Perspective Projection (cont.) Transformations of Normal Vectors

Wolfgang Heidrich

Course News

Assignment 1

- Due next Monday

No new homework this week

Homework 2

- Exercise problems for perspective
- Discussed in labs this week

Quiz 1

- Wed, Jan 26. Duration: 40 minutes
- Topics: affine and perspective transformations

Course News (cont.)

Reading list

- Previously published chapters numbers were from an old book version…

Reading for Quiz (new book version):

- Math prereq: Chapter 2.1-2.4, 4
- Intro: Chapter 1
- Affine transformations: Ch. 6 (was: Ch. 5, old book)
- Perspective: Ch 7 (was: Ch. 6, old book)
  - *Also reading for this week…*

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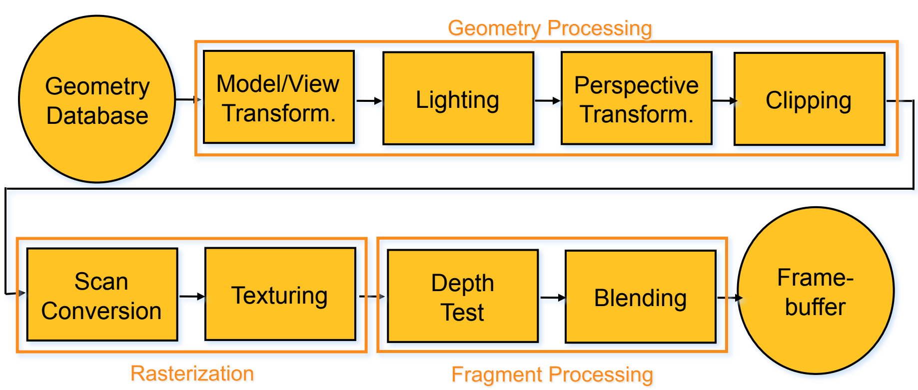
The Rendering Pipeline



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Projective Rendering Pipeline

object **OCS** → **O2W** **modeling transformation** → world **WCS** → **W2V** **viewing transformation** → viewing **VCS** → **V2C** **projection transformation** → clipping **CCS** → **C2N** **perspective divide** → normalized device **NDCS** → **N2D** **viewport transformation** → device **DCS**

OCS - object/model coordinate system

WCS - world coordinate system

VCS - viewing/camera/eye coordinate system

CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device/display/screen coordinate system

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Projective Transformations

Convention:

- Viewing frustum is mapped to a specific parallelpiped
  - *Normalized Device Coordinates (NDC)*
- Only objects inside the parallelpiped get rendered
- Which parallelpied is used depends on the rendering system

OpenGL:

- Left and right image boundary are mapped to x=-1 and x=+1
- Top and bottom are mapped to y=-1 and y=+1
- Near and far plane are mapped to -1 and 1

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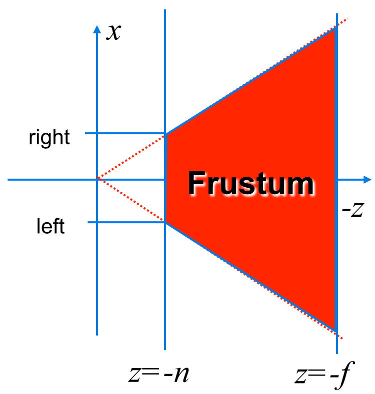
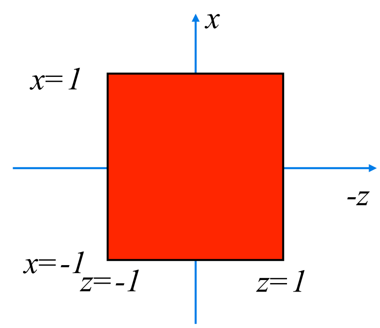
Projective Transformations

OpenGL Convention

Camera coordinates



Clipping Coordinates



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Perspective Derivation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = Ex + Az$$
$$y' = Fy + Bz$$
$$z' = Cz + D$$
$$w' = -z$$

$$x = left \rightarrow x'/w' = 1$$
$$x = right \rightarrow x'/w' = -1$$
$$y = top \rightarrow y'/w' = 1$$
$$y = bottom \rightarrow y'/w' = -1$$
$$z = -near \rightarrow z'/w' = 1$$
$$z = -far \rightarrow z'/w' = -1$$

$$y' = Fy + Bz, \quad \frac{y'}{w'} = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{-z},$$

$$1 = F\frac{y}{-z} + B\frac{z}{-z}, \quad 1 = F\frac{y}{-z} - B, \quad 1 = F\frac{top}{-(-near)} - B,$$

$$1 = F\frac{top}{near} - B$$

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Perspective Derivation

similarly for other 5 planes

6 planes, 6 unknowns

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

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Perspective Example

view volume
- *left = -1, right = 1*
- *bot = -1, top = 1*
- *near = 1, far = 4*

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/3 & -8/3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

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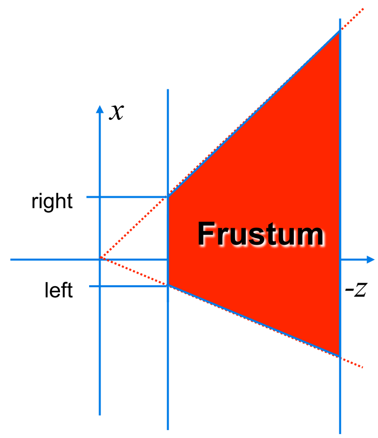
Projective Transformations

Asymmetric Viewing Frusta



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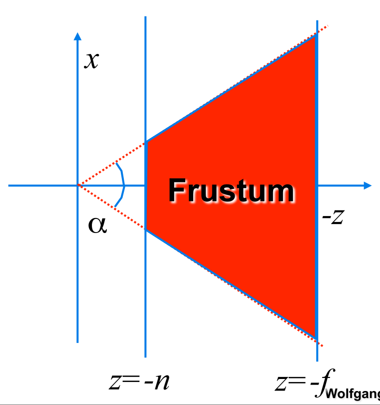
Projective Transformations

Alternative specification of symmetric frusta

- Field-of-view (fov) α
- Fov/2
- Field-of-view in y-direction (fovy) + aspect ratio



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Perspective Matrices in OpenGL

Perspective Matrices:

- glFrustum(left, right, bottom, top, near, far)
  - *Specifies perspective transform (near, far are always positive)*

Convenience Function:

- gluPerspective(fovy, aspect, near, far)
  - *Another way to do perspective*

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Projective Transformations

Properties:

- All transformations that can be expressed as homogeneous 4x4 matrices (in 3D)
- 16 matrix entries, but multiples of the same matrix all describe the same transformation
  - *15 degrees of freedom*
  - *The mapping of 5 points uniquely determines the transformation*

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Projective Transformations

Properties

- Lines are mapped to lines and triangles to triangles
- Parallel lines do **not** remain parallel
  - *E.g. rails vanishing at infinity*
- Affine combinations are **not** preserved
  - *E.g. center of a line does not map to center of projected line (perspective foreshortening)*
  - *The center of a line segment does **not**, in general map to the center of the transformed line segment*
    - Same for other points in triangles

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Orthographic Camera Projection

- Camera's back plane parallel to lens
- Infinite focal length
- No perspective convergence
- Just throw away z values
- OpenGL:
  - *glOrtho*
  - *gluOrtho2D*

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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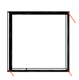
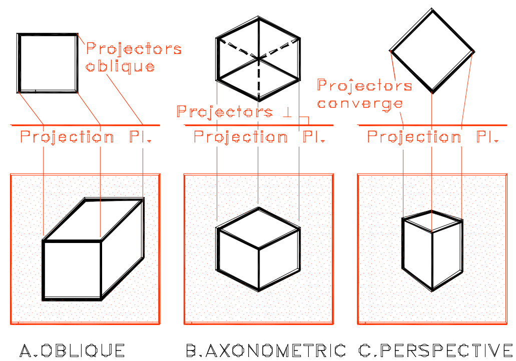
Projection Taxonomy

- planar projections
  - perspective: 1,2,3-point
  - parallel
    - oblique
      - cabinet
      - cavalier
    - orthographic
      - top, front, side
      - axonometric: isometric, dimetric, trimetric





http://ceprofs.tamu.edu/tkramer/ENGR%20111/5.1/20

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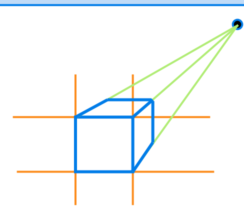
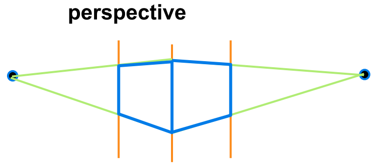
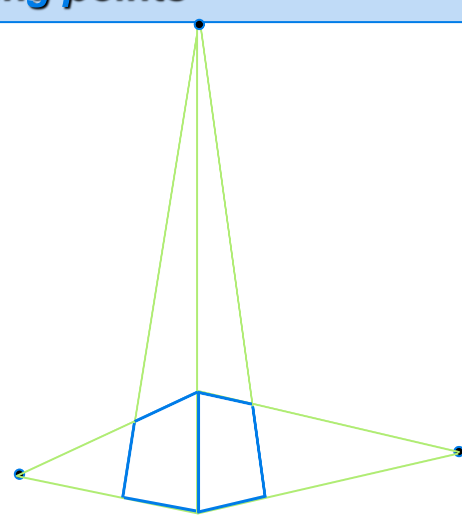
Perspective Projections
classified by vanishing points



one-point perspective



two-point perspective



three-point perspective

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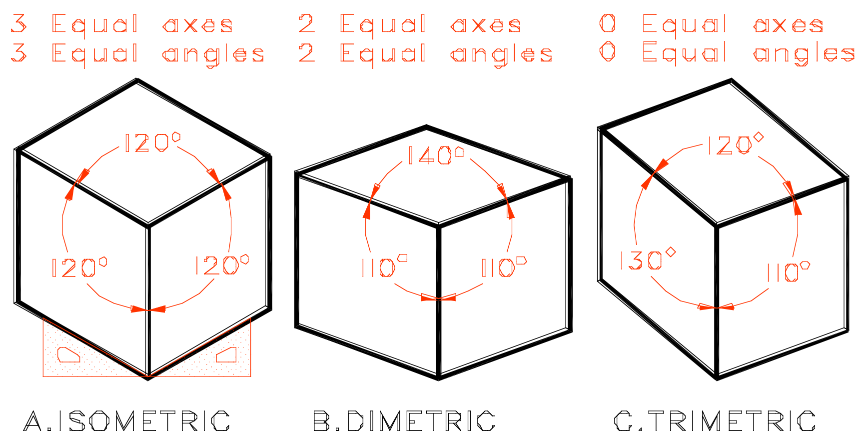
Axonometric Projections

- projectors perpendicular to image plane



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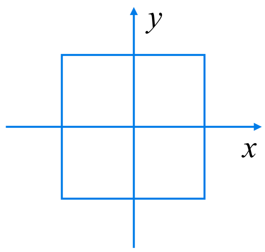
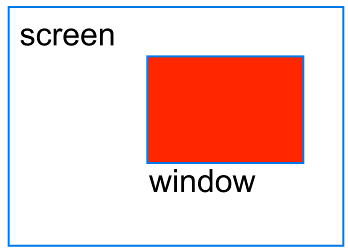
Window-To-Viewport Transformation

Generate pixel coordinates

- Map x, y from range –1…1 (*normalized device coordinates*) to pixel coordinates on the screen
- Map z from -1…1 to 0…1 (used later for visibility)
- Involves 2D scaling and translation





Homogeneous Planes & Normals

Normals & Affine Transformations

Question:

- If we transform some geometry with an affine transformation, how does that affect the normal vector?

Consider

- Rotation
- Translation
- Scaling
- Shear

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Normals & Affine Transformations

Want:

- Representation for normals that allows us to easily describe how they change under affine transformation

Why?

- Normal vectors will be of special interest when we talk about lighting (next week)

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Homogeneous Planes And Normals

Planes in Cartesian Coordinates:

$$\{(x, y, z)^T \mid n_x x + n_y y + n_z z + d = 0\}$$

- n_x, n_y, n_z, and d are the parameters of the plane (normal and distance from origin)

Planes in Homogeneous Coordinates:

$$\{[x, y, z, w]^T \mid n_x x + n_y y + n_z z + dw = 0\}$$

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Homogeneous Planes And Normals

Planes in homogeneous coordinates are represented as row vectors

- E=$[n_x, n_y, n_z, d]$
- Condition that a point $[x,y,z,w]^T$ is located in E

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \in E = [n_x, n_y, n_z, d] \Leftrightarrow [n_x, n_y, n_z, d] \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = 0$$

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Homogeneous Planes And Normals

Transformations of planes

$$[n_x, n_y, n_z, d] \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = 0 \Leftrightarrow T([n_x, n_y, n_z, d]) \cdot (\mathbf{A} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}) = 0$$

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Homogeneous Planes And Normals

Transformations of planes

$$[n_x, n_y, n_z, d] \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = 0 \Leftrightarrow ([n_x, n_y, n_z, d] \cdot \mathbf{A}^{-1}) \cdot (\mathbf{A} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}) = 0$$

- Works for $T([n_x, n_y, n_z, d]) = [n_x, n_y, n_z, d]\mathbf{A}^{-1}$
- Thus: planes have to be transformed by the *inverse* of the affine transformation (multiplied from left as a row vector)!

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Homogeneous Planes And Normals

Homogeneous Normals

- The plane definition also contains its normal
- Normal written as a vector $[n_x, n_y, n_z, 0]^T$

$$\left(\begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix} \cdot \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix}\right) = 0 \Leftrightarrow \left(\left(\mathbf{A}^{-T} \cdot \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}\right) \cdot \left(\mathbf{A} \cdot \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix}\right)\right) = 0$$

- Thus: the normal to any surface has to be transformed by the inverse transpose of the affine transformation (multiplied from the right as a column vector)!

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Transforming Homogeneous Normals

Inverse Transpose of

- Rotation by α
  - *Rotation by α*
- Scale by s
  - *Scale by 1/s*
- Translation by t
  - *Identity matrix!*
- Shear by a along x axis
  - *Shear by -a along y axis*

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Coming Up:

Wednesday:

- Quiz…!

Friday

- Lighting/shading