

University of British Columbia CPSC 314 Computer Graphics Jan-Apr 2010

Tamara Munzner

Math Review

Week 1, Wed Jan 6

http://www.ugrad.cs.ubc.ca/~cs314/Vjan2010

News

- no class this Friday (Jan 8)!
- UBC CS Dept news

Department of Computer Science Undergraduate Events

Events this week

How to Prepare for the Tech Career Fair

Date: Wed. Jan 6

Time: 5 - 6:30 pm

Location: DMP 110

Resume Writing Workshop (for non-coop students)

Date: Thurs. Jan 7

Time: 12:30 – 2 pm

Location: DMP 201

CSSS Movie Night

Date: Thurs, Jan 7

Time: 6 - 10 pm

Location: DMP 310

Movies: "Up" & "The Hangover"

(Free Popcorn & Pop)

Drop-In Resume Edition Session

Date: Mon. Jan 11

Time: 11 am – 2 pm

Location: Rm 255, ICICS/CS Bldg

Industry Panel

Speakers: Managers from Google,

IBM, Microsoft, TELUS, etc.

Date: Tues. Jan 12

Time: Panel: 5:15 – 6:15 pm; Networking: 6:15 – 7:15 pm

Location: Panel: DMP 110;

Networking: X-wing Undergrad

Lounge

Tech Career Fair

Date: Wed. Jan 13

Time: 10 am – 4 pm

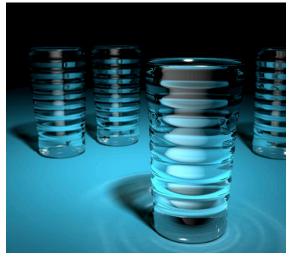
Location: SUB Ballroom

Review: Computer Graphics Defined

- CG uses
 - movies, games, art/design, ads, VR, visualization
- CG state of the art
 - photorealism achievable (in some cases)



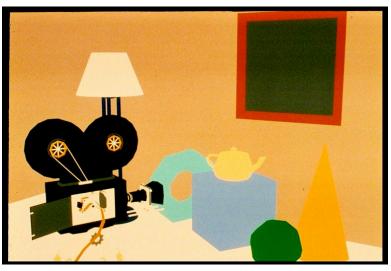






Review: Rendering Capabilities









www.siggraph.org/education/materials/HyperGraph/shutbug.htm

Today's Readings

- FCG Chapter 2: Miscellaneous Math
 - except 2.7 (2.11 in 2nd edition)
- FCG Chapter 5: Linear Algebra
 - except 5.4 (not in 2nd edition)

Notation: Scalars, Vectors, Matrices

- scalar
 - (lower case, italic)
- vector
 - (lower case, bold)
- matrix
 - (upper case, bold)

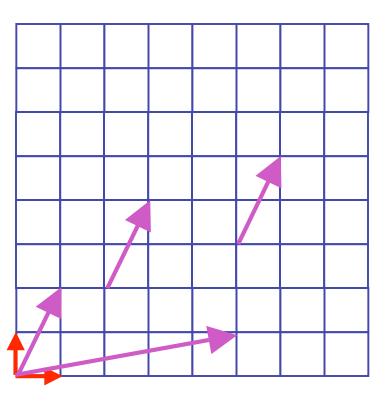
 \boldsymbol{a}

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Vectors

- arrow: length and direction
 - oriented segment in nD space
- offset / displacement
 - location if given origin



Column vs. Row Vectors

• row vectors
$$\mathbf{a}_{row} = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}$$

• column vectors
$$\mathbf{a}_{col} = \begin{bmatrix} a_1 \\ a_2 \\ \cdots \\ a_n \end{bmatrix}$$

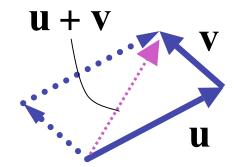
switch back and forth with transpose

$$\mathbf{a}_{col}^T = \mathbf{a}_{row}$$

Vector-Vector Addition

- add: vector + vector = vector
- parallelogram rule
 - tail to head, complete the triangle

geometric



algebraic

$$\mathbf{u} + \mathbf{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \end{bmatrix}$$

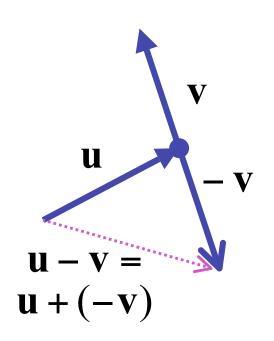
examples:

$$(3,2) + (6,4) = (9,6)$$

 $(2,5,1) + (3,1,-1) = (5,6,0)$

Vector-Vector Subtraction

subtract: vector - vector = vector



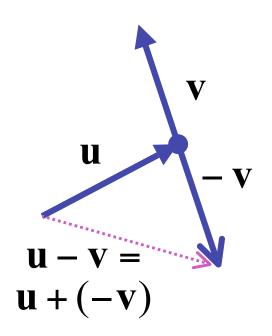
$$\mathbf{u} - \mathbf{v} = \begin{vmatrix} u_1 - v_1 \\ u_2 - v_2 \\ u_3 - v_3 \end{vmatrix}$$

$$(3,2) - (6,4) = (-3,-2)$$

 $(2,5,1) - (3,1,-1) = (-1,4,2)$

Vector-Vector Subtraction

subtract: vector - vector = vector



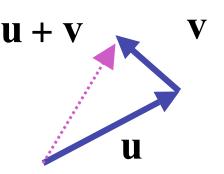
$$\mathbf{u} - \mathbf{v} = \begin{bmatrix} u_1 - v_1 \\ u_2 - v_2 \\ u_3 - v_3 \end{bmatrix}$$

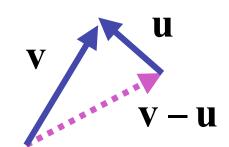
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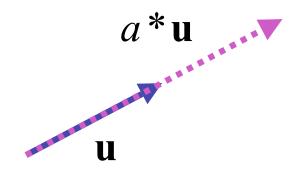
argument reversal





Scalar-Vector Multiplication

- multiply: scalar * vector = vector
 - vector is scaled



$$a * \mathbf{u} = (a * u_1, a * u_2, a * u_3)$$

$$2*(3,2) = (6,4)$$

$$.5*(2,5,1) = (1,2.5,.5)$$

Vector-Vector Multiplication

- multiply: vector * vector = scalar
- dot product, aka inner product

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \bullet \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = (u_1 * v_1) + (u_2 * v_2) + (u_3 * v_3)$$

Vector-Vector Multiplication

- multiply: vector * vector = scalar
- dot product, aka inner product

	V
u	v

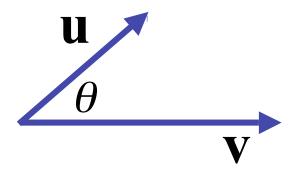
	u_1		v_1		
Г	u_2	•	v_2	5	$= (u_1 * v_1) + (u_2 * v_2) + (u_3 * v_3)$
	$ u_3 $				

Vector-Vector Multiplication

- multiply: vector * vector = scalar
- dot product, aka inner product

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \bullet \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = (u_1 * v_1) + (u_2 * v_2) + (u_3 * v_3)$$
• geometric interpretation
$$\mathbf{u} \bullet \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$$

- geometric interpretation
 - lengths, angles
 - can find angle between two vectors



Dot Product Geometry

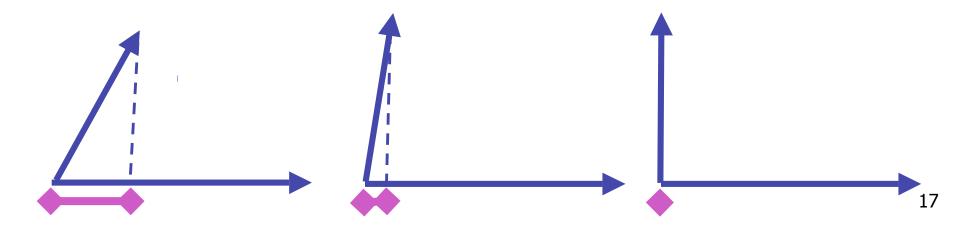
can find length of projection of u onto v

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$$

$$\|\mathbf{u}\| \cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|}$$

$$\|\mathbf{u}\| \cos \theta$$

• as lines become perpendicular, $\mathbf{u} \cdot \mathbf{v} \rightarrow 0$



Dot Product Example

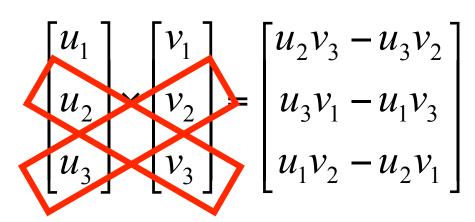
$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \bullet \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = (u_1 * v_1) + (u_2 * v_2) + (u_3 * v_3)$$

$$\begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 7 \\ 3 \end{bmatrix} = (6*1) + (1*7) + (2*3) = 6 + 7 + 6 = 19$$

- multiply: vector * vector = vector
- cross product
 - algebraic

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \times \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} u_2 v_3 - u_3 v_2 \\ u_3 v_1 - u_1 v_3 \\ u_1 v_2 - u_2 v_1 \end{bmatrix}$$

- multiply: vector * vector = vector
- cross product
 - algebraic



- multiply: vector * vector = vector
- cross product
 - algebraic

3
$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_2v_3 - u_3v_2 \\ u_3v_1 - u_1v_3 \\ u_1v_2 - u_2v_1 \end{bmatrix}$$

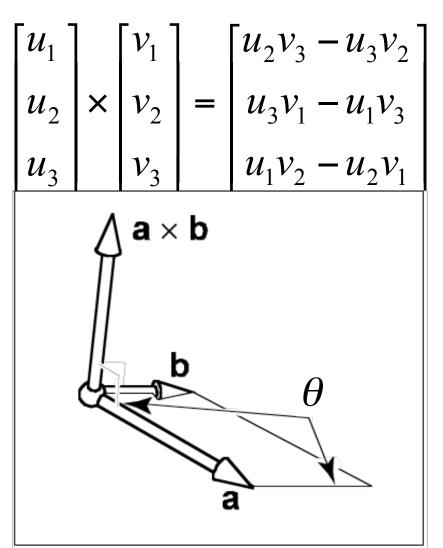
2 $\begin{bmatrix} u_1v_2 - u_2v_1 \\ u_1v_2 - u_2v_1 \end{bmatrix}$

blah blah 21

- multiply: vector * vector = vector
- cross product
 - algebraic
 - geometric

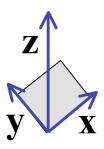
$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{u}\| \|\mathbf{v}\| \sin \theta$$

- $\|\mathbf{a} \times \mathbf{b}\|$ parallelogram area
- a×b perpendicular to parallelogram



RHS vs. LHS Coordinate Systems

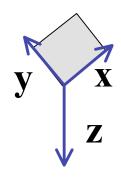
right-handed coordinate system convention



right hand rule: index finger x, second finger y; right thumb points up

$$z = x \times y$$

left-handed coordinate system



left hand rule: index finger x, second finger y; left thumb points down

$$z = x \times y$$

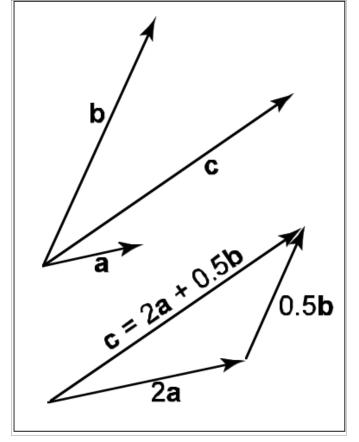
Basis Vectors

 take any two vectors that are linearly independent (nonzero and nonparallel)

· can use linear combination of these to define

any other vector:

$$\mathbf{c} = w_1 \mathbf{a} + w_2 \mathbf{b}$$

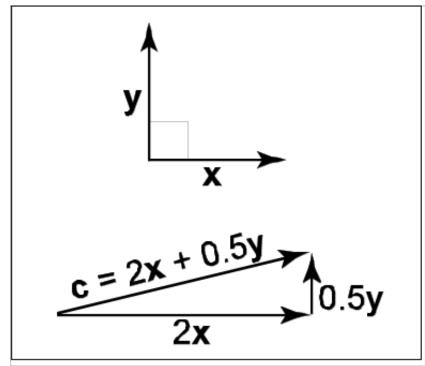


Orthonormal Basis Vectors

- if basis vectors are orthonormal (orthogonal (mutually perpendicular) and unit length)
 - we have Cartesian coordinate system
 - familiar Pythagorean definition of distance

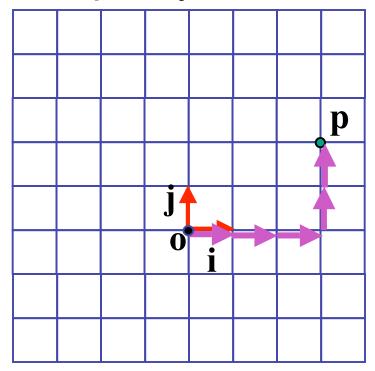
orthonormal algebraic properties

$$\|\mathbf{x}\| = \|\mathbf{y}\| = 1,$$
$$\mathbf{x} \cdot \mathbf{y} = 0$$

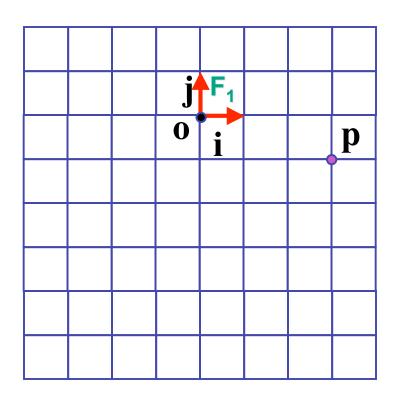


Basis Vectors and Origins

- coordinate system: just basis vectors
 - can only specify offset: vectors
- coordinate frame: basis vectors and origin
 - can specify location as well as offset: points

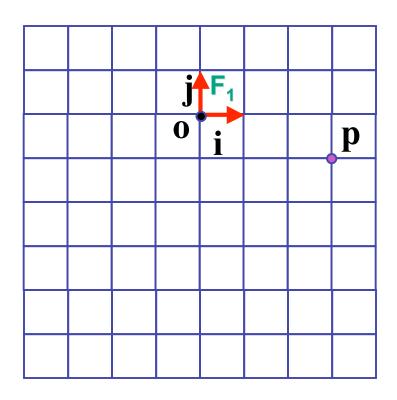


$$\mathbf{p} = \mathbf{o} + x\mathbf{i} + y\mathbf{j}$$



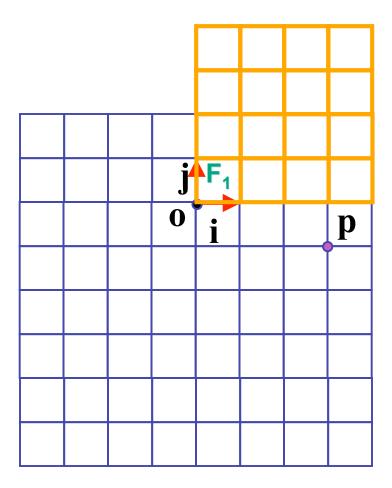
$$\mathbf{p} = \mathbf{o} + x\mathbf{i} + y\mathbf{j}$$

F₁



$$\mathbf{p} = \mathbf{o} + x\mathbf{i} + y\mathbf{j}$$

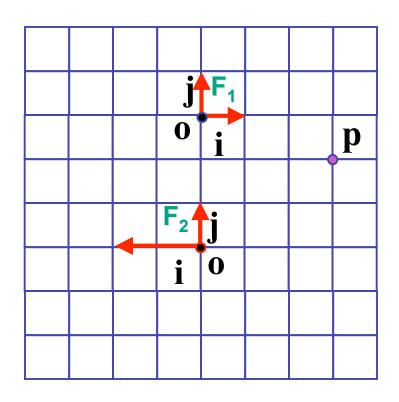
$$F_1$$
 p = (3,-1)



$$\mathsf{F}_1$$

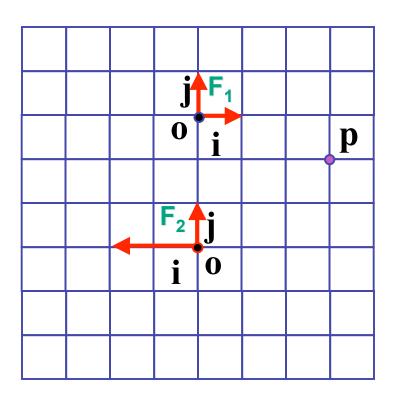
$$\mathbf{p} = \mathbf{o} + x\mathbf{i} + y\mathbf{j}$$

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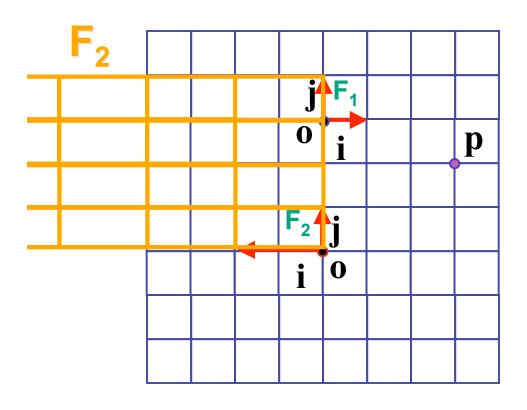
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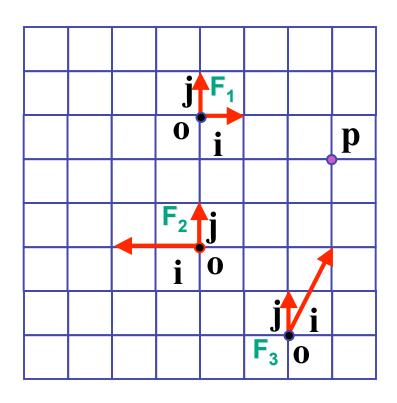
$$F_2$$
 p = (-1.5,2)



$$\mathbf{p} = \mathbf{o} + x\mathbf{i} + y\mathbf{j}$$

$$F_1$$
 p = (3,-1)

$$F_2$$
 p = (-1.5,2)

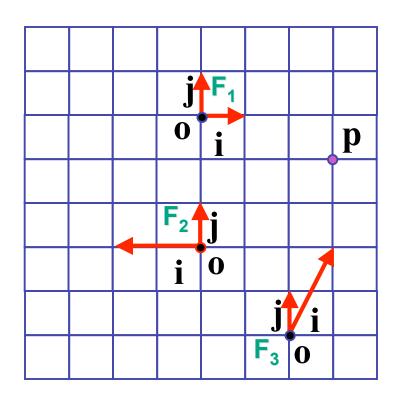


$$\mathbf{p} = \mathbf{o} + x\mathbf{i} + y\mathbf{j}$$

$$F_1$$
 p = (3,-1)

$$F_2$$
 p = (-1.5,2)

$$F_3$$

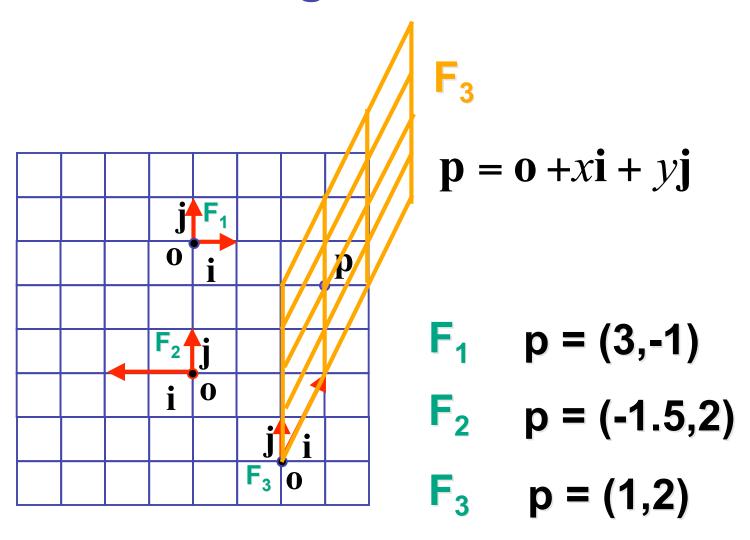


$$\mathbf{p} = \mathbf{o} + x\mathbf{i} + y\mathbf{j}$$

$$F_1$$
 p = (3,-1)

$$F_2$$
 p = (-1.5,2)

$$F_3$$
 p = (1,2)

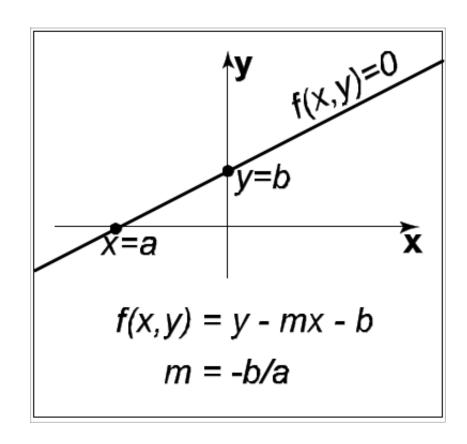


Named Coordinate Frames

- origin and basis vectors $\mathbf{p} = \mathbf{0} + a\mathbf{x} + b\mathbf{y} + c\mathbf{z}$
- pick canonical frame of reference
 - then don't have to store origin, basis vectors
 - just ${\bf p} = (a, b, c)$
 - convention: Cartesian orthonormal one on previous slide
- handy to specify others as needed
 - airplane nose, looking over your shoulder, ...
 - really common ones given names in CG
 - object, world, camera, screen, ...

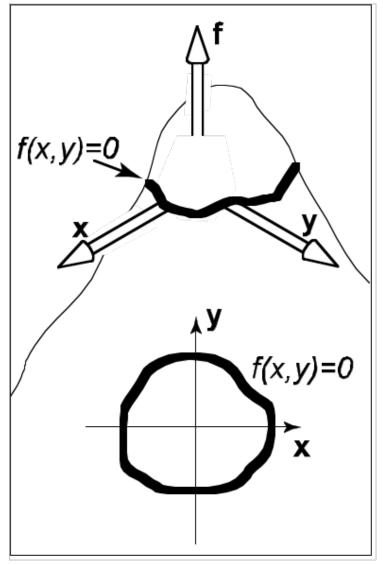
Lines

- slope-intercept form
 - y = mx + b
- implicit form
 - y mx b = 0
 - Ax + By + C = 0
 - f(x,y) = 0



Implicit Functions

- find where function is 0
 - plug in (x,y), check if
 - 0: on line
 - < 0: inside
 - > 0: outside
- analogy: terrain
 - sea level: f=0
 - altitude: function value
 - topo map: equal-value contours (level sets)



Implicit Circles

- $f(x,y) = (x-x_c)^2 + (y-y_c)^2 r^2$
 - circle is points (x,y) where f(x,y) = 0
- $p = (x, y), c = (x_c, y_c) : (\mathbf{p} \mathbf{c}) \cdot (\mathbf{p} \mathbf{c}) r^2 = 0$
 - points p on circle have property that vector from c to p dotted with itself has value r²
- $\|\mathbf{p} \mathbf{c}\|^2 r^2 = 0$
 - points points p on the circle have property that squared distance from c to p is r²
- $\|\mathbf{p} \mathbf{c}\| r = 0$
 - points p on circle are those a distance r from center point c

Parametric Curves

- parameter: index that changes continuously
 - (x,y): point on curve
 - t: parameter
- vector form

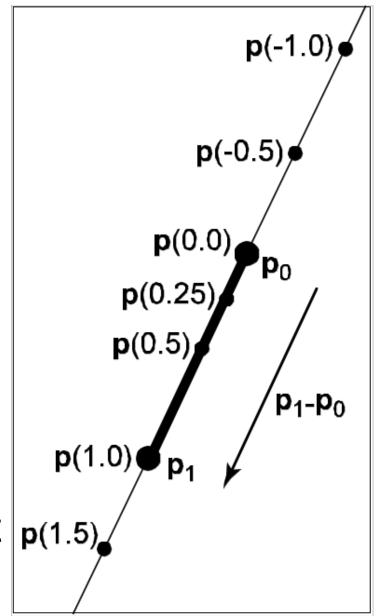
•
$$\mathbf{p} = f(t)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} g(t) \\ h(t) \end{bmatrix}$$

2D Parametric Lines

•
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 + t(x_1 - x_0) \\ y_0 + t(y_1 - y_0) \end{bmatrix}$$

- $\mathbf{p}(t) = \mathbf{p}_0 + t(\mathbf{p}_1 \mathbf{p}_0)$
- $\mathbf{p}(t) = \mathbf{o} + t(\mathbf{d})$
- start at point **p**_{0,}
 go towards **p**₁,
 according to parameter t
 - $\mathbf{p}(0) = \mathbf{p}_0, \ \mathbf{p}(1) = \mathbf{p}_1$



Linear Interpolation

- parametric line is example of general concept
 - $\mathbf{p}(t) = \mathbf{p}_0 + t(\mathbf{p}_1 \mathbf{p}_0)$
 - interpolation
 - **p** goes through **a** at *t* = 0
 - **p** goes through **b** at *t* = 1
 - linear
 - weights t, (1-t) are linear polynomials in t

Matrix-Matrix Addition

add: matrix + matrix = matrix

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} + \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} = \begin{bmatrix} n_{11} + m_{11} & n_{12} + m_{12} \\ n_{21} + m_{21} & n_{22} + m_{22} \end{bmatrix}$$

example

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 5 \\ 7 & 1 \end{bmatrix} = \begin{bmatrix} 1 + (-2) & 3 + 5 \\ 2 + 7 & 4 + 1 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 9 & 5 \end{bmatrix}$$

Scalar-Matrix Multiplication

multiply: scalar * matrix = matrix

$$a \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} a * m_{11} & a * m_{12} \\ a * m_{21} & a * m_{22} \end{bmatrix}$$

example

$$3\begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 3*2 & 3*4 \\ 3*1 & 3*5 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 3 & 15 \end{bmatrix}$$

can only multiply (n,k) by (k,m):
 number of left cols = number of right rows

$$\begin{bmatrix} a & b & c \\ e & f & g \end{bmatrix} \begin{bmatrix} h & l \\ j & k \\ l & m \end{bmatrix}$$

undefined

$$\begin{bmatrix} a & b & c \\ e & f & g \\ o & p & q \end{bmatrix} \begin{bmatrix} h & i \\ j & k \end{bmatrix}$$

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

$$p_{11} = m_{11}n_{11} + m_{12}n_{21}$$

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

$$p_{11} = m_{11}n_{11} + m_{12}n_{21}$$
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$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

$$p_{11} = m_{11}n_{11} + m_{12}n_{21}$$

$$p_{21} = m_{21}n_{11} + m_{22}n_{21}$$

$$p_{12} = m_{11}n_{12} + m_{12}n_{22}$$

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

$$p_{11} = m_{11}n_{11} + m_{12}n_{21}$$

$$p_{21} = m_{21}n_{11} + m_{22}n_{21}$$

$$p_{12} = m_{11}n_{12} + m_{12}n_{22}$$

$$p_{22} = m_{21}n_{12} + m_{22}n_{22}$$

row by column

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

$$p_{11} = m_{11}n_{11} + m_{12}n_{21}$$

$$p_{21} = m_{21}n_{11} + m_{22}n_{21}$$

$$p_{12} = m_{11}n_{12} + m_{12}n_{22}$$

$$p_{22} = m_{21}n_{12} + m_{22}n_{22}$$

noncommutative: AB != BA

Matrix-Vector Multiplication

points as column vectors: postmultiply

$$\begin{bmatrix} x' \\ y' \\ z' \\ h' \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ h \end{bmatrix}$$

$$\mathbf{p'} = \mathbf{Mp}$$

points as row vectors: premultiply

$$\begin{bmatrix} x' & y' & z' & h' \end{bmatrix} = \begin{bmatrix} x & y & z & h \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}^{T} \mathbf{p}^{\dagger T} = \mathbf{p}^{T} \mathbf{M}^{T}$$

Matrices

• transpose
$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}^T = \begin{bmatrix} m_{11} & m_{21} & m_{31} & m_{41} \\ m_{12} & m_{22} & m_{32} & m_{42} \\ m_{13} & m_{23} & m_{33} & m_{43} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}$$

identity

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

• inverse $AA^{-1} = I$

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$$

not all matrices are invertible

Matrices and Linear Systems

linear system of n equations, n unknowns

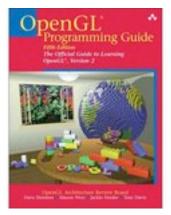
$$3x + 7y + 2z = 4$$
$$2x - 4y - 3z = -1$$
$$5x + 2y + z = 1$$

matrix form Ax=b

$$\begin{bmatrix} 3 & 7 & 2 \\ 2 & -4 & -3 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

Readings for Next Time

- Mon Jan 11
 - RB Chap Introduction to OpenGL
 - RB Chap State Management and Drawing Geometric Objects
 - RB App Basics of GLUT (Aux in v 1.1)
- RB = Red Book = OpenGL Programming Guide
- http://fly.cc.fer.hr/~unreal/theredbook/



Re-Reminder

no class this Friday (Jan 8)!