



University of British Columbia  
CPSC 314 Computer Graphics  
Jan-Apr 2010

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**Lighting/Shading I**

**Week 6, Fri Feb 12**

<http://www.ugrad.cs.ubc.ca/~cs314/Vjan2010>

# Correction: W2V vs. V2W

slide 38 week3.day3 (Fri Jan 22)

- $M_{W2V} = TR$       $T = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$       $R = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- we derived position of camera in world
  - invert for world with respect to camera
- $M_{V2W} = (M_{W2V})^{-1} = R^{-1}T^{-1}$

$$M_{view2world} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z & -\mathbf{e} \cdot \mathbf{u} \\ v_x & v_y & v_z & -\mathbf{e} \cdot \mathbf{v} \\ w_x & w_y & w_z & -\mathbf{e} \cdot \mathbf{w} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Correction: W2V vs. V2W

- $M_{V2W} = (M_{W2V})^{-1} = R^{-1}T^{-1}$

$$M_{view2world} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z & -\mathbf{e} \cdot \mathbf{u} \\ v_x & v_y & v_z & -\mathbf{e} \cdot \mathbf{v} \\ w_x & w_y & w_z & -\mathbf{e} \cdot \mathbf{w} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{V2W} = \begin{bmatrix} u_x & u_y & u_z & -e_x * u_x + -e_y * u_y + -e_z * u_z \\ v_x & v_y & v_z & -e_x * v_x + -e_y * v_y + -e_z * v_z \\ w_x & w_y & w_z & -e_x * w_x + -e_y * w_y + -e_z * w_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Correction: Perspective Derivation

slide 30 week4.day3 (Fri Jan 29)

z axis flip!

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = Ex + Az$$

$$y' = Fy + Bz$$

$$z' = Cz + D$$

$$w' = -z$$

$$x = \text{left} \rightarrow x'/w' = -1$$

$$x = \text{right} \rightarrow x'/w' = 1$$

$$y = \text{top} \rightarrow y'/w' = 1$$

$$y = \text{bottom} \rightarrow y'/w' = -1$$

$$z = -\text{near} \rightarrow z'/w' = 1$$

$$z = -\text{far} \rightarrow z'/w' = -1$$

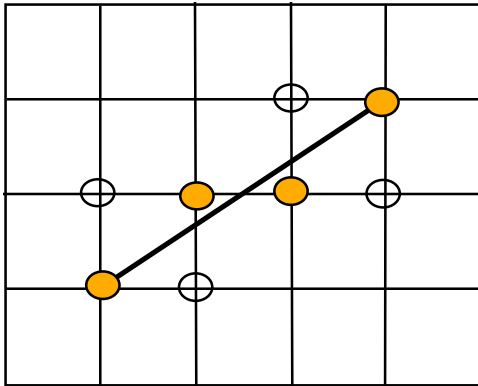
$$y' = Fy + Bz, \quad \frac{y'}{w'} = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{-z},$$

$$1 = F \frac{y}{-z} + B \frac{z}{-z}, \quad 1 = F \frac{y}{-z} - B, \quad 1 = F \frac{\text{top}}{-(-\text{near})} - B,$$

$$1 = F \frac{\text{top}}{\text{near}} - B$$

# News

- P2 due date extended to Tue Mar 2 5pm
  - V2W correction affects Q1 and thus cascades to Q4-Q7
  - perspective correction affects Q8
- TA office hours in lab for P2/H2 questions Fri 2-4 (Garrett)



# PPT Fix: Basic Line Drawing

$$y = mx + b$$

$$y = \frac{(y_1 - y_0)}{(x_1 - x_0)}(x - x_0) + y_0$$

- goals
  - integer coordinates
  - thinnest line with no gaps
- assume
  - $x_0 < x_1$ , slope  $0 < \frac{dy}{dx} < 1$
  - one octant, other cases symmetric
- how can we do this more quickly?

```

Line (  $x_0, y_0, x_1, y_1$  )
begin
float  $dx, dy, x, y, slope$  ;
 $dx \Leftarrow x_1 - x_0$ ;
 $dy \Leftarrow y_1 - y_0$ ;
 $slope \Leftarrow \frac{dy}{dx}$ ;

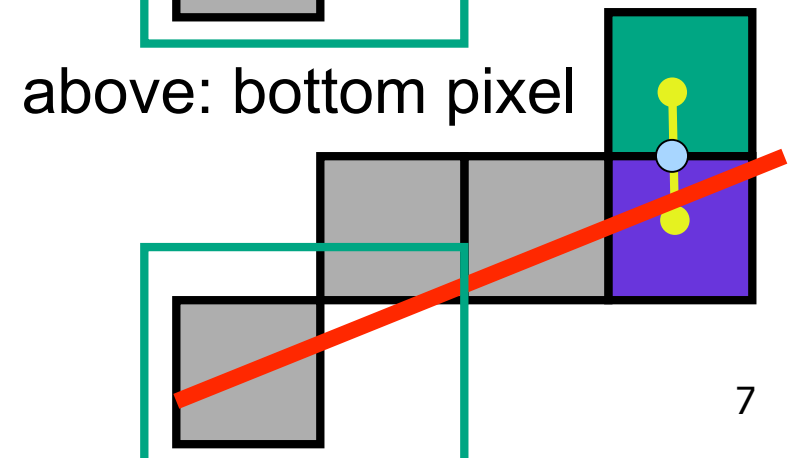
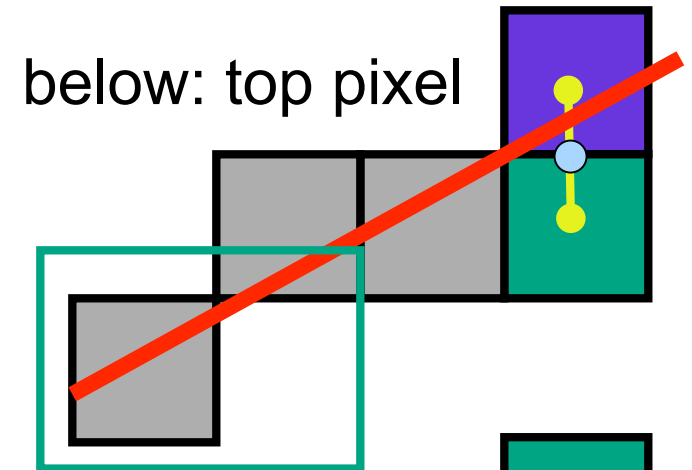
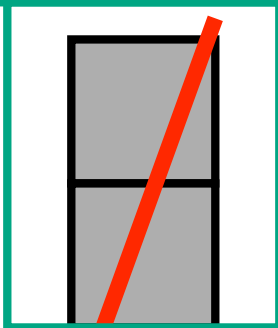
 $y \Leftarrow y_0$ 
for  $x$  from  $x_0$  to  $x_1$  do
  begin
    PlotPixel (  $x, \mathbf{Round}(y)$  ) ;
     $y \Leftarrow y + slope$  ;
  end ;
end ;

```

# Clarification/Correction II: Midpoint

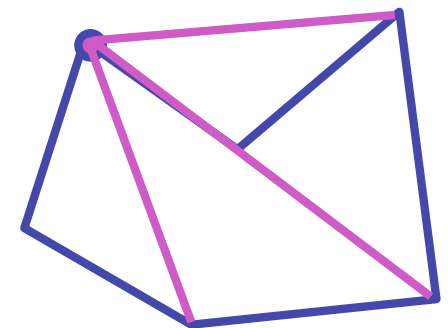
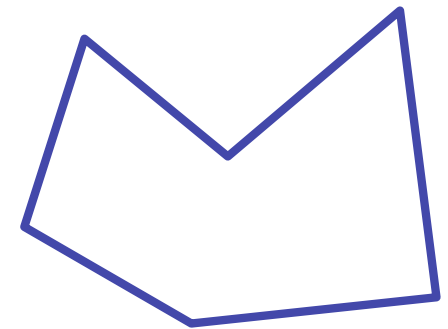
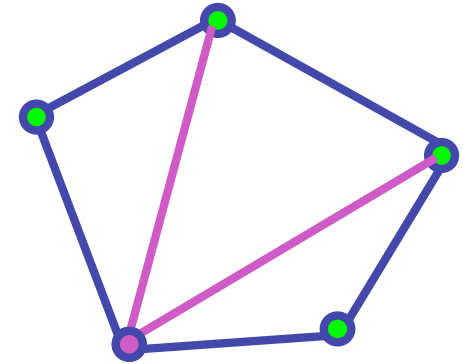
- we're moving horizontally along x direction (first octant)
  - only two choices: draw at current y value, or move up vertically to  $y+1$ ?
    - check if midpoint between two possible pixel centers above or below line
  - candidates
    - top pixel:  $(x+1, y+1)$
    - bottom pixel:  $(x+1, y)$
  - midpoint:  $(x+1, y+.5)$
- check if midpoint above or below line
  - below: pick top pixel
  - above: pick bottom pixel

- other octants: different tests
  - octant II: y loop, check x left/right



# Review: Triangulating Polygons

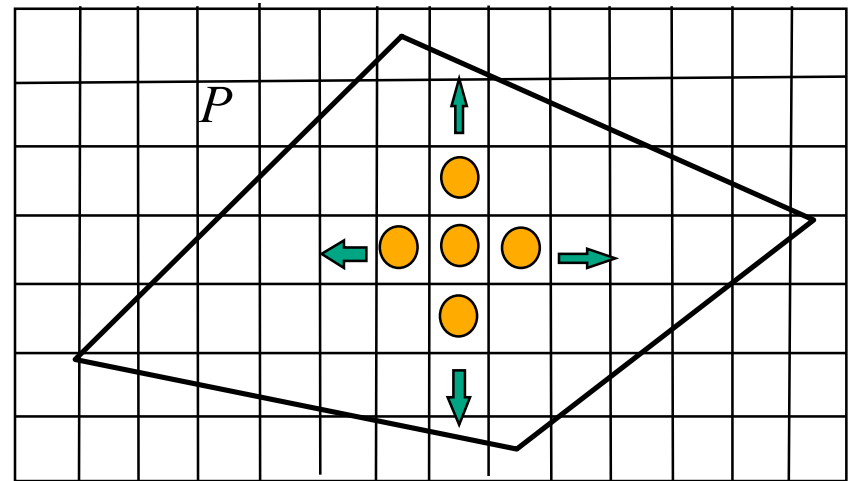
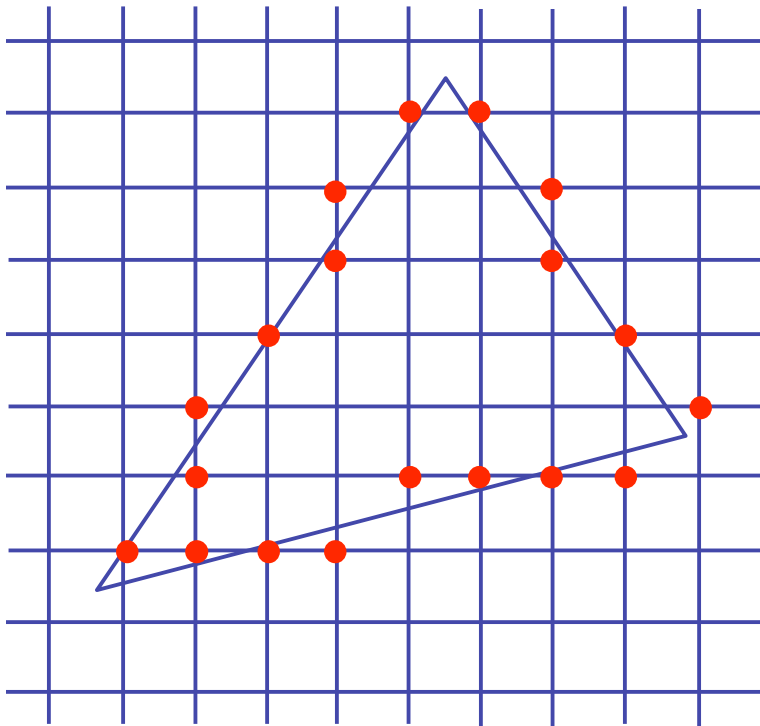
- simple convex polygons
  - trivial to break into triangles
  - pick one vertex, draw lines to all others not immediately adjacent
  - OpenGL supports automatically
    - `glBegin(GL_POLYGON) ... glEnd()`
- concave or non-simple polygons
  - more effort to break into triangles
  - simple approach may not work
  - OpenGL can support at extra cost
    - `gluNewTess(), gluTessCallback(), ...`





# Review: Flood Fill

- simple algorithm
  - draw edges of polygon
  - use flood-fill to draw interior



# PPT Fix: Flood Fill

- draw edges
- run:

**FloodFill**(Polygon P, int  $x$ , int  $y$ , Color  $C$ )

if not (**OnBoundary**( $x,y,P$ ) or **Colored**( $x,y,C$ ))

begin

**PlotPixel**( $x,y,C$ );

**FloodFill**(P, $x + 1,y,C$ );

**FloodFill**(P, $x,y + 1,C$ );

**FloodFill**(P, $x,y - 1,C$ );

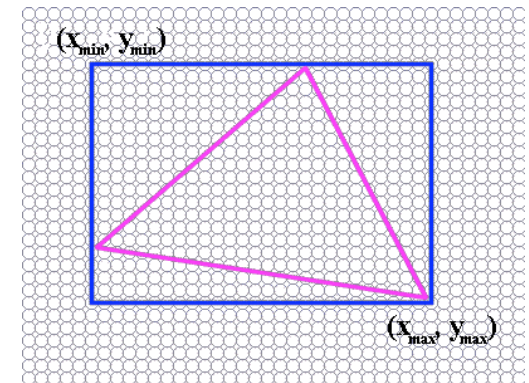
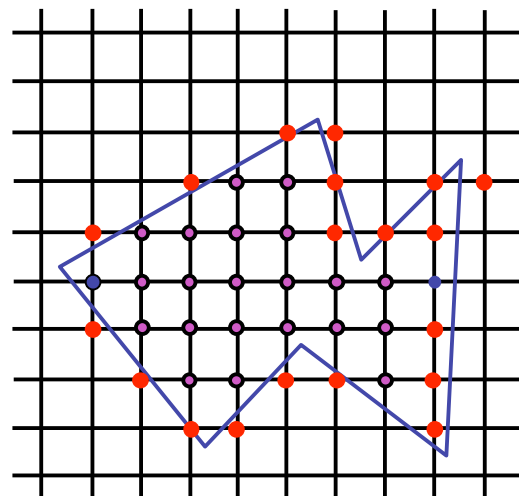
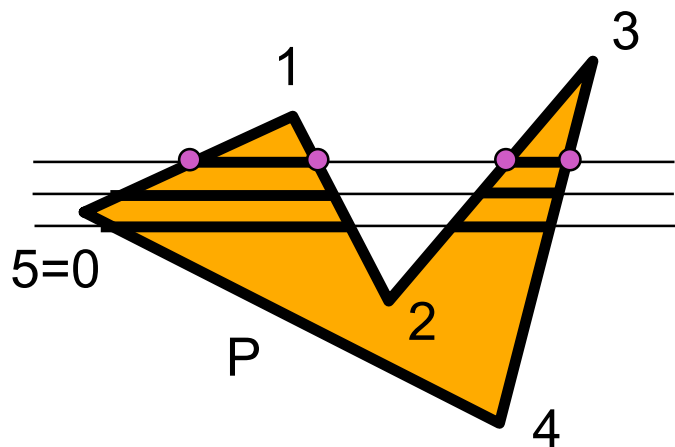
**FloodFill**(P, $x - 1,y,C$ );

end ;

- drawbacks?

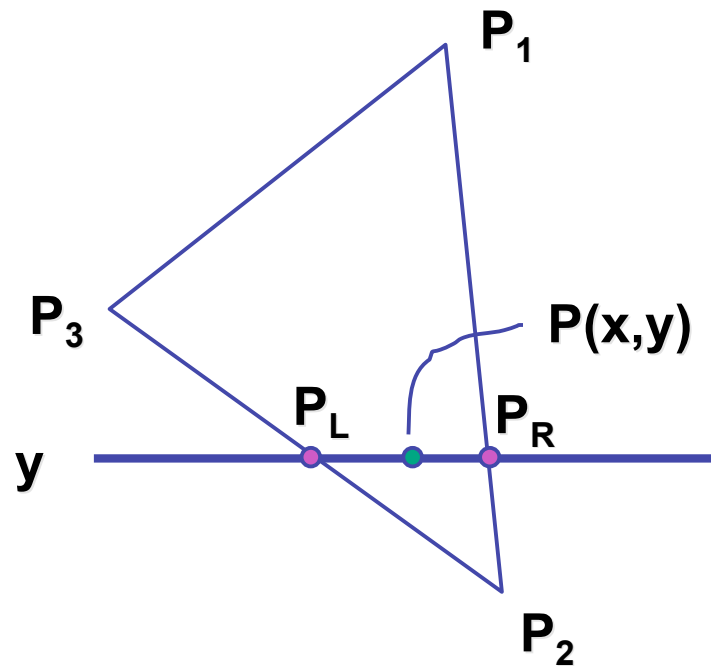
# Review: Scanline Algorithms

- **scanline**: a line of pixels in an image
  - set pixels inside polygon boundary along horizontal lines one pixel apart vertically
    - parity test: draw pixel if edgecount is odd
    - optimization: only loop over axis-aligned bounding box of  $x_{min}/x_{max}$ ,  $y_{min}/y_{max}$



# Review: Bilinear Interpolation

- interpolate quantity along  $L$  and  $R$  edges, as a function of  $y$ 
  - then interpolate quantity as a function of  $x$



# Review: Barycentric Coordinates

- non-orthogonal coordinate system based on triangle itself
  - origin:  $P_1$ , basis vectors:  $(P_2 - P_1)$  and  $(P_3 - P_1)$

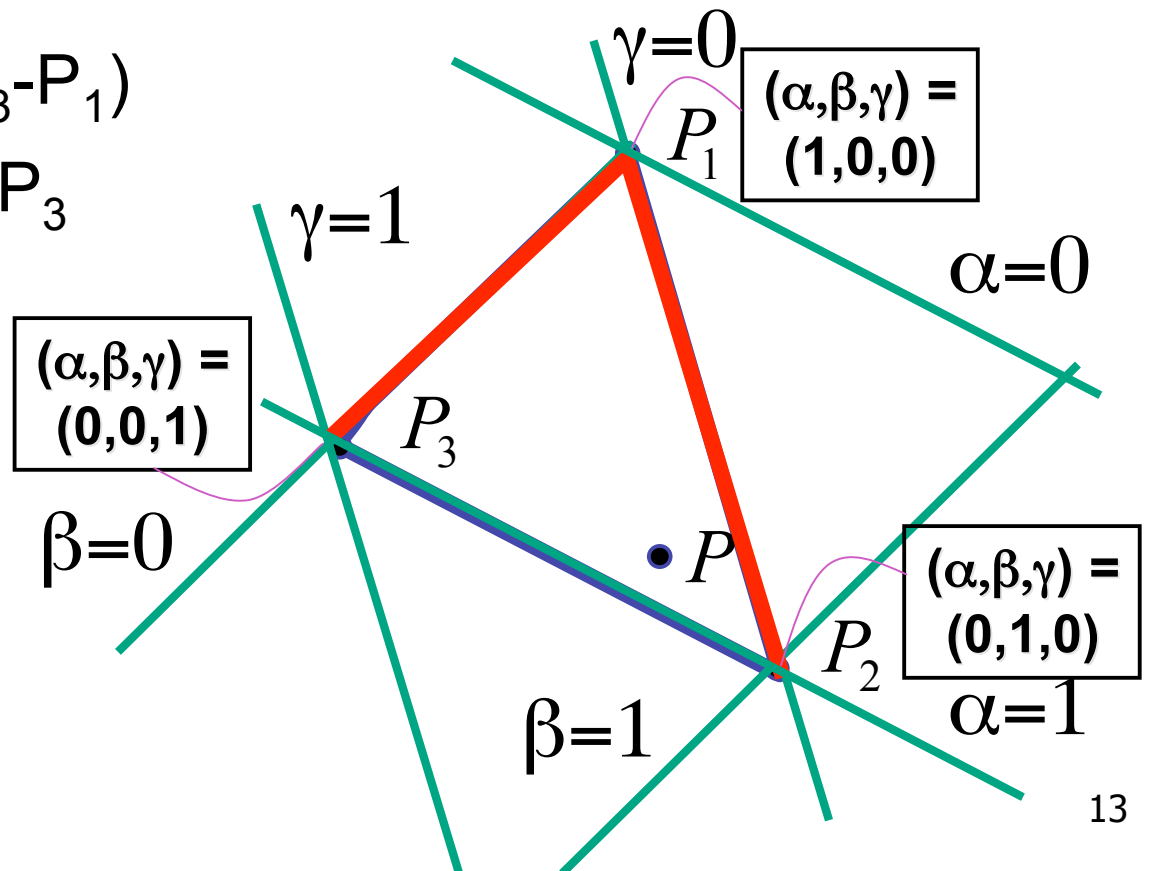
$$P = P_1 + \beta(P_2 - P_1) + \gamma(P_3 - P_1)$$

$$P = (1 - \beta - \gamma)P_1 + \beta P_2 + \gamma P_3$$

$$P = \alpha P_1 + \beta P_2 + \gamma P_3$$

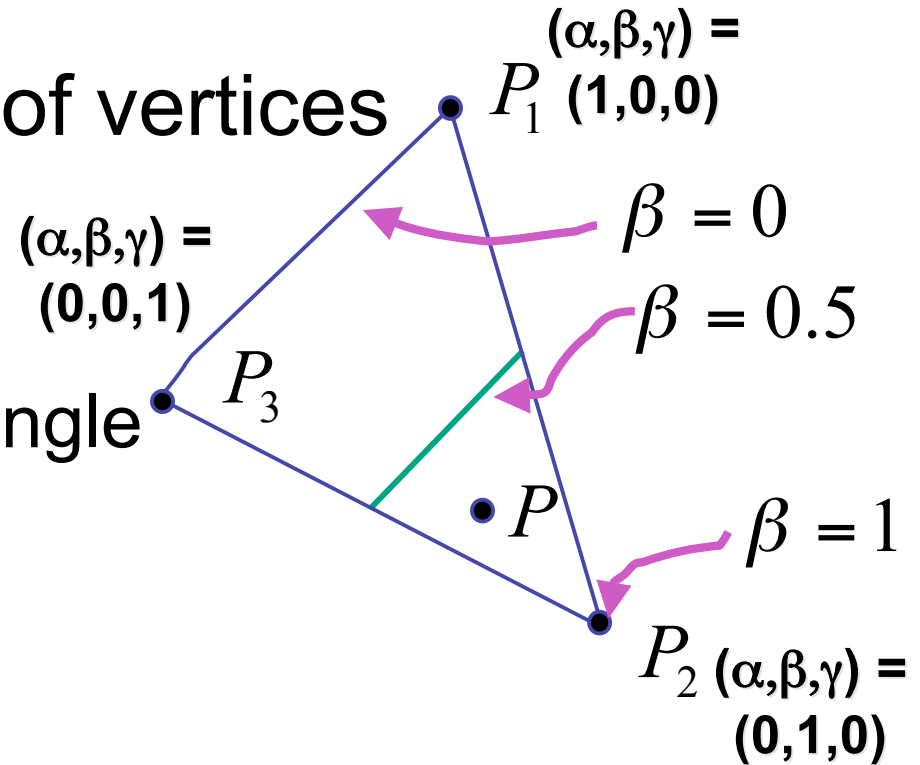
$$\alpha + \beta + \gamma = 1$$

$$0 \leq \alpha, \beta, \gamma \leq 1$$



# Using Barycentric Coordinates

- weighted combination of vertices
- smooth mixing
- speedup
  - compute once per triangle



$$\left\{ \begin{array}{l} P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3 \\ \alpha + \beta + \gamma = 1 \\ 0 \leq \alpha, \beta, \gamma \leq 1 \text{ for points inside triangle} \end{array} \right.$$

“convex combination of points”

- demo

<http://www.cut-the-knot.org/Curriculum/Geometry/Barycentric.shtml>

# Computing Barycentric Coordinates

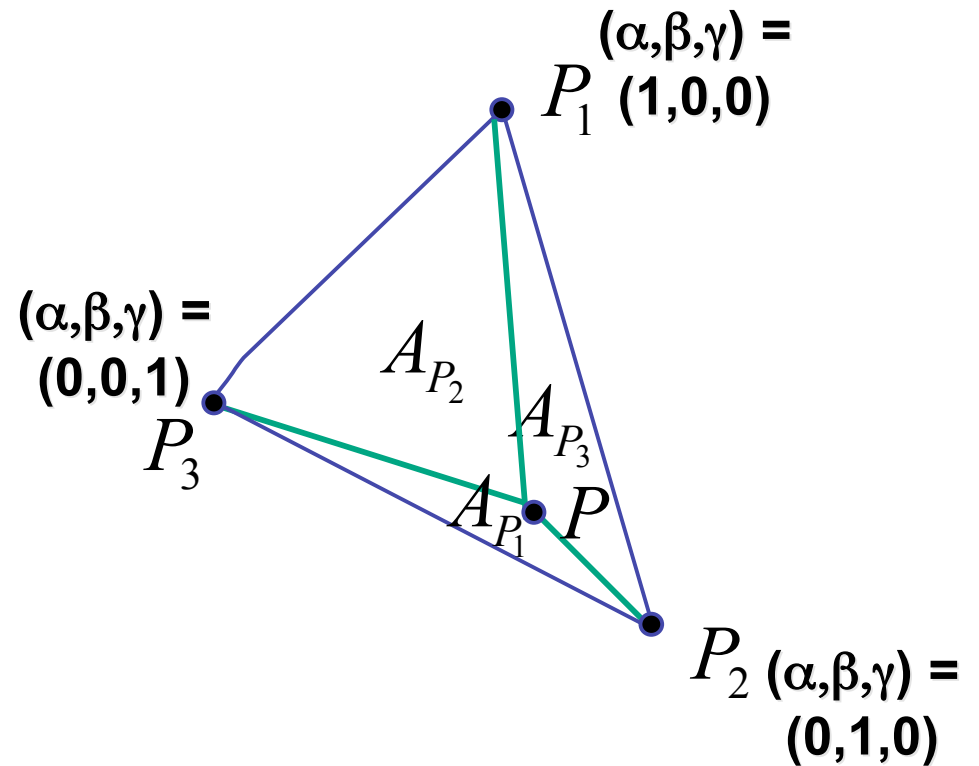
- 2D triangle area
  - half of parallelogram area
    - from cross product

$$A = A_{P_1} + A_{P_2} + A_{P_3}$$

$$\alpha = A_{P_1} / A$$

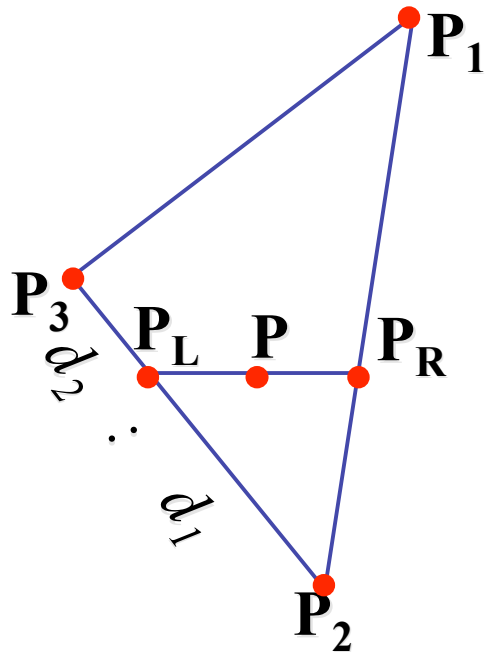
$$\beta = A_{P_2} / A$$

$$\gamma = A_{P_3} / A$$



# Deriving Barycentric From Bilinear

- from bilinear interpolation of point P on scanline

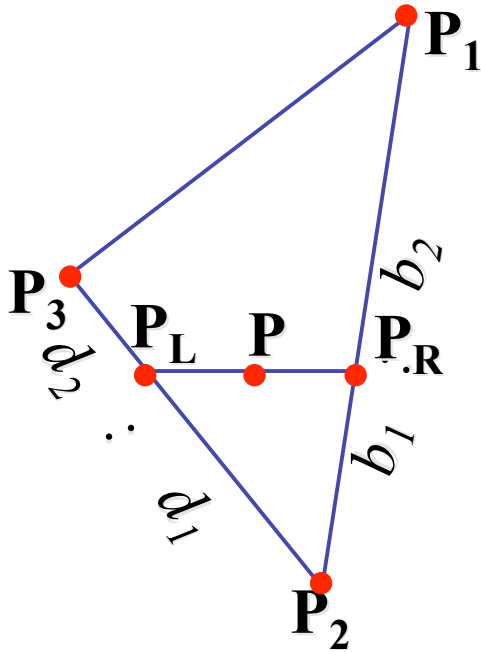


$$\begin{aligned} P_L &= P_2 + \frac{d_1}{d_1 + d_2} (P_3 - P_2) \\ &= \left(1 - \frac{d_1}{d_1 + d_2}\right) P_2 + \frac{d_1}{d_1 + d_2} P_3 = \\ &= \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3 \end{aligned}$$



# Deriving Barycentric From Bilinear

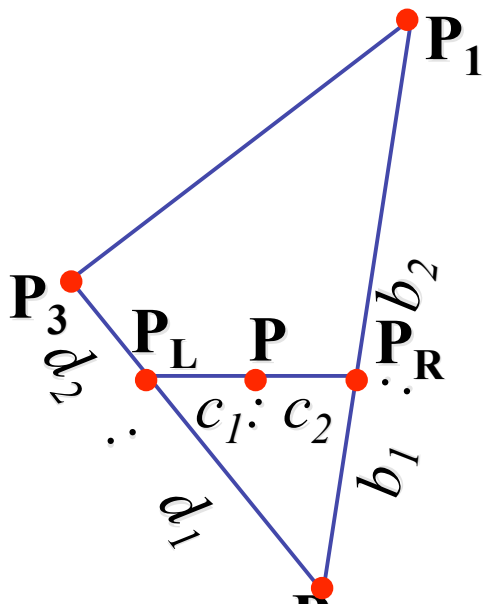
- similarly



$$\begin{aligned}
 P_R &= P_2 + \frac{b_1}{b_1 + b_2} (P_1 - P_2) \\
 &= \left(1 - \frac{b_1}{b_1 + b_2}\right) P_2 + \frac{b_1}{b_1 + b_2} P_1 = \\
 &= \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1
 \end{aligned}$$

# Deriving Barycentric From Bilinear

- combining



$$P = \frac{c_2}{c_1 + c_2} \cdot P_L + \frac{c_1}{c_1 + c_2} \cdot P_R$$

$$P_L = \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3$$

$$P_R = \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1$$

- gives  $P_2$

$$P = \frac{c_2}{c_1 + c_2} \left( \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3 \right) + \frac{c_1}{c_1 + c_2} \left( \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1 \right)$$

# Deriving Barycentric From Bilinear

- thus  $P = \alpha P_1 + \beta P_2 + \gamma P_3$  with

$$\alpha = \frac{c_1}{c_1 + c_2} \frac{b_1}{b_1 + b_2}$$

$$\beta = \frac{c_2}{c_1 + c_2} \frac{d_2}{d_1 + d_2} + \frac{c_1}{c_1 + c_2} \frac{b_2}{b_1 + b_2}$$

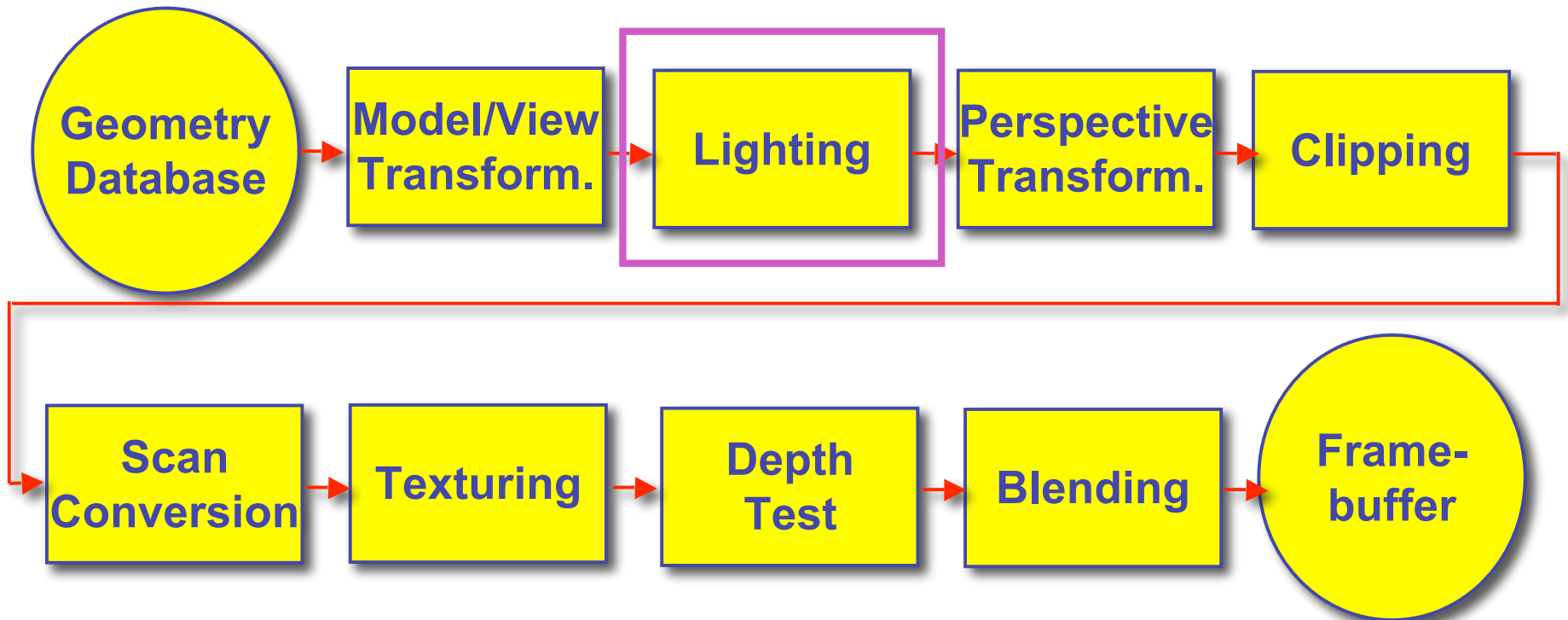
$$\gamma = \frac{c_2}{c_1 + c_2} \frac{d_1}{d_1 + d_2}$$

- can verify barycentric properties

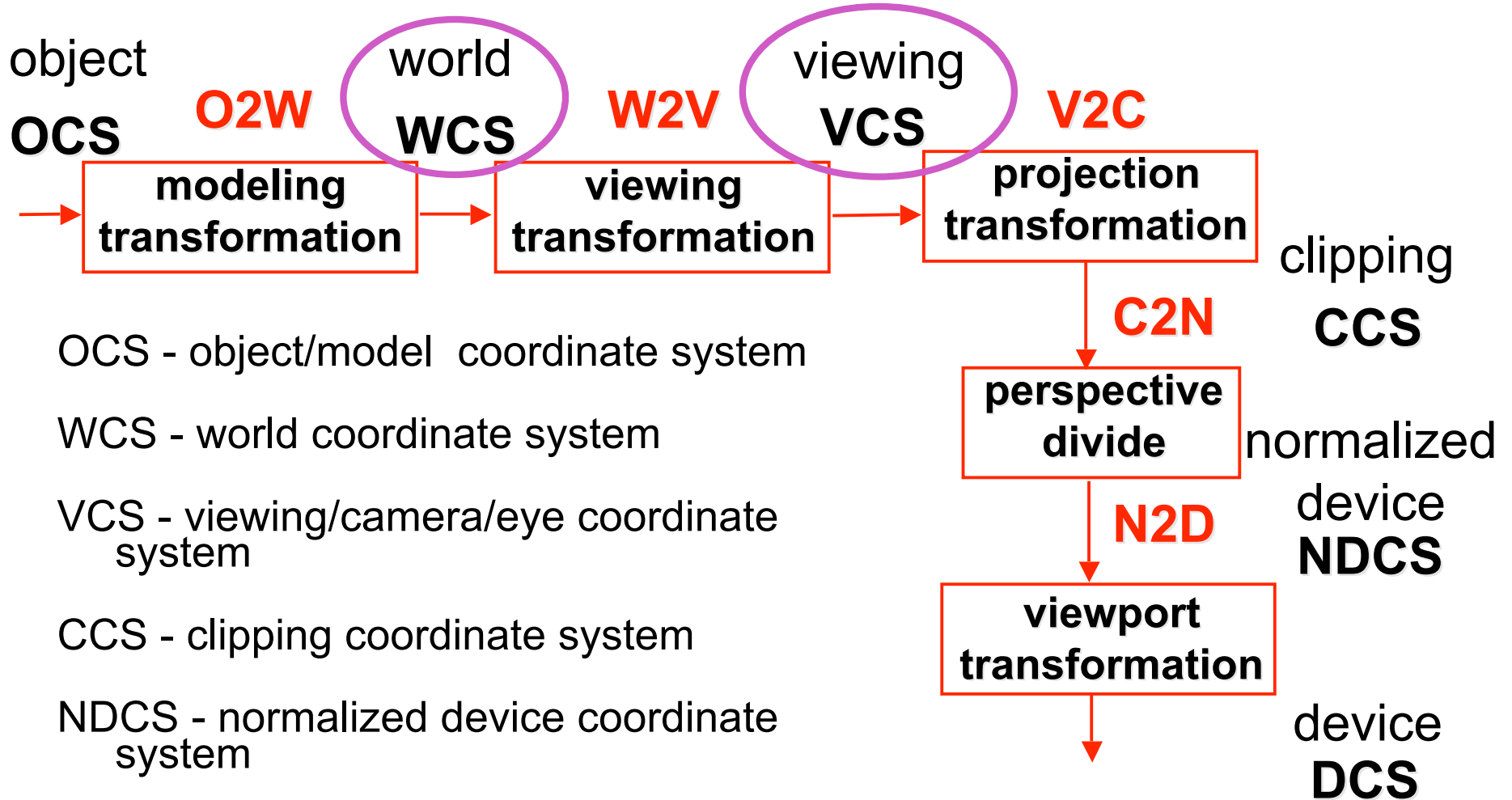
$$\alpha + \beta + \gamma = 1, \quad 0 \leq \alpha, \beta, \gamma \leq 1$$

# Lighting I

# Rendering Pipeline



# Projective Rendering Pipeline



OCS - object/model coordinate system

WCS - world coordinate system

VCS - viewing/camera/eye coordinate system

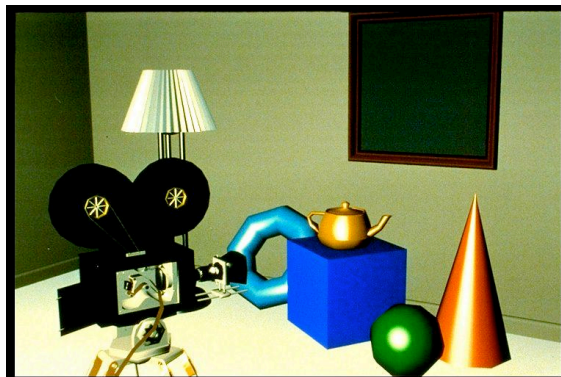
CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device/display/screen coordinate system

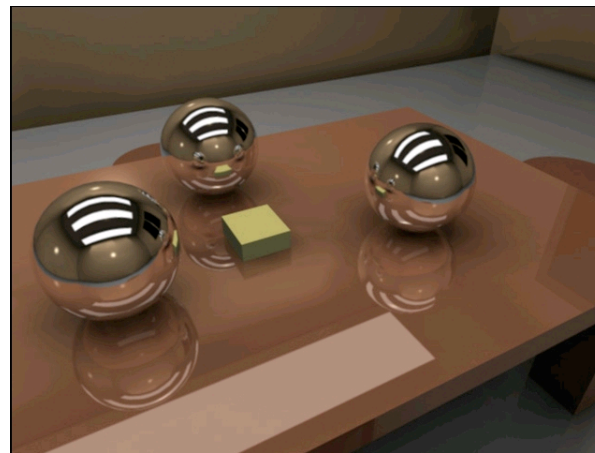
# Goal

- simulate interaction of light and objects
- fast: fake it!
  - approximate the look, ignore real physics
- get the physics (more) right
  - BRDFs: Bidirectional Reflection Distribution Functions
- local model: interaction of each object with light
- global model: interaction of objects with each other

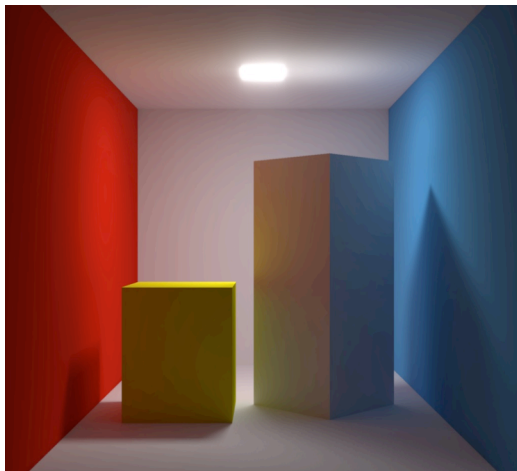


# Photorealistic Illumination

- transport of energy from light sources to surfaces & points
  - global includes direct and indirect illumination – more later



[electricimage.com]



Henrik Wann Jensen



# Illumination in the Pipeline

- local illumination
  - only models light arriving directly from light source
  - no interreflections or shadows
    - can be added through tricks, multiple rendering passes
- light sources
  - simple shapes
- materials
  - simple, non-physical reflection models

# Light Sources

- types of light sources

- `glLightfv(GL_LIGHT0, GL_POSITION, light[])`

- directional/parallel lights

- real-life example: sun

- infinitely far source: homogeneous coord  $w=0$

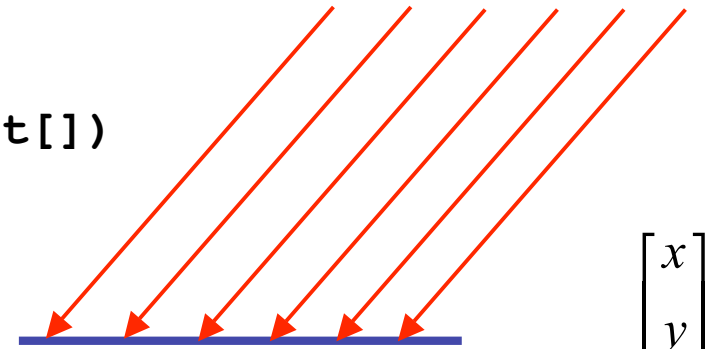
- point lights

- same intensity in all directions

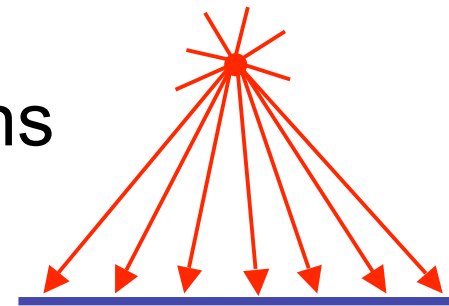
- spot lights

- limited set of directions:

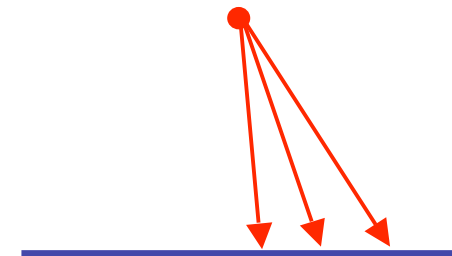
- point+direction+cutoff angle



$$\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

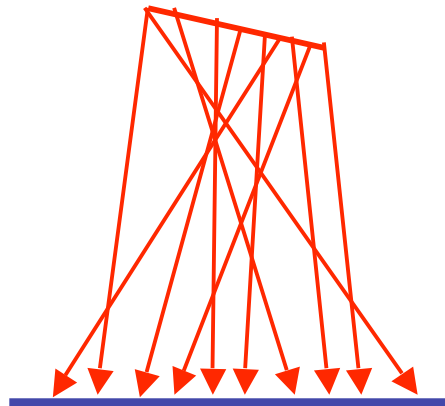


$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



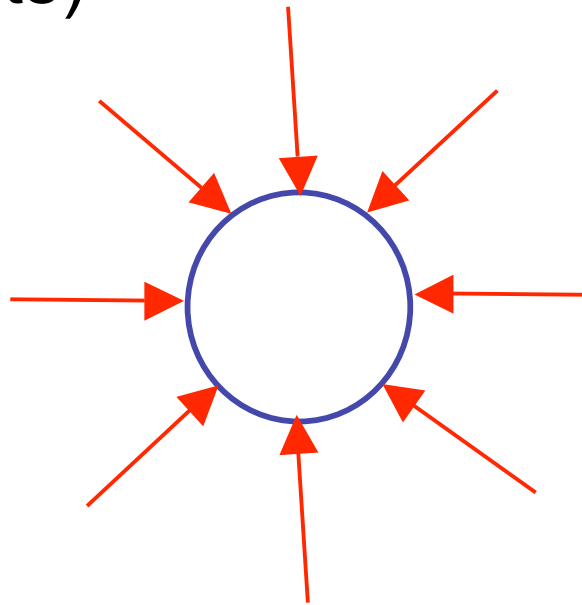
# Light Sources

- area lights
  - light sources with a finite area
  - more realistic model of many light sources
  - not available with projective rendering pipeline (i.e., not available with OpenGL)



# Light Sources

- ambient lights
  - no identifiable source or direction
  - hack for replacing true global illumination
    - (diffuse interreflection: light bouncing off from other objects)

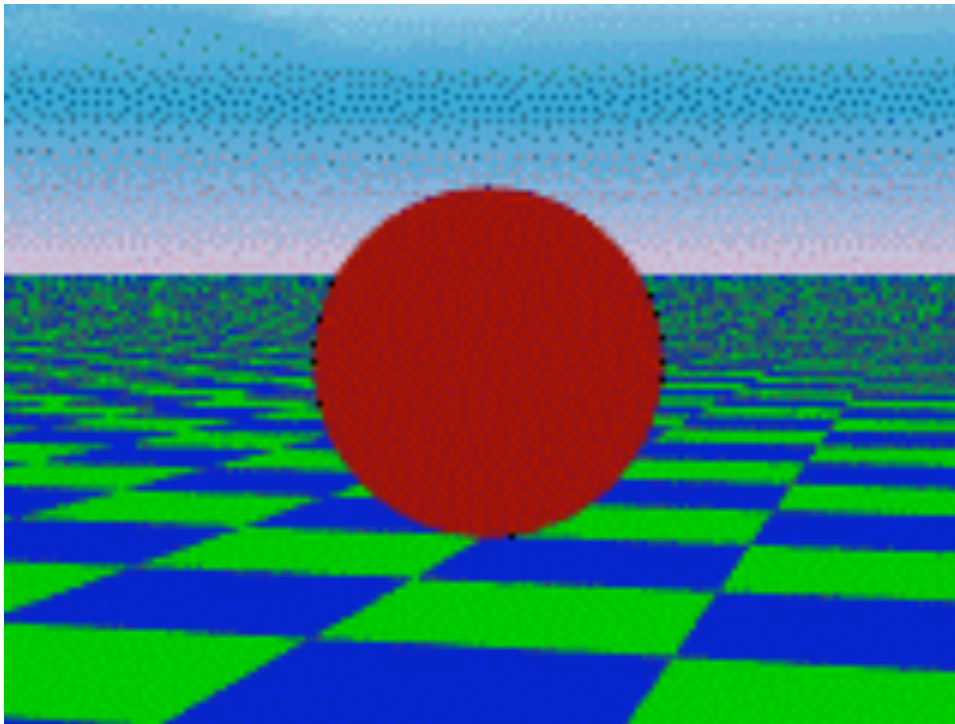


# Diffuse Interreflection



# Ambient Light Sources

- scene lit only with an ambient light source



Light Position  
Not Important

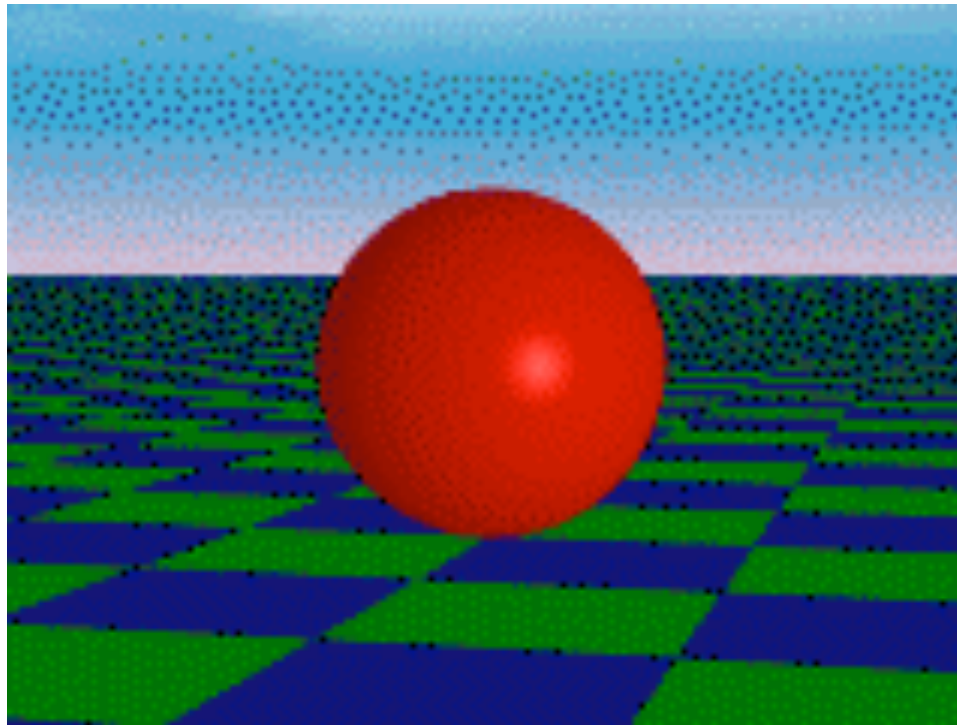
Viewer Position  
Not Important

Surface Angle  
Not Important

# Directional Light Sources

- scene lit with directional and ambient light

Surface Angle  
Important



Light Position  
Not Important

Viewer Position  
Not Important

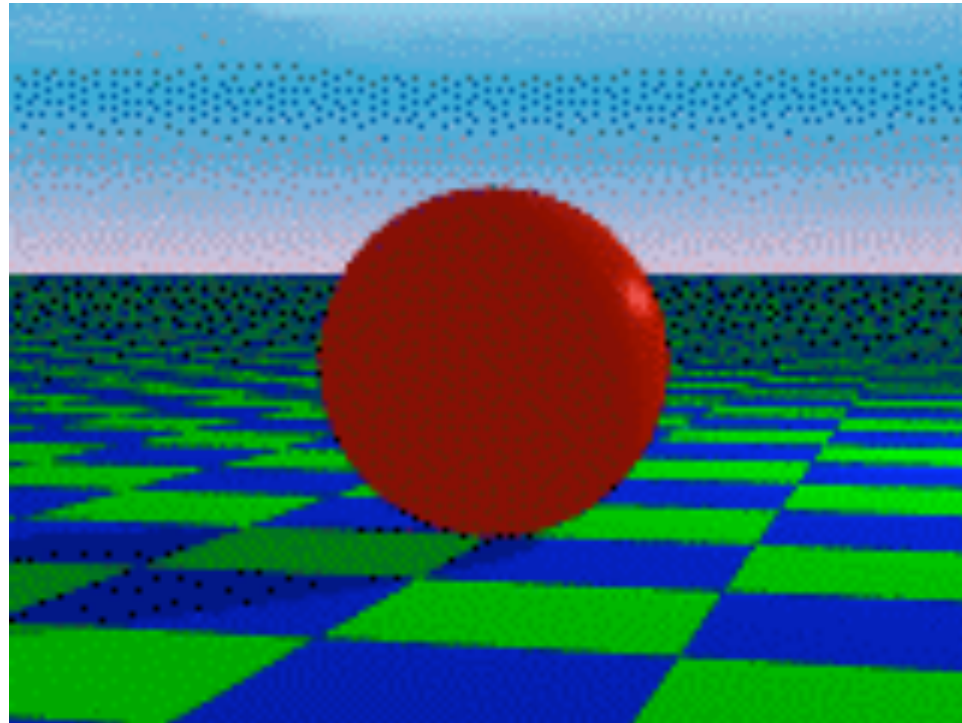
# Point Light Sources

- scene lit with ambient and point light source

Light Position  
Important

Viewer Position  
Important

Surface Angle  
Important



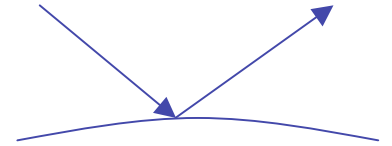


# Light Sources

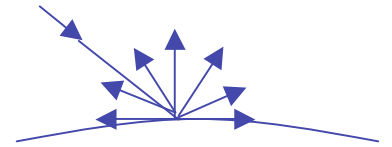
- geometry: positions and directions
  - standard: world coordinate system
    - effect: lights fixed wrt world geometry
    - demo:  
<http://www.xmission.com/~nate/tutors.html>
  - alternative: camera coordinate system
    - effect: lights attached to camera (car headlights)
  - points and directions undergo normal model/view transformation
- illumination calculations: camera coords

# Types of Reflection

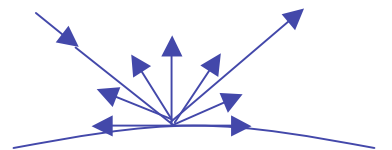
- *specular* (a.k.a. *mirror* or *regular*) reflection causes light to propagate without scattering.



- *diffuse* reflection sends light in all directions with equal energy.



- *mixed* reflection is a weighted combination of specular and diffuse.

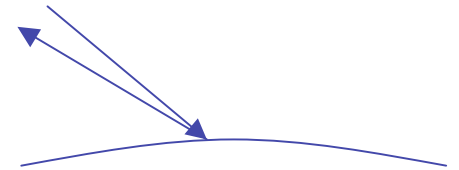


# Specular Highlights

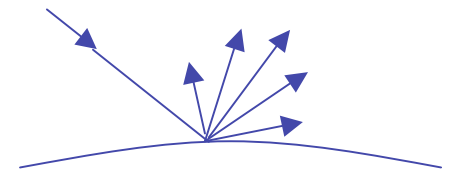


# Types of Reflection

- *retro-reflection* occurs when incident energy reflects in directions close to the incident direction, for a wide range of incident directions.

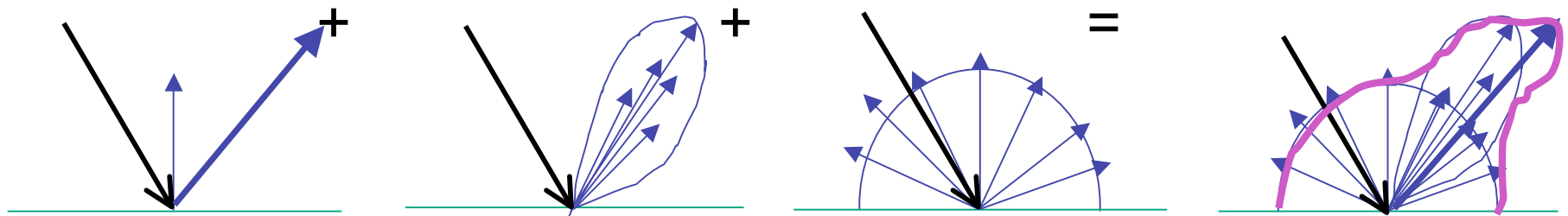


- *gloss* is the property of a material surface that involves mixed reflection and is responsible for the mirror like appearance of rough surfaces.



# Reflectance Distribution Model

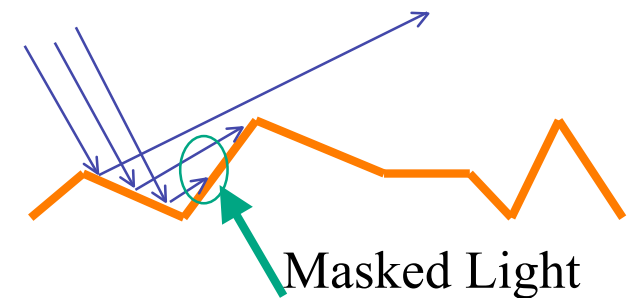
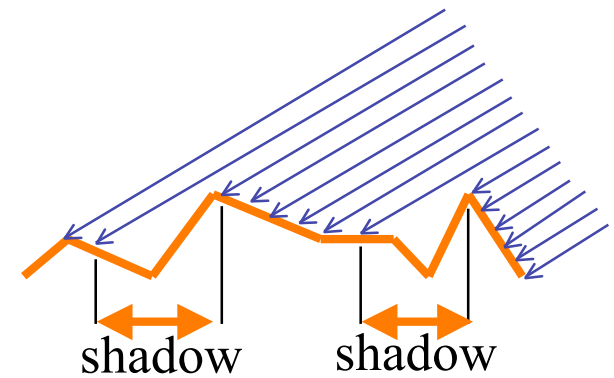
- most surfaces exhibit complex reflectances
  - vary with incident and reflected directions.
  - model with combination



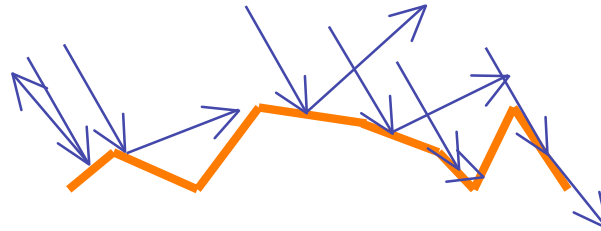
specular + glossy + diffuse =  
reflectance distribution

# Surface Roughness

- at a microscopic scale, all real surfaces are rough
- cast shadows on themselves
- “mask” reflected light:



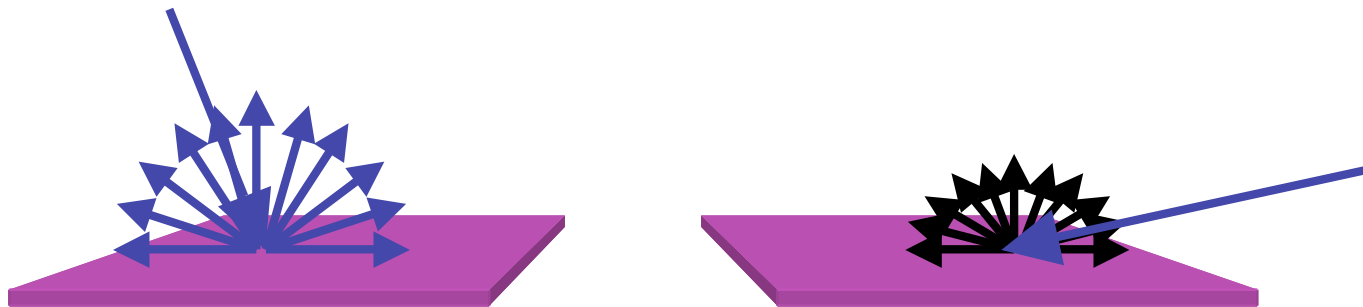
# Surface Roughness



- notice another effect of roughness:
  - each “microfacet” is treated as a perfect mirror.
  - incident light reflected in different directions by different facets.
  - end result is mixed reflectance.
    - smoother surfaces are more specular or glossy.
    - random distribution of facet normals results in diffuse reflectance.

# Physics of Diffuse Reflection

- ideal diffuse reflection
  - very rough surface at the microscopic level
    - real-world example: chalk
  - microscopic variations mean incoming ray of light equally likely to be reflected in any direction over the hemisphere
  - what does the reflected intensity depend on?





# Lambert's Cosine Law

- ideal diffuse surface reflection

the energy reflected by a small portion of a surface from a light source in a given direction is proportional to the cosine of the angle between that direction and the surface normal

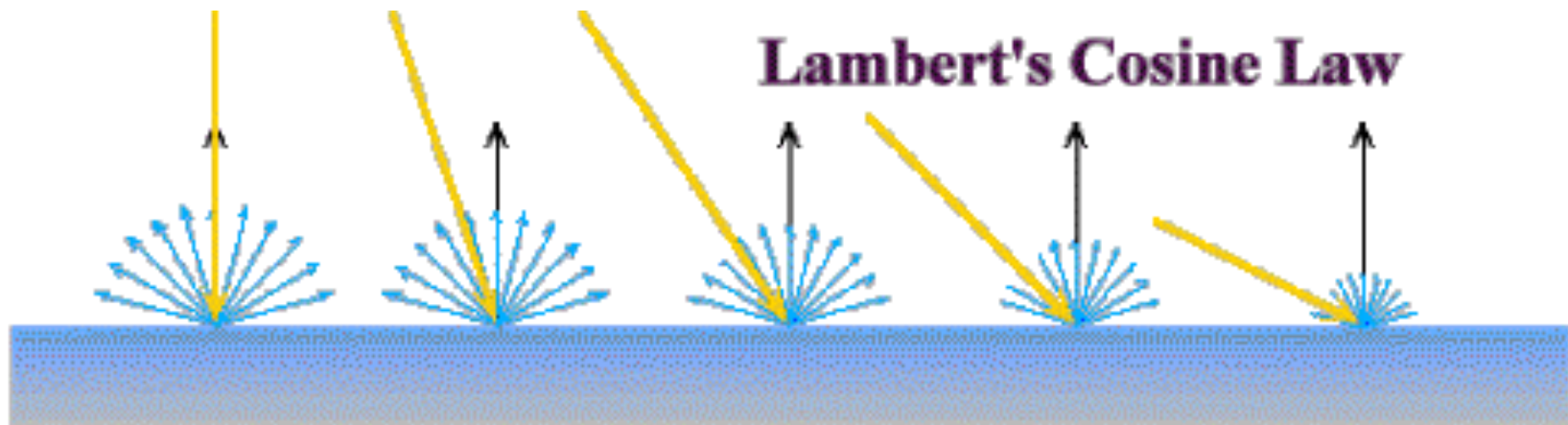
- **reflected** intensity

- independent of **viewing** direction

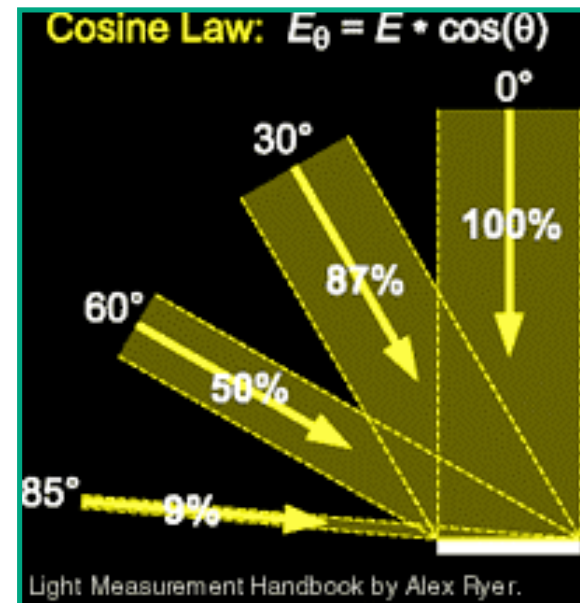
- depends on surface orientation wrt light

- often called **Lambertian surfaces**

# Lambert's Law



intuitively: cross-sectional area of the “beam” intersecting an element of surface area is smaller for greater angles with the normal.



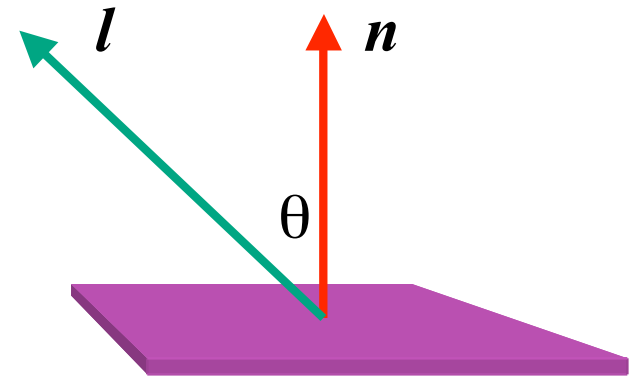
# Computing Diffuse Reflection

- depends on **angle of incidence**: angle between surface normal and incoming light

- $I_{\text{diffuse}} = k_d I_{\text{light}} \cos \theta$

- in practice use vector arithmetic

- $I_{\text{diffuse}} = k_d I_{\text{light}} (\mathbf{n} \cdot \mathbf{l})$



- always normalize vectors used in lighting!!!

- $\mathbf{n}$ ,  $\mathbf{l}$  should be unit vectors

- scalar (B/W intensity) or 3-tuple or 4-tuple (color)

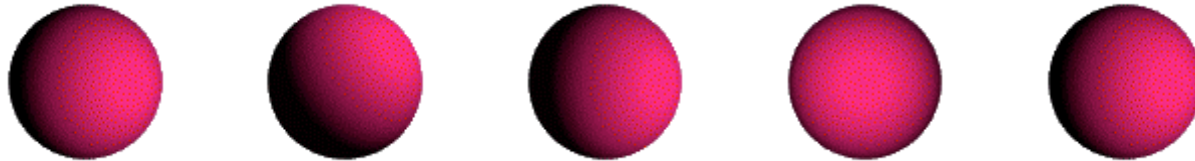
- $k_d$ : diffuse coefficient, surface color

- $I_{\text{light}}$ : incoming light intensity

- $I_{\text{diffuse}}$ : outgoing light intensity (for diffuse reflection)

# Diffuse Lighting Examples

- Lambertian sphere from several lighting angles:



- need only consider angles from  $0^\circ$  to  $90^\circ$
- *why?*
- *demo: Brown exploratory on reflection*
- [http://www.cs.brown.edu/exploratories/freeSoftware/repository/edu/brown/cs/exploratories/applets/reflection2D/reflection\\_2d\\_java\\_browser.html](http://www.cs.brown.edu/exploratories/freeSoftware/repository/edu/brown/cs/exploratories/applets/reflection2D/reflection_2d_java_browser.html)