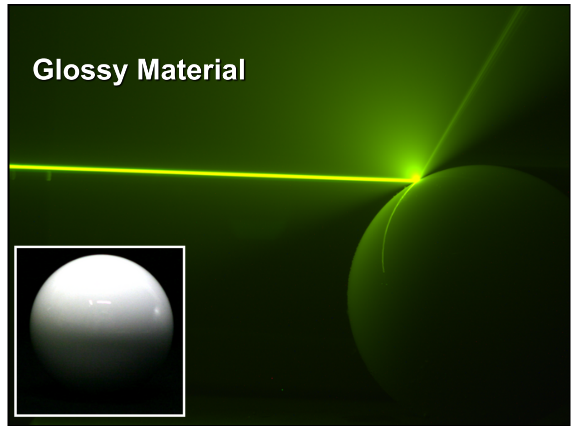

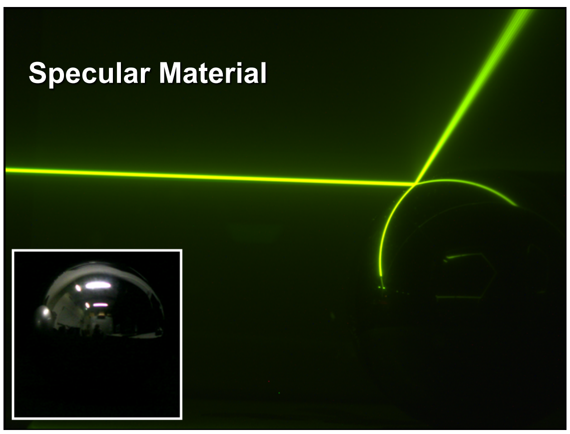

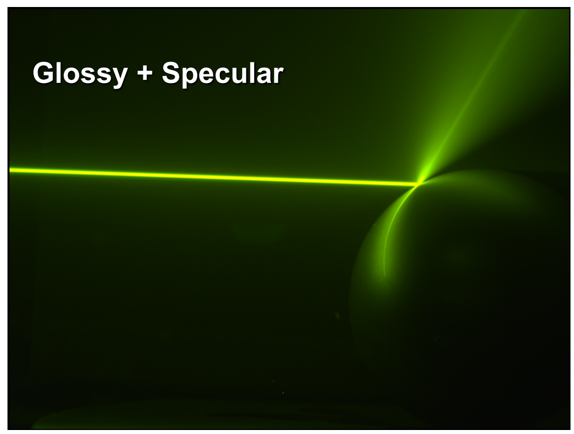

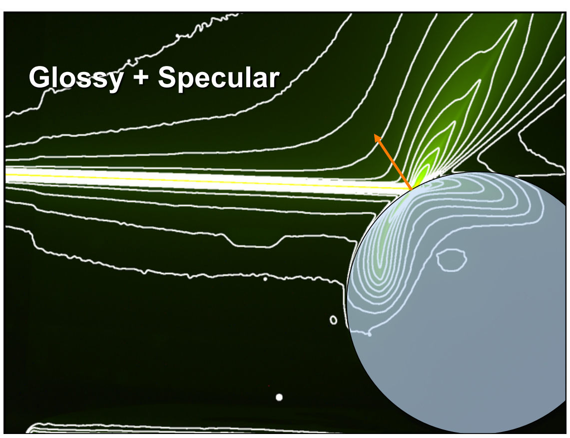

BRDF

Model for all these effects:

- **B**i-directional
  - *i.e. dependent on 2 directions: incident, exitant*
- **R**eflectance
  - *A model for surface reflection (not transmission)*
- **D**istribution
  - *Light is distributed over different exitant directions*
- **F**unction

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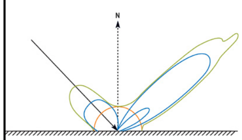
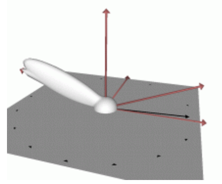
BRDF lobe plots

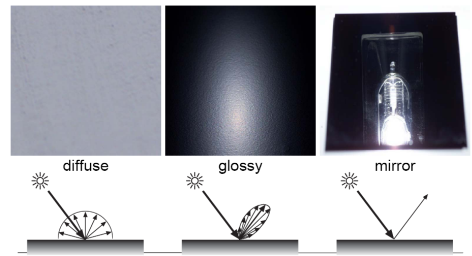
2D slice



3D surface

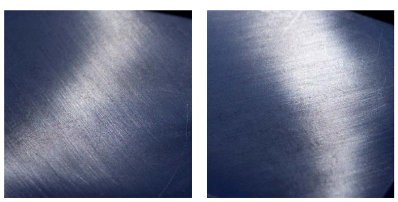


BRDF lobes and appearance

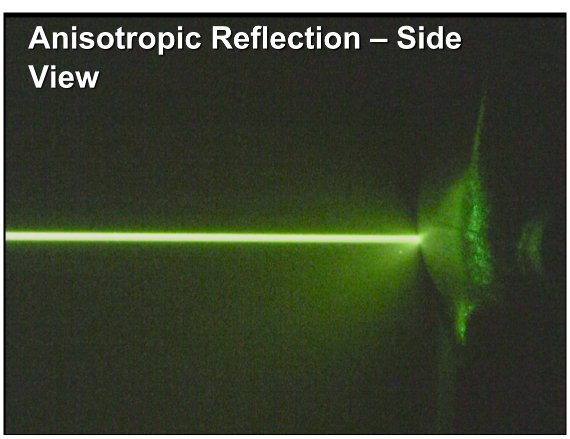


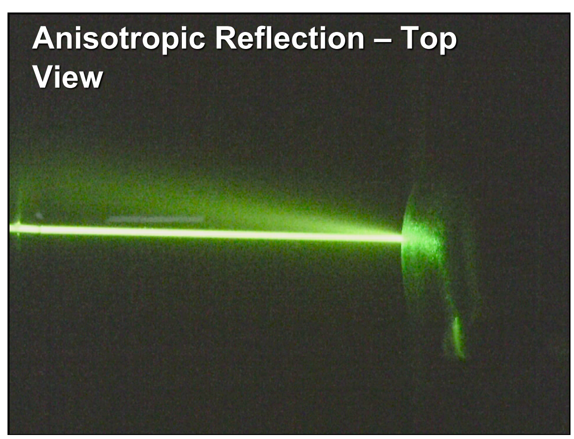
diffuse glossy mirror

BRDF lobes and appearance

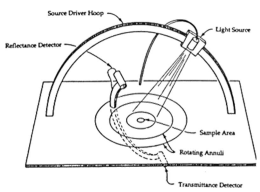


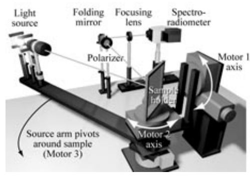
anisotropic





BRDF measurement





Limitations of the BRDF Model

BRDFs cannot describe

- Light of one wavelength that gets absorbed and re-emitted at a different wavelength
  - *(fluorescence)*
- Light that gets absorbed and emitted much later
  - *(phosphorescence)*
- Light that penetrates the object surface, scatters in the interior of the object, and exits at a different point form where it entered
  - *(subsurface scattering)*

Materials

Practical Considerations

- In practice, we often simplify the BRDF model further
- Derive specific formulas that describe different reflectance behaviors
  - *E.g. diffuse, glossy, specular*
- Computational efficiency is also a concern

Coming Up:

Next week

- More on lighting / shading