Home Work 1

① For the ~~first~~ 4 coordinate frames you can simply solve this problem graphically. You'll get

$$P = \frac{1}{3} \cdot x_1 + \left(-\frac{3}{2}\right) y_1 \qquad P = -2 \cdot x_2 - 7 y_2$$

$$\text{i.e } \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ -3/2 \end{pmatrix} \qquad \text{i.e } \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} -2 \\ -7 \end{pmatrix}$$

$$P = 4 x_4 + 2 y_4 \qquad P = -6 x_5 + 5 y_5$$

For coordinate frame #3, the situation is a bit harder, so you need to use the general algebraic approach. i.e:

$$P = a_3 \cdot x_3 + b_3 \cdot y_3$$

$$\begin{pmatrix} 3 \\ 7 \end{pmatrix} = a_3 \begin{pmatrix} 3 \\ 1 \end{pmatrix} + b_3 \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

This gives you 2 equations, 2 unknowns. Solving gives:

$$P = 3 x_3 + 2 y_3$$

Note that this algebraic approach is the general solution, you can solve the other examples the same way.

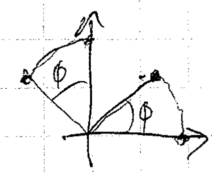
HW 1

(2) For 2x2 linear transformations, you have 4 matrix entries. To determine these 4 entries, you need to observe <u>2</u> points (other than the origin) and how they are transformed by the matrix. (The origin always maps to itself in linear transforms).

a) rotation:

Point $\begin{pmatrix}1\\0\end{pmatrix}$ gets rotated to $\begin{pmatrix}\cos\phi\\ \sin\phi\end{pmatrix}$

Point $\begin{pmatrix}0\\1\end{pmatrix}$ gets rotated to $\begin{pmatrix}-\sin\phi\\ \cos\phi\end{pmatrix}$



i.e: the rotation matrix is $M = \begin{pmatrix}a & b\\ c & d\end{pmatrix}$

with $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\cos\phi\\ \sin\phi\end{pmatrix} \Rightarrow a = \cos\phi \quad c = \sin\phi$

and $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-\sin\phi\\ \cos\phi\end{pmatrix} \Rightarrow b = -\sin\phi \quad d = \cos\phi$

Note: you could choose any points (other than $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$), but the equations would end up being harder to solve. Use points with as many 0s and 1s as possible...

b) scaling:

$$\left.\begin{array}{ll}\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\alpha\\0\end{pmatrix} & \text{(scaling by } \alpha \text{ in } x)\\ \begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\ \beta\end{pmatrix} & \text{(scaling by } \beta \text{ in } y)\end{array}\right\} \Rightarrow \begin{array}{l} a = \alpha\\ d = \beta\\ b = c = 0\end{array}$$

c) shear

$\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \Rightarrow a = 1, \; c = 0$

$\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1/2\\1\end{pmatrix} \Rightarrow b = \frac{1}{2}, \; d = 1$

HW 1

(3) You need 3 points (2 for the linear part, 1 for the translation)

(4) a) Just do this for 2 points:

$$T\left((1-a)x_1 + a x_2\right)$$
$$= M\left((1-a)x_1 + a x_2\right) + t \qquad \text{since } T(x) = Mx + t$$

$\uparrow$ cartesian matrix (~~Not~~ Not homogenous)

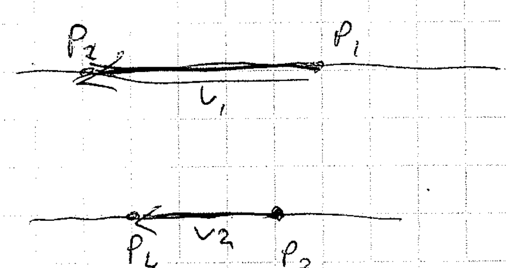
$$= (1-a)Mx_1 + aMx_2 + (1-a)t + a\cdot t$$
$$= (1-a)(Mx_1 + t) + a(Mx_2 + t)$$
$$= (1-a)\,T(x_1) + a\,T(x_2) .$$

For more than one point, this works the same way.

You can always write t as $t = \sum_{i=0}^{n} a_i t$ <u>iff</u> $\sum_{i=0}^{n} a_i = 1$!

b) Parallel lines are given as pairs of 2 points:

$\boxed{\text{parallel means: } v_1 = k\cdot v_2}$



i.e $x_2 - x_1 = k\cdot(x_4 - x_3)$

$\Leftrightarrow M(x_2 - x_1) = k\cdot M(x_4 - x_3)$

$\Rightarrow Mx_2 - Mx_1 = k(Mx_4 - Mx_3)$

$\Rightarrow \underbrace{(Mx_2 + t)}_{T(x_2)} - \underbrace{(Mx_1 + t)}_{T(x_1)} = k\left(\underbrace{(Mx_4 + t)}_{T(x_4)} - \underbrace{(Mx_3 + t)}_{T(x_3)}\right)$

$T(x_2) - T(x_1) = k\cdot\left(T(x_4) - T(x_3)\right)$

$\rightarrow$ i.e, the transformed points are still parallel!