



University of British Columbia
CPSC 314 Computer Graphics
Jan-Apr 2010

Tamara Munzner

Rasterization II

Week 6, Wed Feb 10

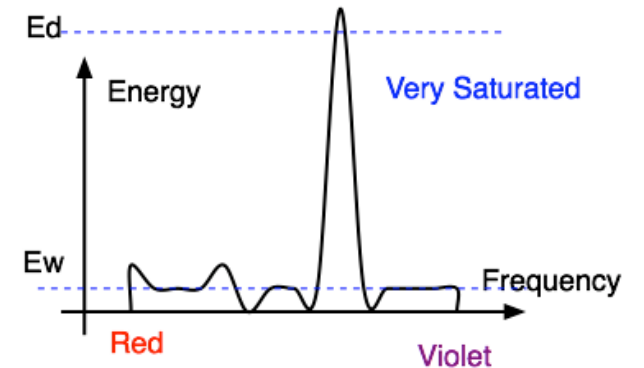
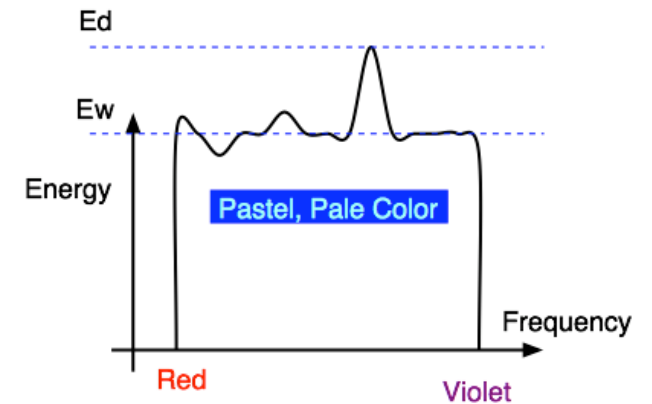
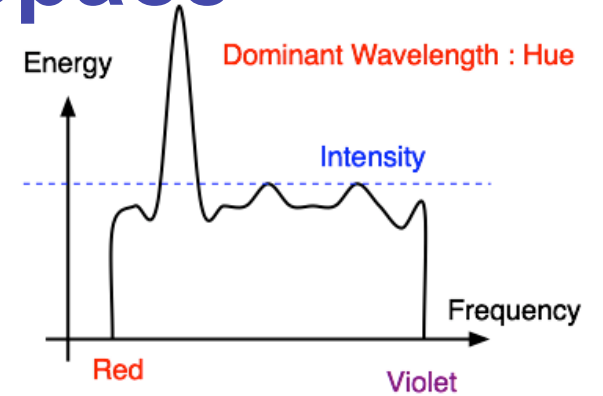
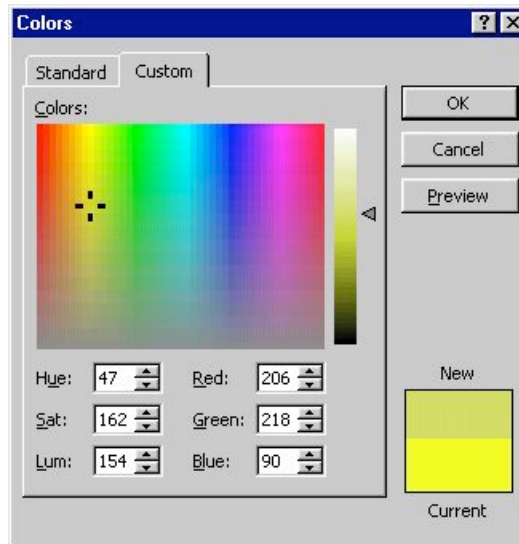
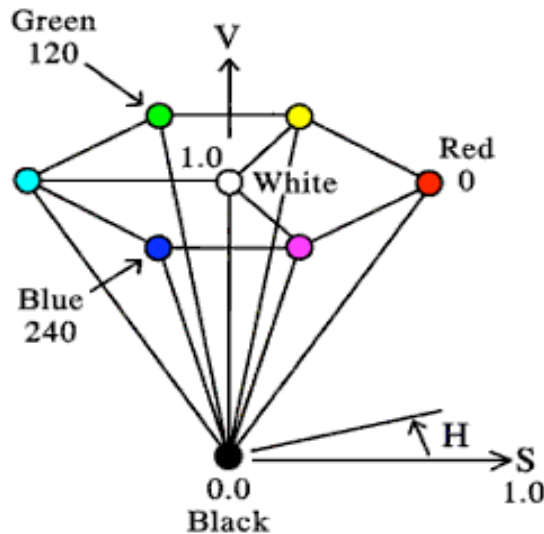
<http://www.ugrad.cs.ubc.ca/~cs314/Vjan2010>

Correction: News

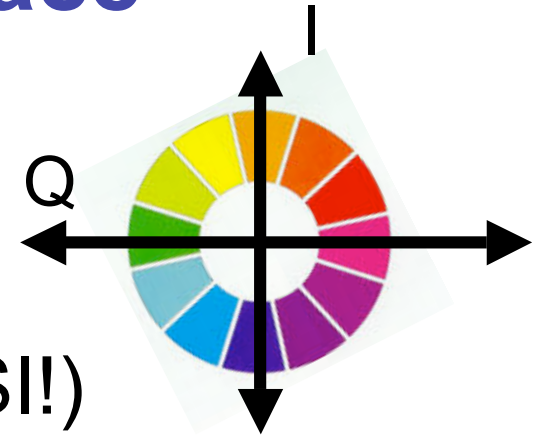
- TA office hours in lab for P2/H2 questions this week
 - Mon 3-5 (Shailen)
 - Tue 3:30-5 (Kai)
 - Wed 2-4 (Shailen)
 - Thu 3-5 (Kai)
 - Fri 2-4 (Garrett)
- again - start **now**, do not put off until late in break!

Review: HSV Color Space

- hue: dominant wavelength, “color”
- saturation: how far from grey
- value/brightness: how far from black/white
- cannot convert to RGB with matrix alone



Review: YIQ Color Space



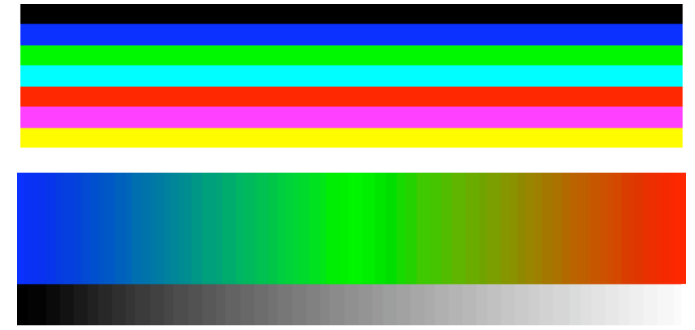
- color model used for color TV
 - Y is luminance (same as CIE)
 - I & Q are color (not same I as HSI!)
 - using Y backwards compatible for B/W TVs
 - conversion from RGB is linear

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.30 & 0.59 & 0.11 \\ 0.60 & -0.28 & -0.32 \\ 0.21 & -0.52 & 0.31 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- green is much lighter than red, and red lighter than blue

Review: Luminance vs. Intensity

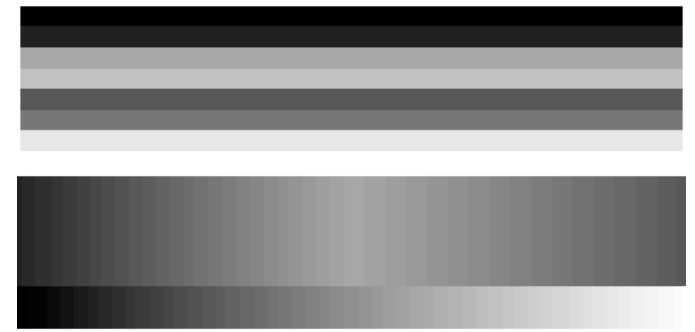
- luminance
 - Y of YIQ
 - $0.299R + 0.587G + 0.114B$
- intensity/brightness
 - I/V/B of HSI/HSV/HSB
 - $0.333R + 0.333G + 0.333B$



(a) Colour Image



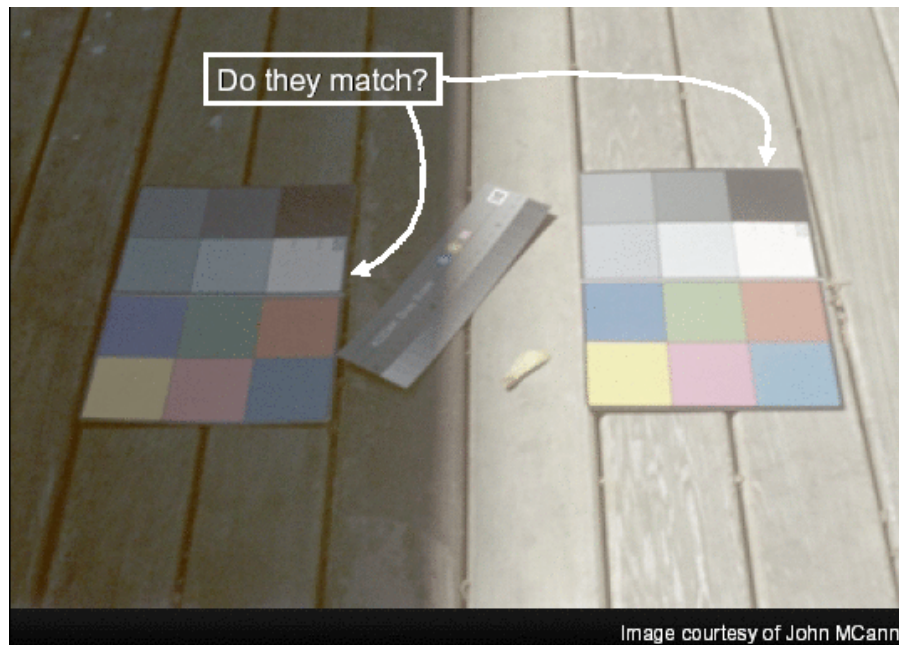
(b) Intensity Image



(c) Luminance Image

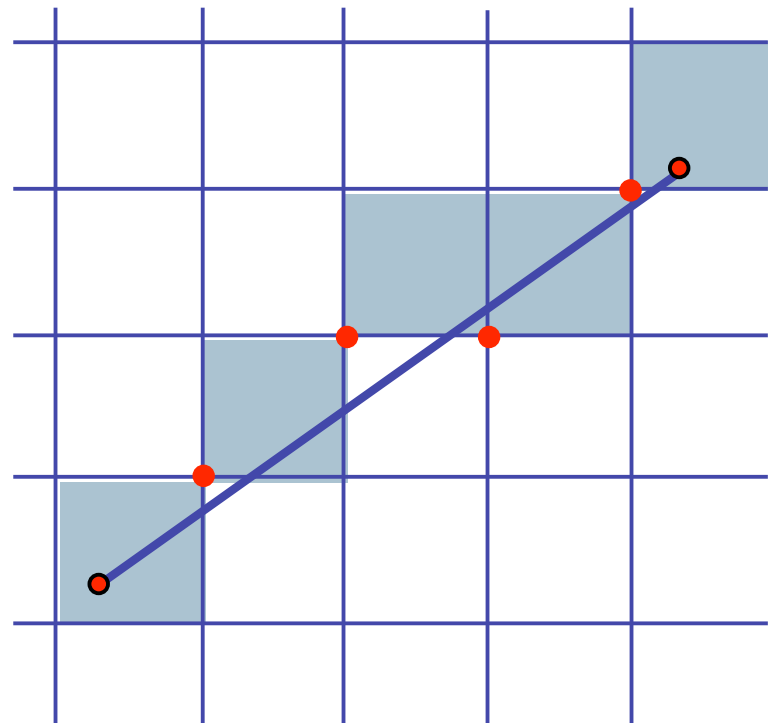
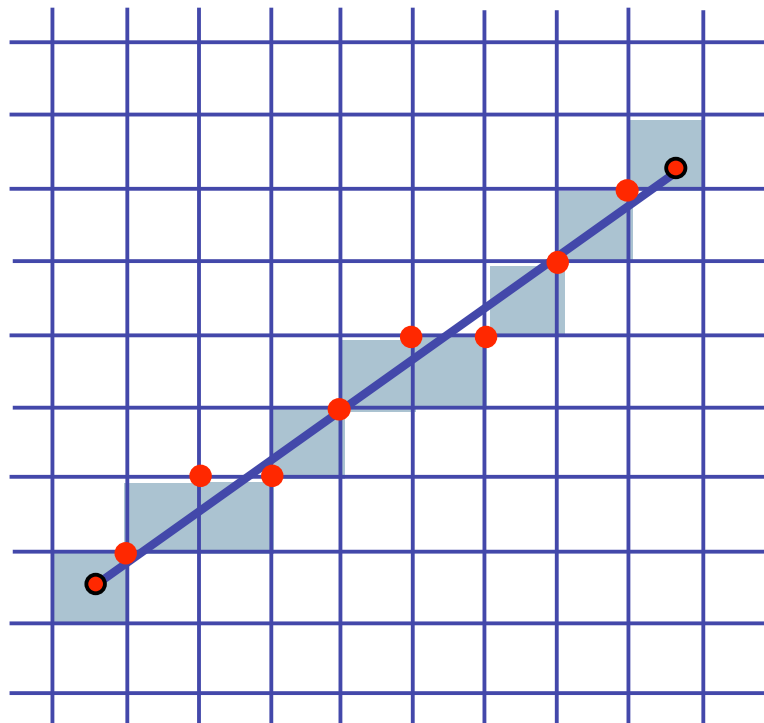
Review: Color Constancy

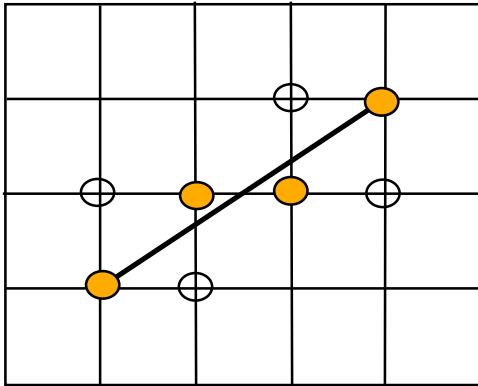
- automatic “white balance” from change in illumination
- vast amount of processing behind the scenes!
- colorimetry vs. perception



Review: Scan Conversion

- convert continuous rendering primitives into discrete fragments/pixels
 - given vertices in DCS, fill in the pixels
- display coordinates required to provide scale for discretization





Review: Basic Line Drawing

$$y = mx + b$$

$$y = \frac{(y_1 - y_0)}{(x_1 - x_0)}(x - x_0) + y_0$$

- goals
 - integer coordinates
 - thinnest line with no gaps

• assume



- one octant, other cases symmetric
- how can we do this more quickly?

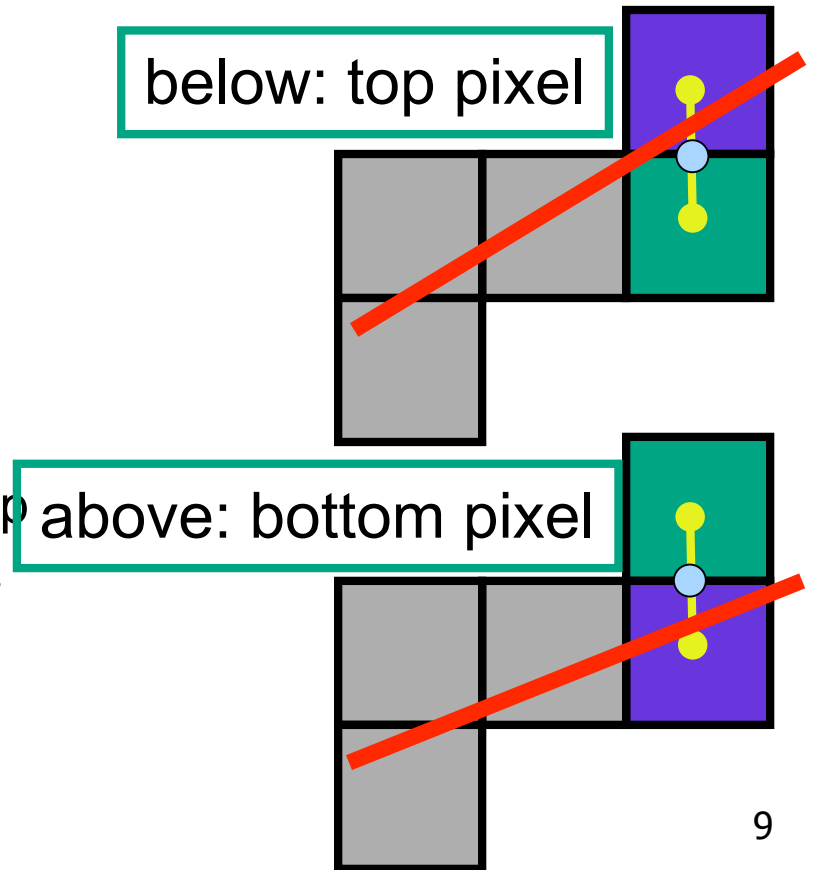
```

int dx = x1 - x0, dy = y1 - y0;
int slope;
if (abs(dx) >= abs(dy)) slope = dx / dy;
else slope = dy / dx;
int x = x0, y = y0;
while (x <= x1 && y <= y1) {
  plot(x, y);
  if (slope > 0) y++;
  else y--;
  x++;
}

```


Review/Correction: Midpoint Algorithm

- we're moving horizontally along x direction (first octant)
 - only two choices: draw at current y value, or move up vertically to $y+1$?
 - check if midpoint between two possible pixel centers above or below line
 - candidates
 - top pixel: $(x+1, y+1)$
 - bottom pixel: $(x+1, y)$
 - midpoint: $(x+1, y+.5)$
- check if midpoint above or below line
 - below: pick top pixel
 - above: pick bottom pixel
- key idea behind Bresenham
 - reuse computation from previous step
 - integer arithmetic by doubling values



Making It Fast: Reuse Computation

- midpoint: if $f(x+1, y+.5) < 0$ then $y = y+1$
- on previous step evaluated $f(x-1, y-.5)$ or $f(x-1, y+.5)$
- $f(x+1, y) = f(x,y) + (y_0-y_1)$
- $f(x+1, y+1) = f(x,y) + (y_0-y_1) + (x_1-x_0)$

```
y=y0
```

```
d = f(x0+1, y0+.5)
```

```
for (x=x0; x <= x1; x++) {
```

```
    draw(x, y);
```

```
    if (d<0) then {
```

```
        y = y + 1;
```

```
        d = d + (x1 - x0) + (y0 - y1)
```

```
    } else {
```

```
        d = d + (y0 - y1)
```

```
    }
```

Making It Fast: Integer Only

- avoid dealing with non-integer values by doubling both sides

```
y=y0
d = f(x0+1, y0+.5)
for (x=x0; x <= x1; x++)
{
draw(x,y);
if (d<0) then {
y = y + 1;
d = d + (x1 - x0) +
(y0 - y1)
} else {
d = d + (y0 - y1)
}
}
```

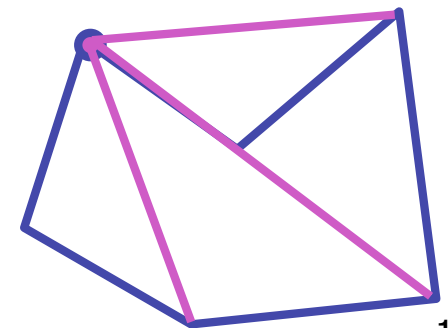
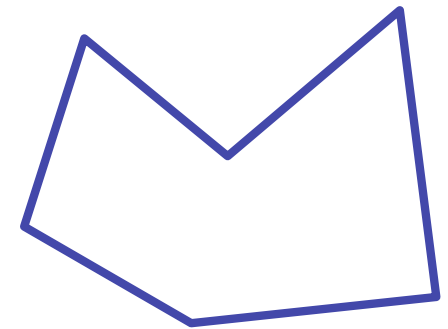
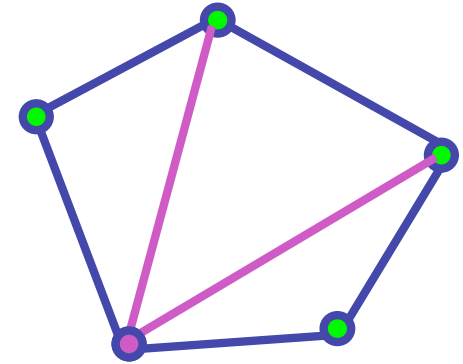
```
y=y0
2d = 2*(y0-y1)(x0+1) +
(x1-x0)(2y0+1) +
2x0y1 - 2x1y0
for (x=x0; x <= x1; x++) {
draw(x,y);
if (d<0) then {
y = y + 1;
d = d + 2(x1 - x0) +
2(y0 - y1)
} else {
d = d + 2(y0 - y1)
}
}
```

Rasterizing Polygons/Triangles

- basic surface representation in rendering
- why?
 - lowest common denominator
 - can approximate any surface with arbitrary accuracy
 - all polygons can be broken up into triangles
 - guaranteed to be:
 - planar
 - triangles - convex
 - simple to render
 - can implement in hardware

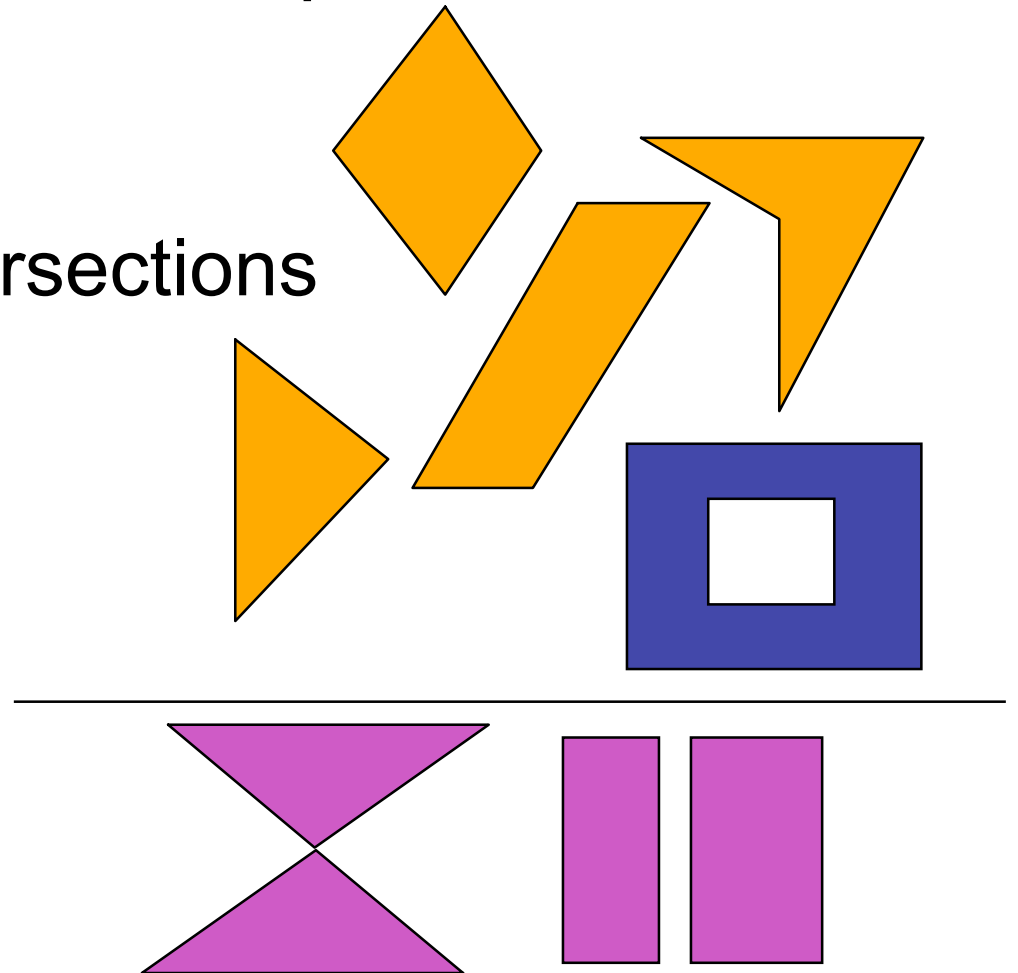
Triangulating Polygons

- simple convex polygons
 - trivial to break into triangles
 - pick one vertex, draw lines to all others not immediately adjacent
 - OpenGL supports automatically
 - `glBegin(GL_POLYGON) ... glEnd()`
- concave or non-simple polygons
 - more effort to break into triangles
 - simple approach may not work
 - OpenGL can support at extra cost
 - `gluNewTess(), gluTessCallback(), ...`



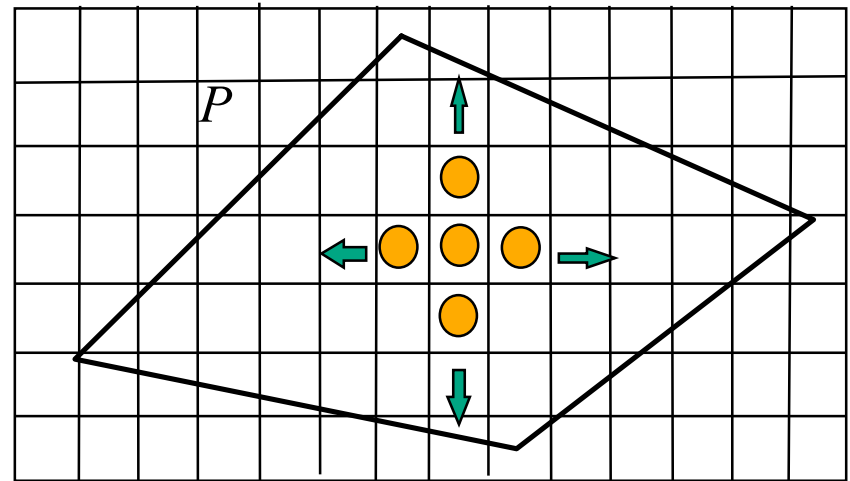
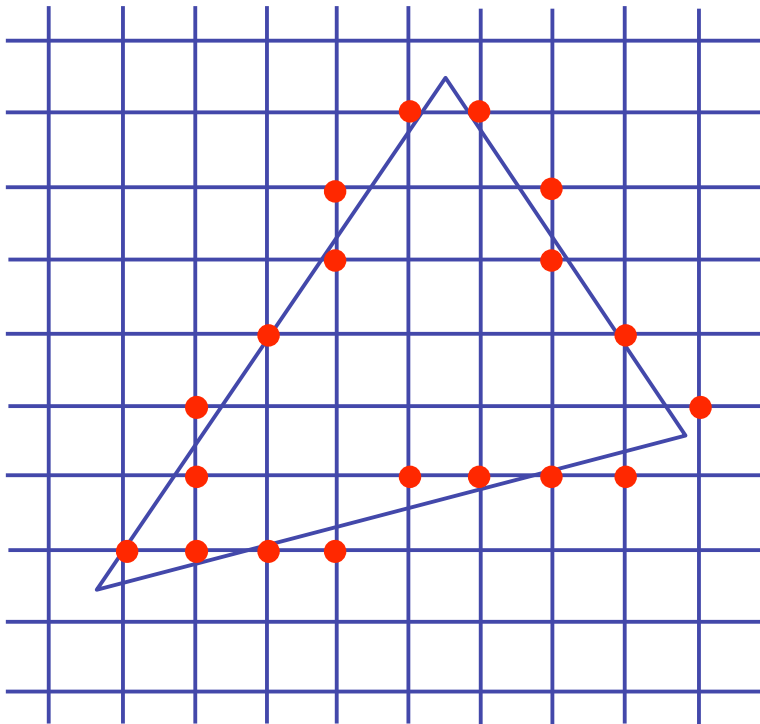
Problem

- input: closed 2D polygon
- problem: fill its interior with specified color on graphics display
- assumptions
 - simple - no self intersections
 - simply connected
- solutions
 - flood fill
 - edge walking



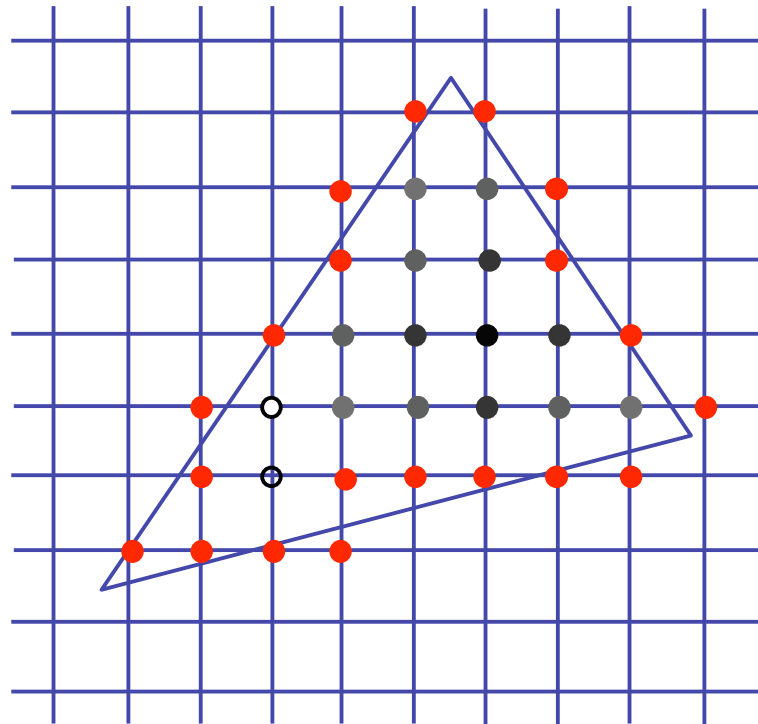
Flood Fill

- simple algorithm
 - draw edges of polygon
 - use flood-fill to draw interior



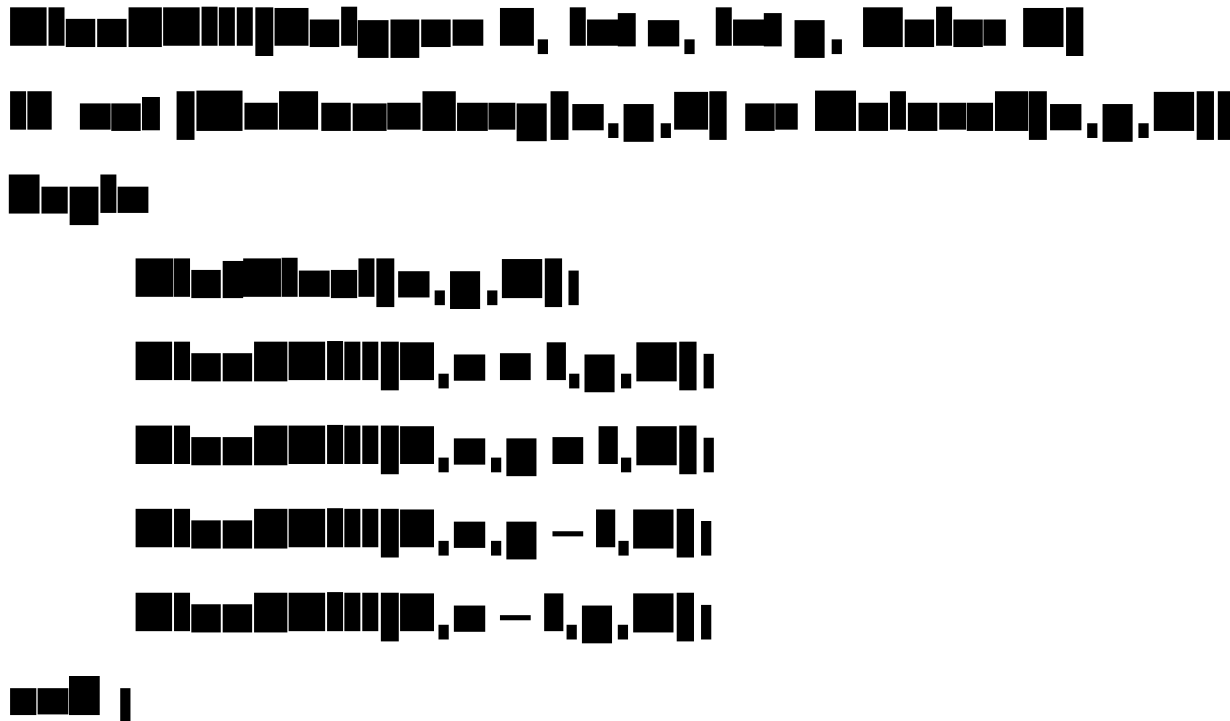
Flood Fill

- start with **seed point**
- recursively set all neighbors until boundary is hit



Flood Fill

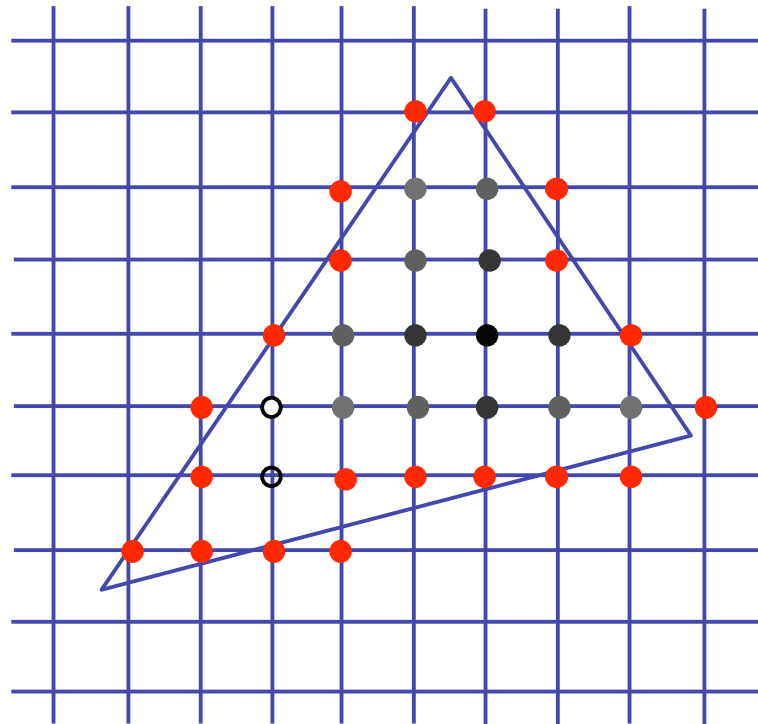
- draw edges
- run:



- drawbacks?

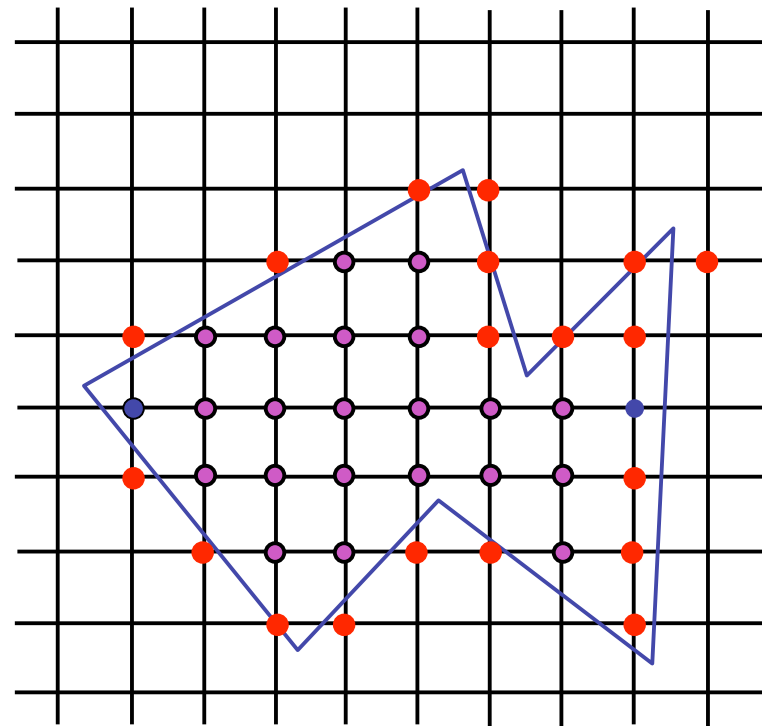
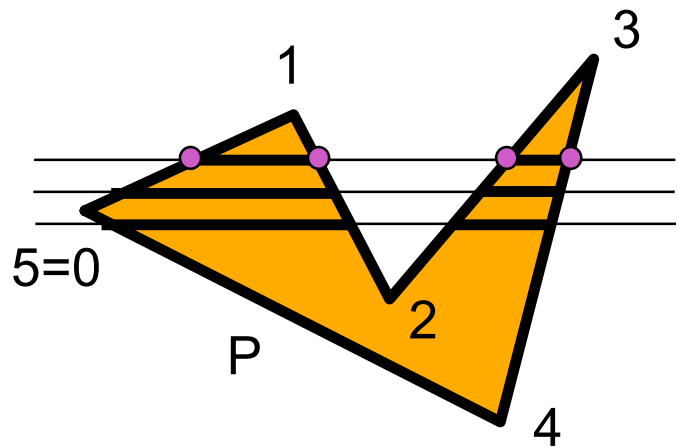
Flood Fill Drawbacks

- pixels visited up to 4 times to check if already set
- need per-pixel flag indicating if set already
 - must clear for every polygon!



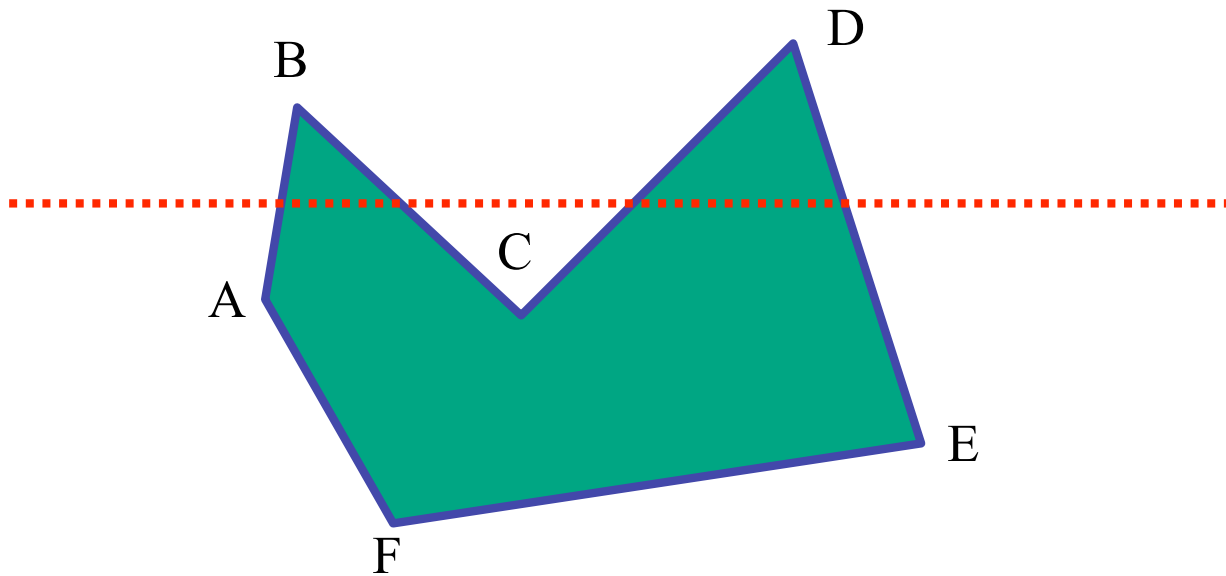
Scanline Algorithms

- **scanline**: a line of pixels in an image
 - set pixels inside polygon boundary along horizontal lines one pixel apart vertically



General Polygon Rasterization

- how do we know whether given pixel on scanline is inside or outside polygon?



General Polygon Rasterization

- idea: use a **parity test**

```
for each scanline
```

```
  edgeCnt = 0;
```

```
  for each pixel on scanline (l to r)
```

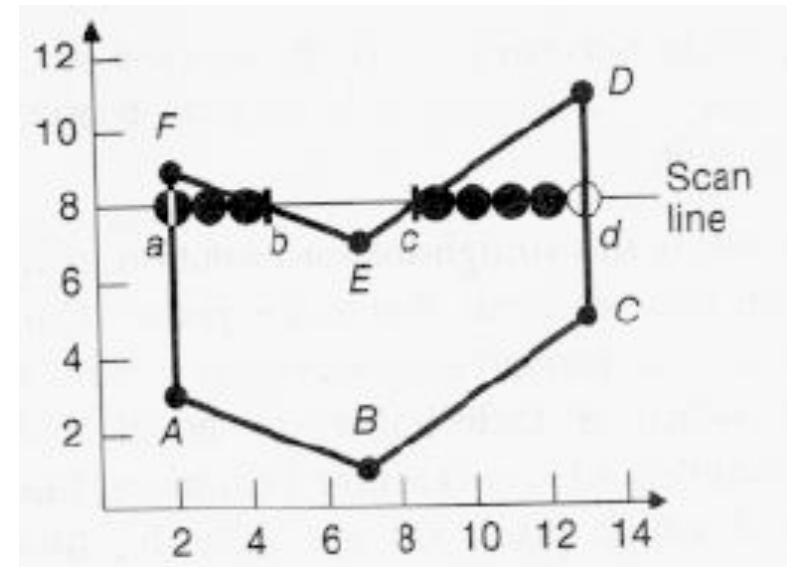
```
    if (oldpixel->newpixel crosses edge)
```

```
      edgeCnt ++;
```

```
    // draw the pixel if edgeCnt odd
```

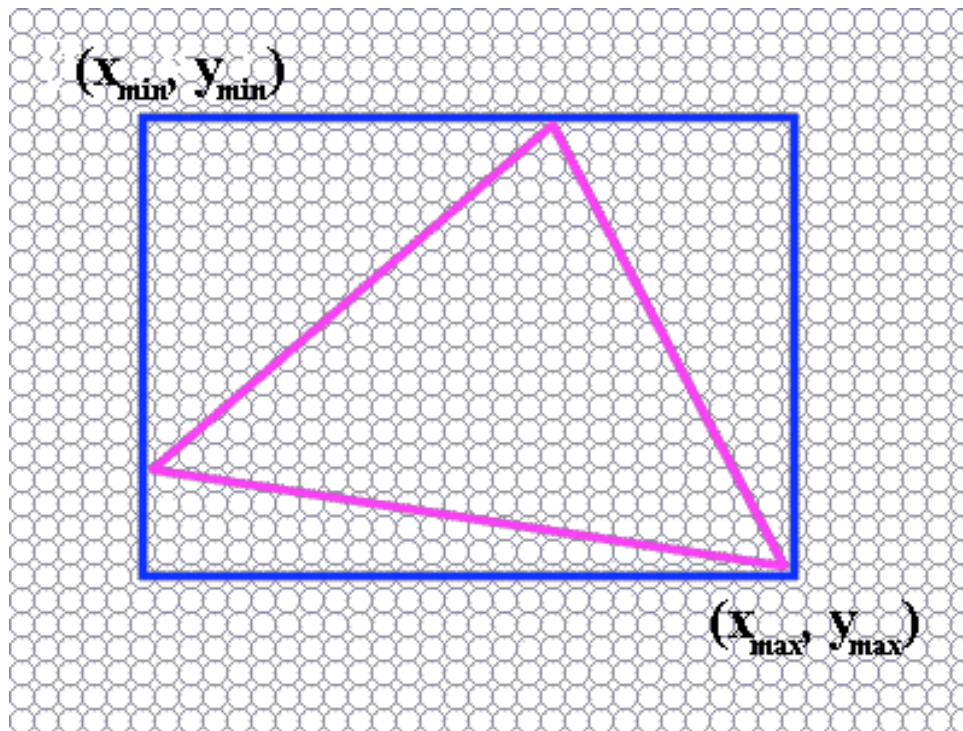
```
    if (edgeCnt % 2)
```

```
      setPixel(pixel);
```



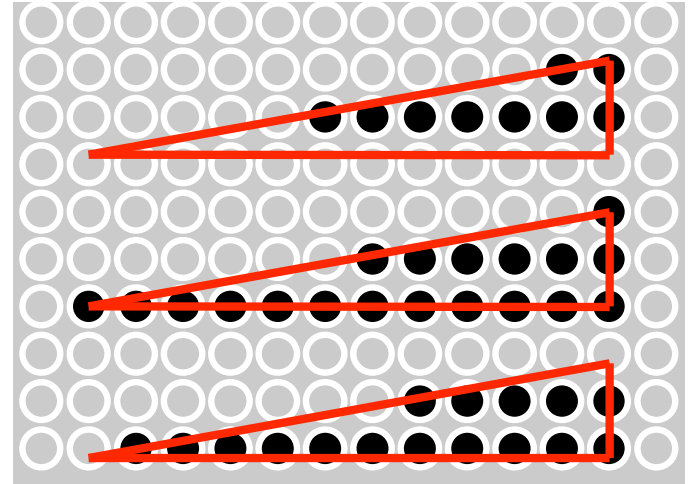
Making It Fast: Bounding Box

- smaller set of candidate pixels
 - loop over x_{\min} , x_{\max} and y_{\min} , y_{\max} instead of all x , all y

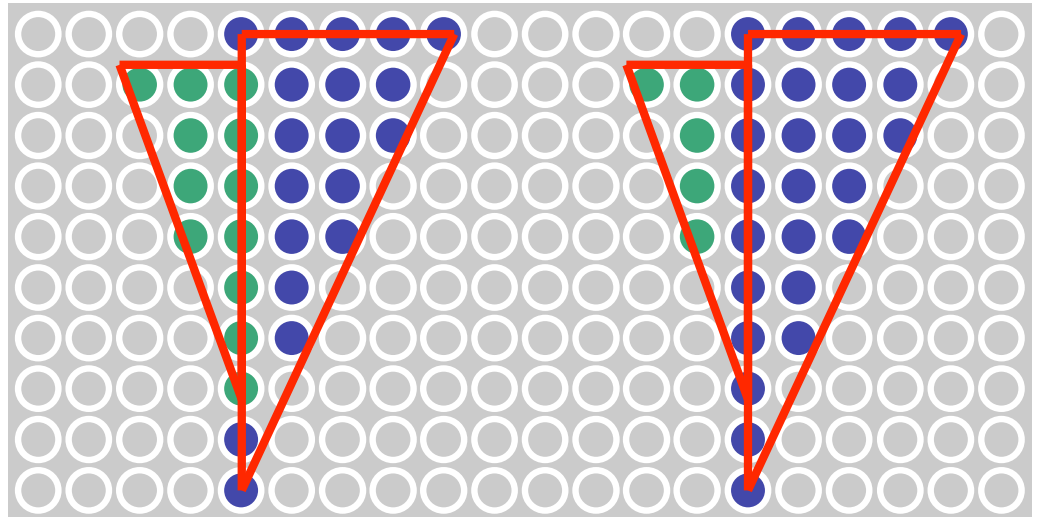


Triangle Rasterization Issues

- moving slivers



- shared edge ordering



Triangle Rasterization Issues

- *exactly which pixels should be lit?*
 - pixels with centers inside triangle edges
- *what about pixels exactly on edge?*
 - draw them: order of triangles matters (it shouldn't)
 - don't draw them: gaps possible between triangles
- need a consistent (if arbitrary) rule
 - example: draw pixels on left or top edge, but not on right or bottom edge
 - example: check if triangle on same side of edge as offscreen point

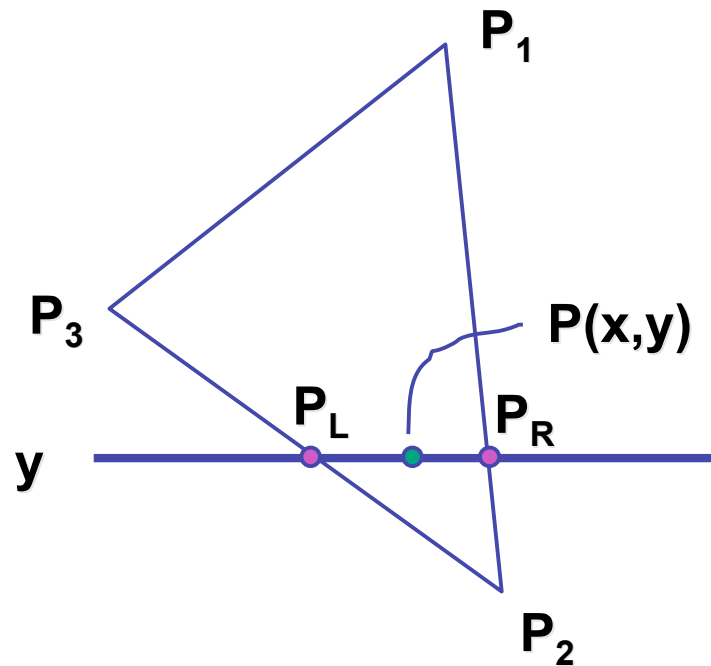
Interpolation

Interpolation During Scan Conversion

- drawing pixels in polygon requires interpolating many values between vertices
 - r,g,b colour components
 - use for shading
 - z values
 - u,v texture coordinates
 - N_x, N_y, N_z surface normals
- equivalent methods (for triangles)
 - bilinear interpolation
 - barycentric coordinates

Bilinear Interpolation

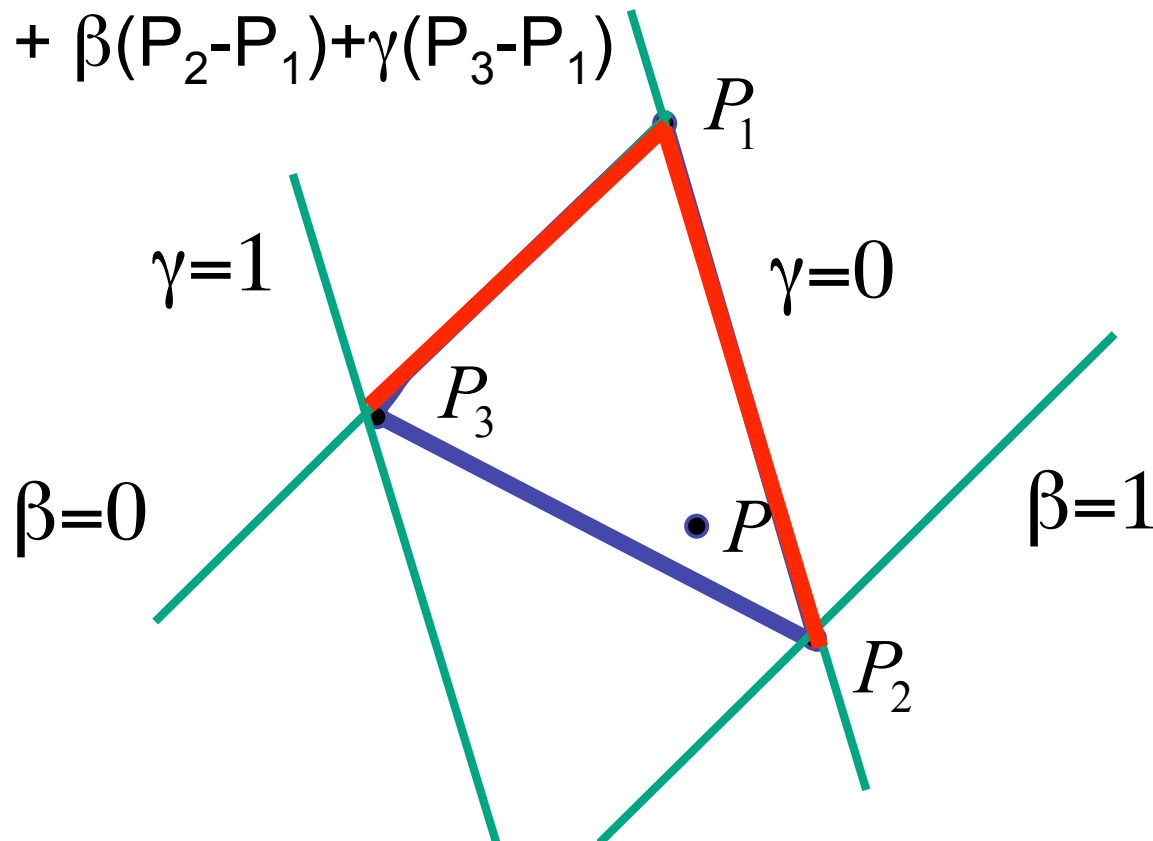
- interpolate quantity along L and R edges, as a function of y
 - then interpolate quantity as a function of x



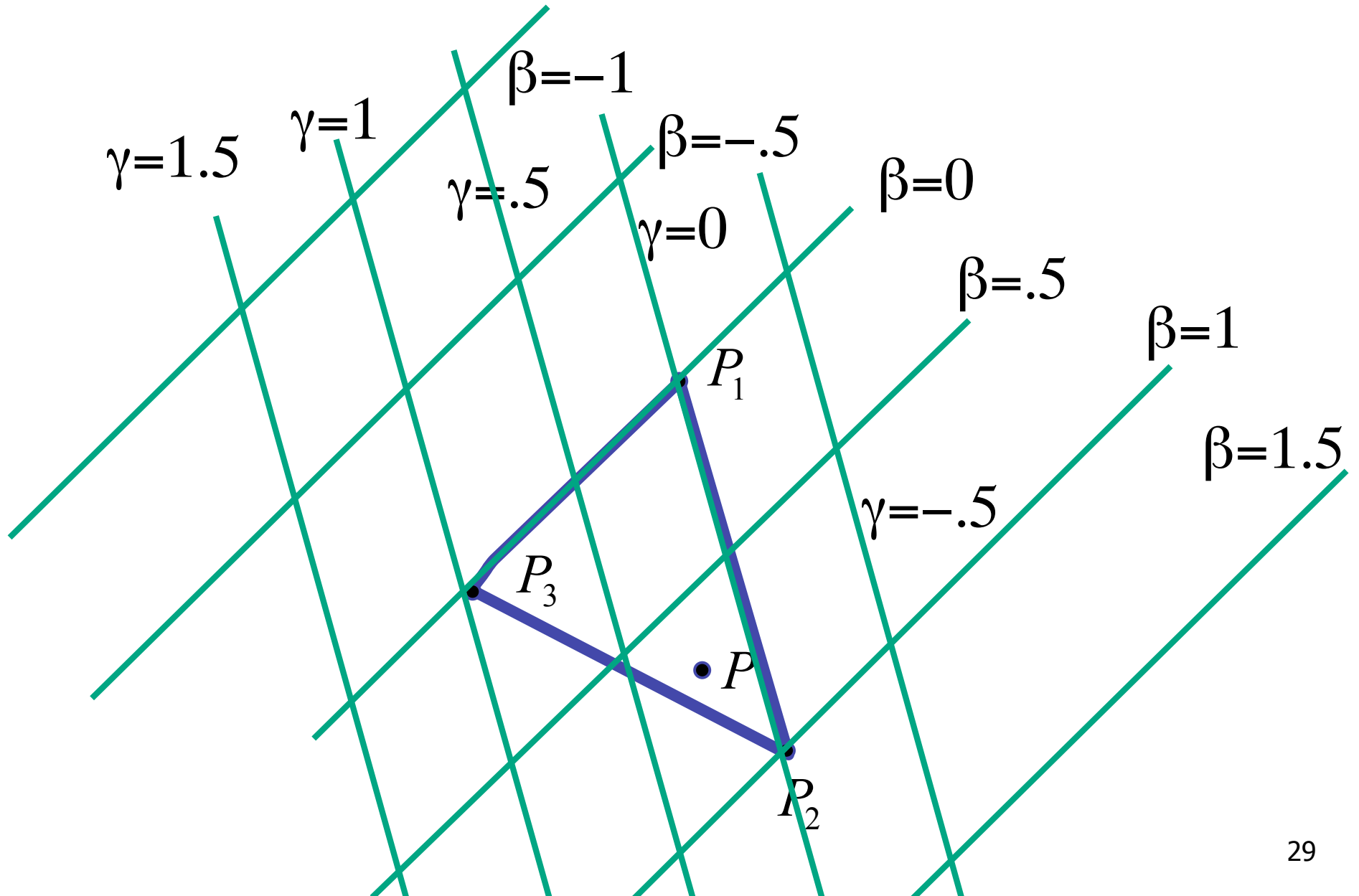
Barycentric Coordinates

- non-orthogonal coordinate system based on triangle itself
 - origin: P_1 , basis vectors: $(P_2 - P_1)$ and $(P_3 - P_1)$

$$P = P_1 + \beta(P_2 - P_1) + \gamma(P_3 - P_1)$$



Barycentric Coordinates



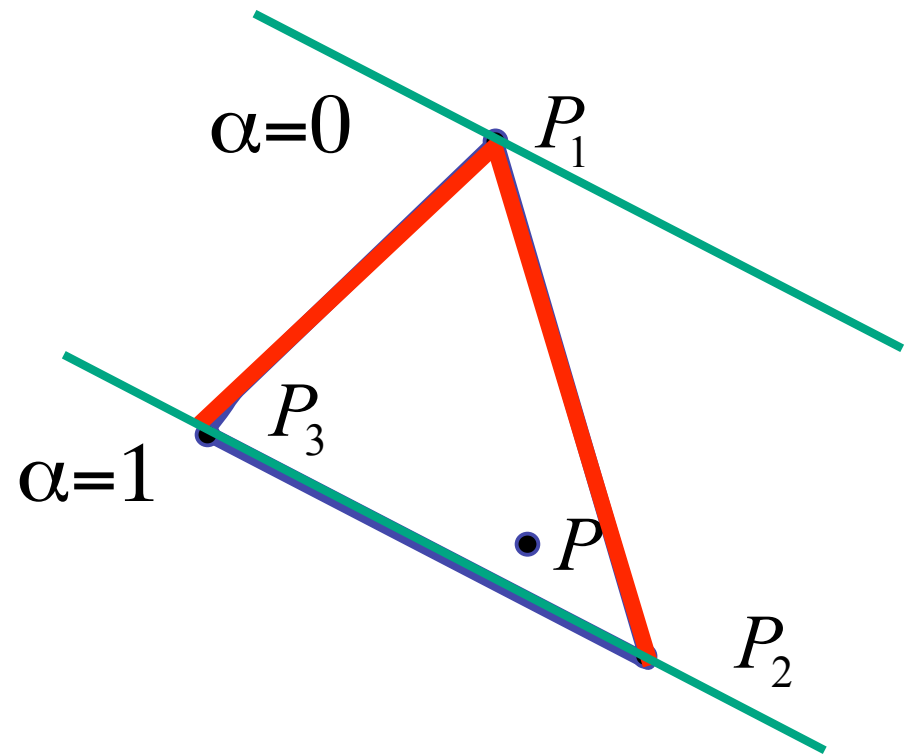
Barycentric Coordinates

- non-orthogonal coordinate system based on triangle itself
 - origin: P_1 , basis vectors: $(P_2 - P_1)$ and $(P_3 - P_1)$

$$P = P_1 + \beta(P_2 - P_1) + \gamma(P_3 - P_1)$$

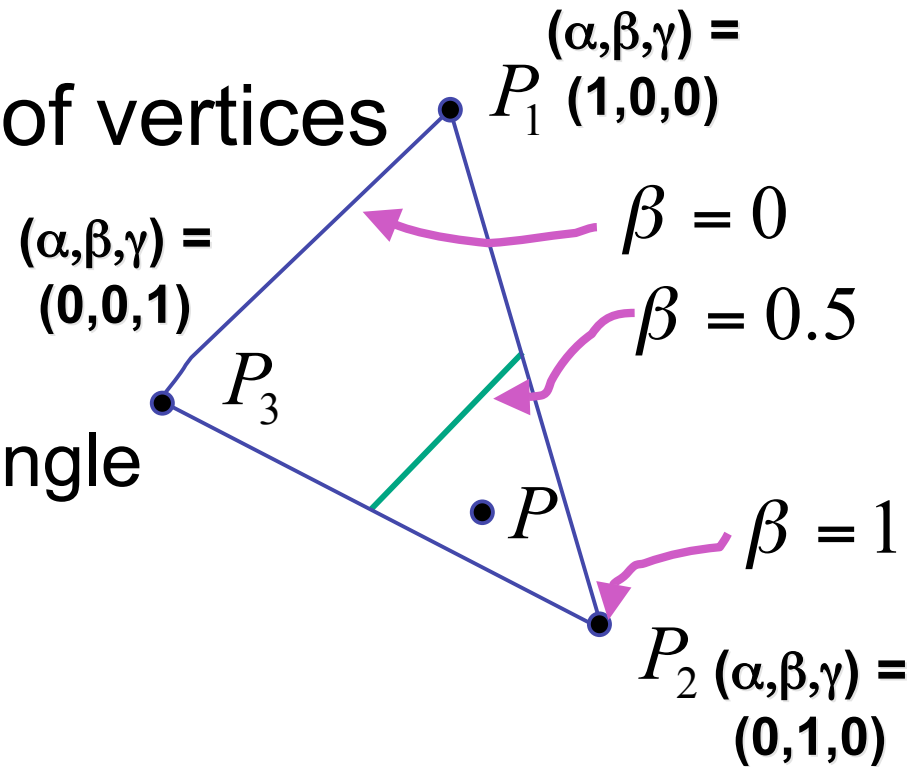
$$P = (1 - \beta - \gamma)P_1 + \beta P_2 + \gamma P_3$$

$$P = \alpha P_1 + \beta P_2 + \gamma P_3$$



Using Barycentric Coordinates

- weighted combination of vertices
- smooth mixing
- speedup
 - compute once per triangle

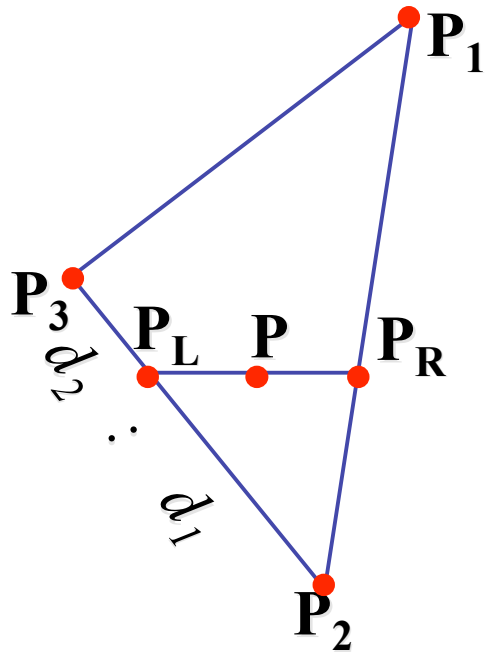


$$\left\{ \begin{array}{l} P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3 \\ \alpha + \beta + \gamma = 1 \\ 0 \leq \alpha, \beta, \gamma \leq 1 \text{ for points inside triangle} \end{array} \right.$$

“convex combination
of points”

Deriving Barycentric From Bilinear

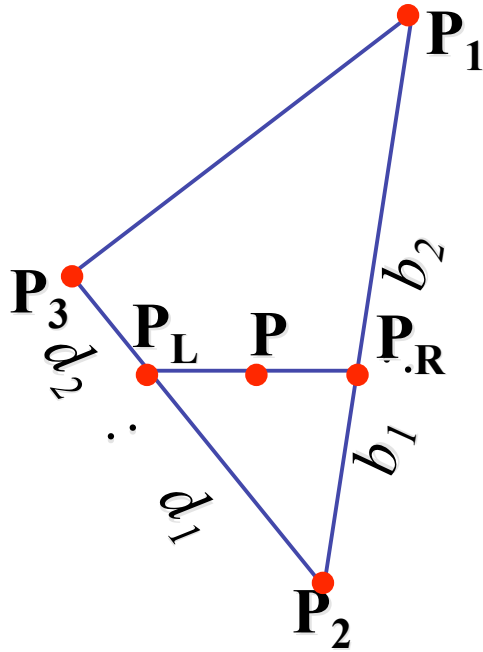
- from bilinear interpolation of point P on scanline



$$\begin{aligned} P_L &= P_2 + \frac{d_1}{d_1 + d_2} (P_3 - P_2) \\ &= \left(1 - \frac{d_1}{d_1 + d_2}\right) P_2 + \frac{d_1}{d_1 + d_2} P_3 = \\ &= \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3 \end{aligned}$$

Deriving Barycentric From Bilinear

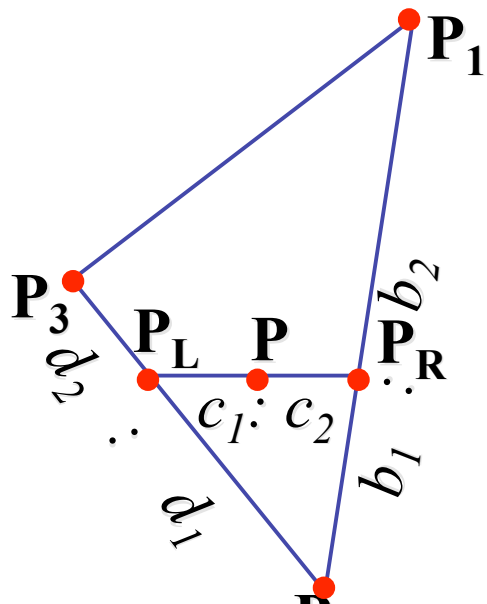
- similarly



$$\begin{aligned}
 P_R &= P_2 + \frac{b_1}{b_1 + b_2} (P_1 - P_2) \\
 &= \left(1 - \frac{b_1}{b_1 + b_2}\right) P_2 + \frac{b_1}{b_1 + b_2} P_1 = \\
 &= \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1
 \end{aligned}$$

Deriving Barycentric From Bilinear

- combining



- gives P_2

$$P_L = \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3$$

$$P_R = \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1$$

$$P = \frac{d_1 d_2}{d_1 d_2 + d_1 d_3 + d_2 d_3} P_1 + \frac{d_1 d_3}{d_1 d_2 + d_1 d_3 + d_2 d_3} P_2 + \frac{d_2 d_3}{d_1 d_2 + d_1 d_3 + d_2 d_3} P_3$$

$$P = \frac{d_1 d_2}{d_1 d_2 + d_1 d_3 + d_2 d_3} P_1 + \frac{d_1 d_3}{d_1 d_2 + d_1 d_3 + d_2 d_3} P_2 + \frac{d_2 d_3}{d_1 d_2 + d_1 d_3 + d_2 d_3} P_3$$

Deriving Barycentric From Bilinear

- thus $P = \alpha P_1 + \beta P_2 + \gamma P_3$ with

$$\alpha = \frac{(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)}{(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)}$$

$$\beta = \frac{(x_3 - x_1)(y_2 - y_1) - (x_2 - x_1)(y_3 - y_1)}{(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)}$$

$$\gamma = \frac{(x_1 - x_2)(y_3 - y_1) - (x_1 - x_3)(y_2 - y_1)}{(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)}$$

- can verify barycentric properties

$$\alpha + \beta + \gamma = 1, \quad 0 \leq \alpha, \beta, \gamma \leq 1$$

Computing Barycentric Coordinates

- 2D triangle area
 - half of parallelogram area
 - from cross product

$$A = A_{P_1} + A_{P_2} + A_{P_3}$$

$$\alpha = A_{P_1} / A$$

$$\beta = A_{P_2} / A$$

$$\gamma = A_{P_3} / A$$

