



Tamara Munzner

Rasterization II

Week 6, Wed Feb 10

<http://www.ugrad.cs.ubc.ca/~cs314/Vjan2010>

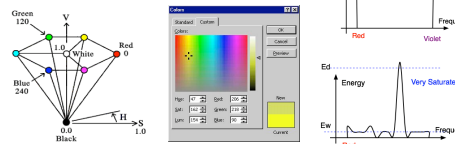
Correction: News

- TA office hours in lab for P2/H2 questions this week
 - Mon 3-5 (Shailen)
 - Tue 3:30-5 (Kai)
 - Wed 2-4 (Shailen)
 - Thu 3-5 (Kai)
 - Fri 2-4 (Garrett)
- again - start **now**, do not put off until late in break!

2

Review: HSV Color Space

- hue: dominant wavelength, "color"
- saturation: how far from grey
- value/brightness: how far from black/white
- cannot convert to RGB with matrix alone



Review: YIQ Color Space

- color model used for color TV
- Y is luminance (same as CIE)
- I & Q are color (not same I as HSI!)
- using Y backwards compatible for B/W TVs
- conversion from RGB is linear

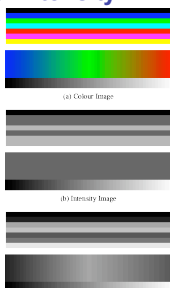
$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.30 & 0.59 & 0.11 \\ 0.60 & -0.28 & -0.32 \\ 0.21 & -0.52 & 0.31 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- green is much lighter than red, and red lighter than blue

4

Review: Luminance vs. Intensity

- luminance
 - Y of YIQ
 - $0.299R + 0.587G + 0.114B$
- intensity/brightness
 - I/V/B of HSI/HSV/HSB
 - $0.333R + 0.333G + 0.333B$

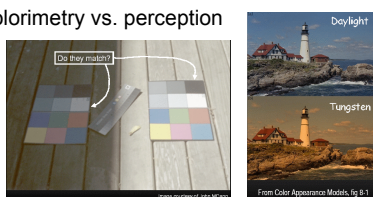


www.csse.uwa.edu.au/~robyn/Visioncourse/colour/lecture/node5.html

5

Review: Color Constancy

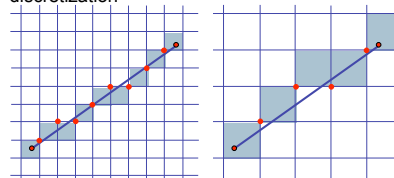
- automatic "white balance" from change in illumination
- vast amount of processing behind the scenes!
- colorimetry vs. perception



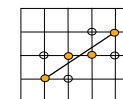
6

Review: Scan Conversion

- convert continuous rendering primitives into discrete fragments/pixels
 - given vertices in DCS, fill in the pixels
- display coordinates required to provide scale for discretization



7

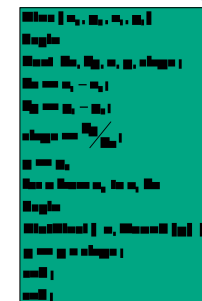


Review: Basic Line Drawing

$$y = mx + b$$

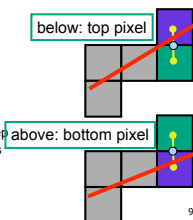
$$y = \frac{(y_1 - y_0)}{(x_1 - x_0)}(x - x_0) + y_0$$

- goals
 - integer coordinates
 - thinnest line with no gaps
- assume
 - one octant, other cases symmetric
- how can we do this more quickly?



Review/Correction: Midpoint Algorithm

- we're moving horizontally along x direction (first octant)
 - only two choices: draw at current y value, or move up vertically to y+1?
 - check if midpoint between two possible pixel centers above or below line
- candidates
 - top pixel: (x+1, y+1)
 - bottom pixel: (x+1, y)
- midpoint: (x+1, y+0.5)
- check if midpoint above or below line
 - below: pick top pixel
 - above: pick bottom pixel
- key idea behind Bresenham
 - reuse computation from previous step
 - integer arithmetic by doubling values



9

Making It Fast: Reuse Computation

- midpoint: if $f(x+1, y+0.5) < 0$ then $y = y+1$
 - on previous step evaluated $f(x-1, y-0.5)$ or $f(x-1, y+0.5)$
 - $f(x+1, y) = f(x, y) + (y_0 - y_1)$
 - $f(x+1, y+1) = f(x, y) + (y_0 - y_1) + (x_1 - x_0)$
- ```

y=y0
d = f(x0+1, y0+0.5)
for (x=x0; x <= x1; x++) {
 draw(x, y);
 if (d<0) then {
 y = y + 1;
 d = d + (x1 - x0) + (y0 - y1);
 } else {
 d = d + (y0 - y1);
 }
}

```

10

### Making It Fast: Integer Only

- avoid dealing with non-integer values by doubling both sides

```

y=y0
d = f(x0+1, y0+0.5)
for (x=x0; x <= x1; x++) {
 draw(x, y);
 if (d<0) then {
 y = y + 1;
 d = d + (x1 - x0) + (y0 - y1);
 } else {
 d = d + (y0 - y1);
 }
}

```

```

y=y0
2d = 2*(y0-y1)*(x0+1) + (x1-x0)*(2y0+1) + 2x0y1 - 2x1y0
for (x=x0; x <= x1; x++) {
 draw(x, y);
 if (d<0) then {
 y = y + 1;
 d = d + 2*(x1 - x0) + 2*(y0 - y1);
 } else {
 d = d + 2*(y0 - y1);
 }
}

```

11

### Rasterizing Polygons/Triangles

- basic surface representation in rendering
- why?
  - lowest common denominator
    - can approximate any surface with arbitrary accuracy
      - all polygons can be broken up into triangles
  - guaranteed to be:
    - planar
    - triangles - convex
  - simple to render
    - can implement in hardware

12

### Triangulating Polygons

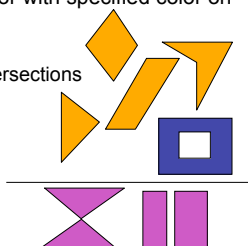
- simple convex polygons
  - trivial to break into triangles
  - pick one vertex, draw lines to all others not immediately adjacent
  - OpenGL supports automatically
    - `glBegin(GL_POLYGON) ... glEnd()`
- concave or non-simple polygons
  - more effort to break into triangles
  - simple approach may not work
  - OpenGL can support at extra cost
    - `gluNewTess(), gluTessCallback(), ...`



13

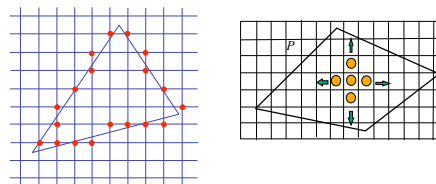
### Problem

- input: closed 2D polygon
- problem: fill its interior with specified color on graphics display
- assumptions
  - simple - no self intersections
  - simply connected
- solutions
  - flood fill
  - edge walking



### Flood Fill

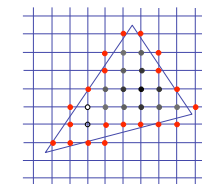
- simple algorithm
  - draw edges of polygon
  - use flood-fill to draw interior



15

### Flood Fill

- start with **seed point**
- recursively set all neighbors until boundary is hit



16

## Flood Fill

- draw edges

• run:

```
function FloodFill(x, y, old, new)
 if not (old == FloodFill(x, y, old, new))
 FloodFill(x, y, old, new)
 FloodFill(x, y, old, new)
 FloodFill(x, y, old, new)
 FloodFill(x, y, old, new)
 FloodFill(x, y, old, new)
 FloodFill(x, y, old, new)
 FloodFill(x, y, old, new)
end

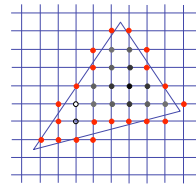
end
```

- drawbacks?

17

## Flood Fill Drawbacks

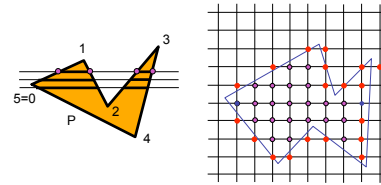
- pixels visited up to 4 times to check if already set
- need per-pixel flag indicating if set already
  - must clear for every polygon!



18

## Scanline Algorithms

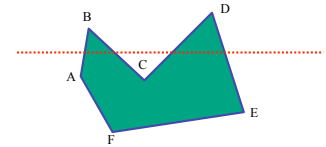
- **scanline:** a line of pixels in an image
  - set pixels inside polygon boundary along horizontal lines one pixel apart vertically



19

## General Polygon Rasterization

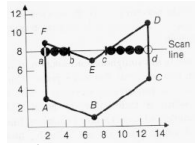
- how do we know whether given pixel on scanline is inside or outside polygon?



20

## General Polygon Rasterization

- idea: use a **parity test**

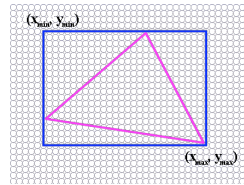


```
for each scanline
 edgeCnt = 0;
 for each pixel on scanline (1 to r)
 if (oldpixel->newpixel crosses edge)
 edgeCnt++;
 // draw the pixel if edgeCnt odd
 if (edgeCnt % 2)
 setPixel(pixel);
```

21

## Making It Fast: Bounding Box

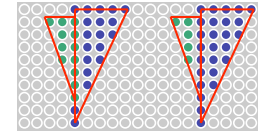
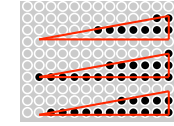
- smaller set of candidate pixels
  - loop over xmin, xmax and ymin,ymax instead of all x, all y



23

## Triangle Rasterization Issues

- moving slivers
- shared edge ordering



23

## Triangle Rasterization Issues

- **exactly which pixels should be lit?**
  - pixels with centers inside triangle edges
- **what about pixels exactly on edge?**
  - draw them: order of triangles matters (it shouldn't)
  - don't draw them: gaps possible between triangles
- need a consistent (if arbitrary) rule
  - example: draw pixels on left or top edge, but not on right or bottom edge
  - example: check if triangle on same side of edge as offscreen point

24

## Interpolation

## Interpolation During Scan Conversion

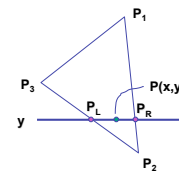
- drawing pixels in polygon requires interpolating many values between vertices
  - r,g,b colour components
    - use for shading
  - z values
  - u,v texture coordinates
  - $N_x, N_y, N_z$  surface normals
- equivalent methods (for triangles)
  - bilinear interpolation
  - barycentric coordinates

25

26

## Bilinear Interpolation

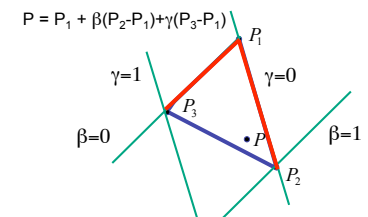
- interpolate quantity along L and R edges, as a function of y
  - then interpolate quantity as a function of x



27

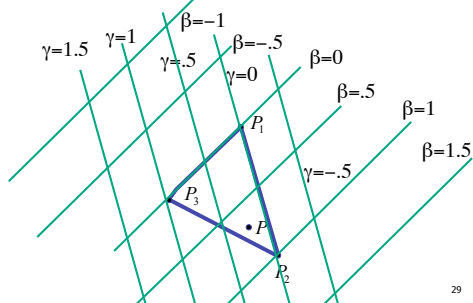
## Barycentric Coordinates

- non-orthogonal coordinate system based on triangle itself
  - origin: P1, basis vectors: (P2-P1) and (P3-P1)



28

## Barycentric Coordinates

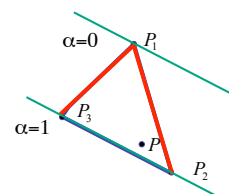


29

## Barycentric Coordinates

- non-orthogonal coordinate system based on triangle itself
  - origin: P1, basis vectors: (P2-P1) and (P3-P1)

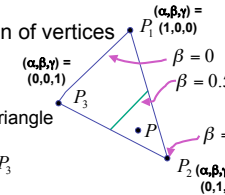
$$P = P_1 + \beta(P_2-P_1) + \gamma(P_3-P_1)$$
$$P = (1-\beta-\gamma)P_1 + \beta P_2 + \gamma P_3$$
$$P = \alpha P_1 + \beta P_2 + \gamma P_3$$



30

## Using Barycentric Coordinates

- weighted combination of vertices
- smooth mixing
- speedup
  - compute once per triangle



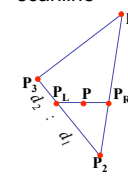
$$\begin{cases} P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3 \\ \alpha + \beta + \gamma = 1 \\ 0 \leq \alpha, \beta, \gamma \leq 1 \text{ for points inside triangle} \end{cases}$$

“convex combination of points”

31

## Deriving Barycentric From Bilinear

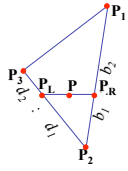
- from bilinear interpolation of point P on scanline


$$P_L = P_2 + \frac{d_1}{d_1 + d_2}(P_3 - P_2)$$
$$= (1 - \frac{d_1}{d_1 + d_2})P_2 + \frac{d_1}{d_1 + d_2}P_3 =$$
$$= \frac{d_2}{d_1 + d_2}P_2 + \frac{d_1}{d_1 + d_2}P_3$$

32

### Deriving Barycentric From Bilinear

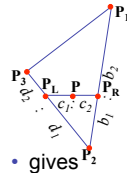
- similarly



$$\begin{aligned}
 P_R &= P_2 + \frac{b_1}{b_1 + b_2} (P_1 - P_2) \\
 &= \left(1 - \frac{b_1}{b_1 + b_2}\right) P_2 + \frac{b_1}{b_1 + b_2} P_1 = \\
 &= \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1
 \end{aligned}$$

### Deriving Barycentric From Bilinear

- combining



$$\begin{aligned}
 P_L &= \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3 \\
 P_R &= \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1
 \end{aligned}$$

- gives

$$\left[ \begin{array}{ccc} \frac{d_2}{d_1 + d_2} & \frac{d_1}{d_1 + d_2} & 0 \\ \frac{b_2}{b_1 + b_2} & \frac{b_1}{b_1 + b_2} & 0 \\ \frac{c_1}{c_1 + c_2 + c_3} & \frac{c_2}{c_1 + c_2 + c_3} & \frac{c_3}{c_1 + c_2 + c_3} \end{array} \right] \cdot \left[ \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} \right] = P$$

### Deriving Barycentric From Bilinear

- thus  $P = \alpha P_1 + \beta P_2 + \gamma P_3$  with

$$\left[ \begin{array}{ccc} \frac{d_2}{d_1 + d_2} & \frac{d_1}{d_1 + d_2} & 0 \\ \frac{b_2}{b_1 + b_2} & \frac{b_1}{b_1 + b_2} & 0 \\ \frac{c_1}{c_1 + c_2 + c_3} & \frac{c_2}{c_1 + c_2 + c_3} & \frac{c_3}{c_1 + c_2 + c_3} \end{array} \right] \cdot \left[ \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} \right] = P$$

- can verify barycentric properties

$$\alpha + \beta + \gamma = 1, \quad 0 \leq \alpha, \beta, \gamma \leq 1$$

### Computing Barycentric Coordinates

- 2D triangle area
- half of parallelogram area
- from cross product

$$A = A_{P_1} + A_{P_2} + A_{P_3}$$

$$\alpha = A_{P_1} / A$$

$$\beta = A_{P_2} / A$$

$$\gamma = A_{P_3} / A$$

