Ray Tracing

Wolfgang Heidrich

Course News

Assignment 3 (project)

- Due April 1

Reading (this week)

- Chapter 20 (color)

Reading (next week)

- Chapter 10 (ray tracing), except 10.8-10.10
- Chapter 14 (global illumination)

Course Topics for the Rest of the Term

Color

- Monday, Today

Ray-tracing & Global Illumination

- Friday, next week

Parametric Curves/Surfaces

- March 30/April 1
- Taught by Robert Bridson - I will be at a conference

Overview of current research

- April 3/6 (Ivo Ihrke – I am still at conference)

April 8 – Final Q&A (I will be back for that)

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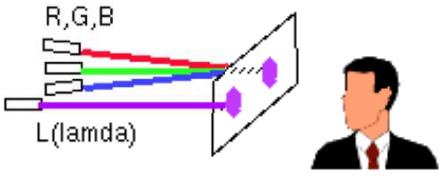
Color Matching Experiments

Performed in the 1930s



Idea: perceptually based measurement

- shine given wavelength (λ) on a screen
- User must control three pure lights producing three other wavelengths (say R=700 nm, G=546 nm, and B=438 nm)
- Adjust intensity of RGB until colors are identical

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Color Matching Experiment

Results

- It was found that any color S(λ) could be matched with three suitable primaries A(λ), B(λ), and C(λ)
  - *Used monochromatic light at 438, 546, and 700 nanometers*
- Also found the space is linear, I.e. if

$$R(\lambda) \equiv S(\lambda)$$

then

$$R(\lambda) + M(\lambda) \equiv S(\lambda) + M(\lambda)$$

and

$$k \cdot R(\lambda) \equiv k \cdot S(\lambda)$$

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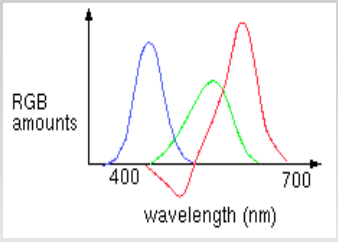
Negative Lobes

Actually:

- Exact target match possible sometimes requires "negative light"



- Some red has to be added to target color to permit exact match using "knobs" on RGB intensity output
- Equivalent mathematically to removing red from RGB output

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Negative Lobes

So:

- Can't generate **all** other wavelengths with **any** set of three **positive** monochromatic lights!

Solution:

- Convert to new synthetic "primaries" to make the color matching easy

$$\begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \\ \mathbf{Z} \end{pmatrix} = \begin{pmatrix} 2.36460 & -0.51515 & 0.00520 \\ -0.89653 & 1.42640 & -0.01441 \\ -0.46807 & 0.08875 & 1.00921 \end{pmatrix} \begin{pmatrix} \mathbf{R} \\ \mathbf{G} \\ \mathbf{B} \end{pmatrix}$$

Note:

- **R, G, B** are the same monochromatic primaries as before
- The corresponding matching functions x(λ), y(λ), z(λ) are now positive everywhere
- But the primaries contain "negative" light contributions, and are therefore not physically realizable

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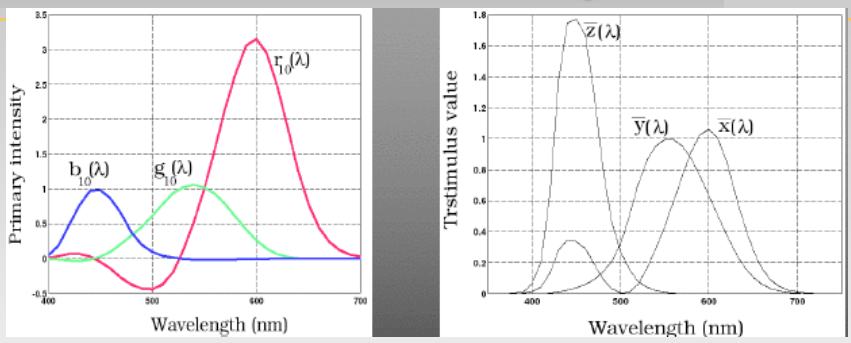
Matching Functions - Measured vs. CIE Color Spaces



Measured basis

- Monochromatic lights
- Physical observations
- Negative lobes

Transformed basis

- "imaginary" lights
- All positive, unit area matching functions
- Y is luminance, no hue
- X,Z no luminance

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Notation

Don't confuse:

- Synthetic primaries **X**, **Y**, **Z**
  - *Contain negative frequencies*
  - *Do not correspond to visible colors*
- Color matching functions x(λ), y(λ), z(λ)
  - *Are non-negative everywhere*
- Coefficients *X, Y, Z*
- Normalized **chromaticity values**

$$x = \frac{X}{X+Y+Z}, y = \frac{Y}{X+Y+Z}, z = \frac{Z}{X+Y+Z}$$

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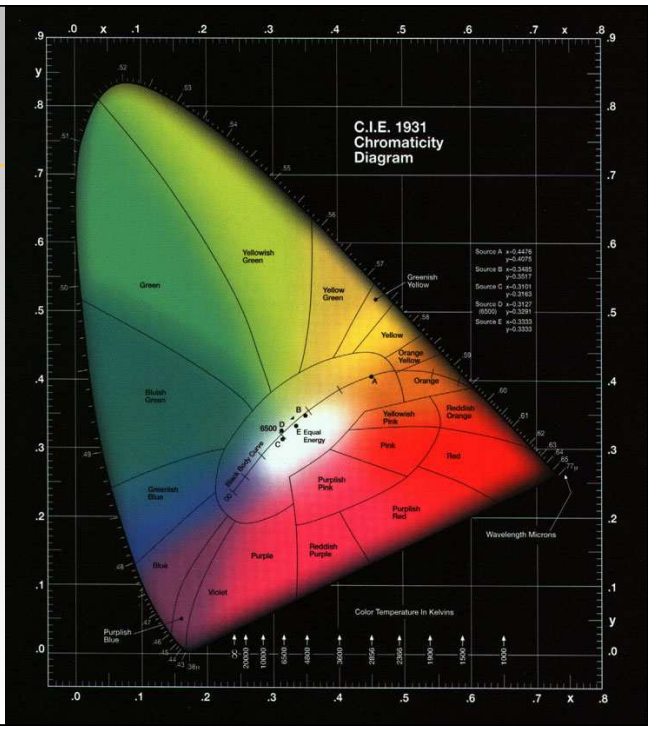
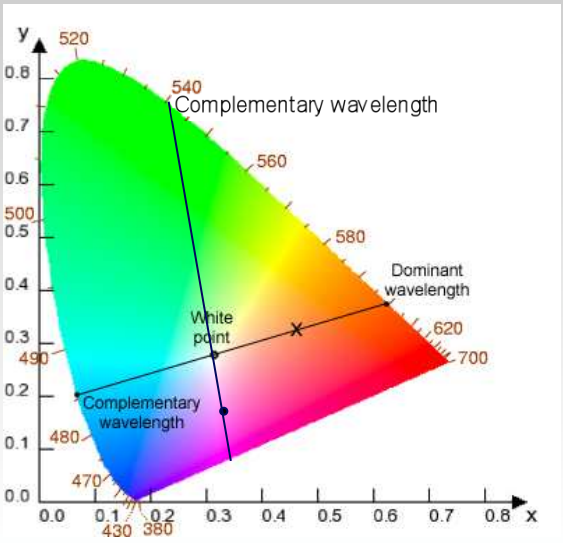
CIE Diagram

- Blackbody curve
- Illumination:
  - *Candle 2000K*
  - *Light bulb 3000K (A)*
  - *Sunset/ sunrise 3200K*
  - *Day light 6500K (D)*
  - *Overcast day 7000K*
  - *Lightning >20,000K*



Color Interpolation, Dominant & Opponent Wavelength



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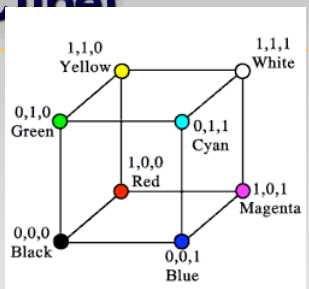
RGB Color Space (Color Cube)

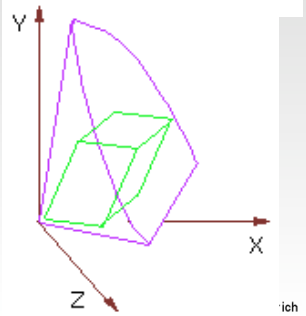
Define colors with (r, g, b) amounts of red, green, and blue

- Used by OpenGL
- Hardware-centric
- Describes the colors that can be generated with specific RGB light sources

RGB color cube sits within CIE color space

- Subset of perceivable colors
- Scaled, rotated, sheared cube

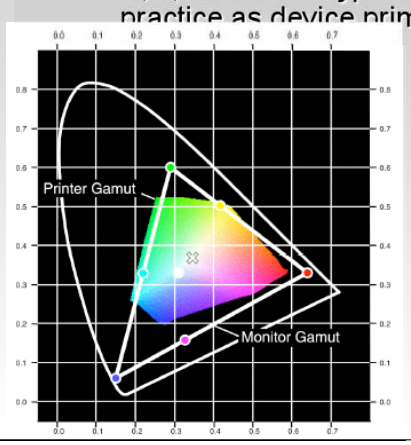


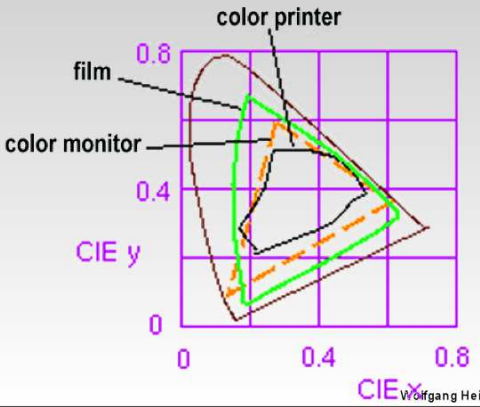


Device Color Gamuts

Use CIE chromaticity diagram to compare the gamuts of various devices

- X, Y, and Z are hypothetical light sources, not used in practice as device primaries





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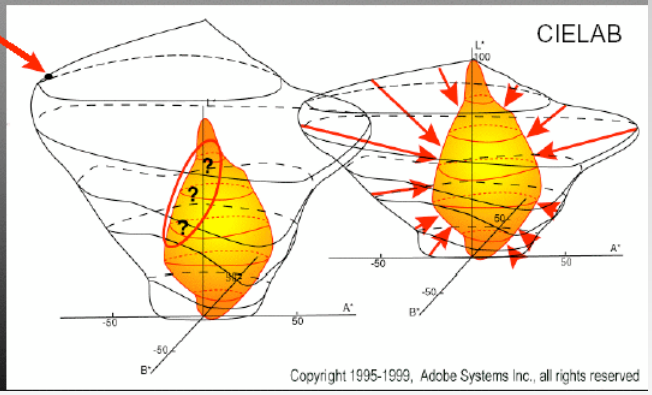
Gamut Mapping

Where does this color go?



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Additive vs. Subtractive Colors

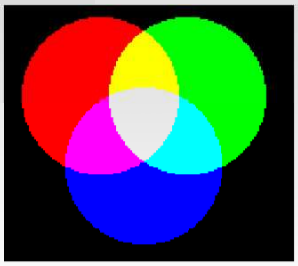
Additive: light

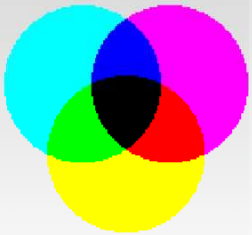
- Monitors, LCDs
- RGB model

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Subtractive: pigment

- Printers
- CMY(K) model





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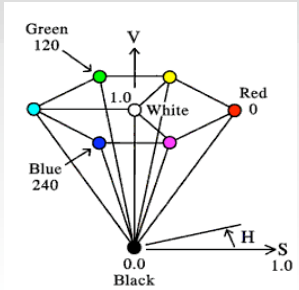
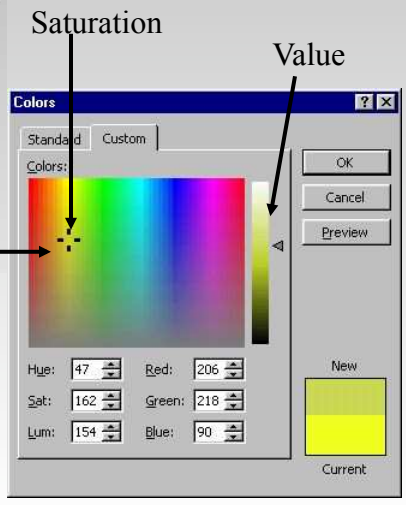
HSV Color Space

More intuitive color space for people

- H = Hue
- S = Saturation
- V = Value
  - *Or brightness B*
  - *Or intensity I*





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Monitors

Monitors have nonlinear response to input

- Characterize by gamma
  - *displayedIntensity* = a^{γ} *(maxIntensity)*

Gamma correction

- displayedIntensity = $\left(a^{1/\gamma}\right)^{\gamma}$ (maxIntensity)

  = a (maxIntensity)

Gamma for CRTs:

- Around 2.4

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Ray Tracing

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Overview

So far

- Real-time/HW rendering w/ Rendering Pipeline
- Rendering algorithms using the Rendering Pipeline

Now

- Ray-Tracing
  - *Simple algorithm for software rendering*
    - Usually offline (e.g. movies etc.)
    - But: research on making this method real-time
  - *Extremely flexible (new effects can easily be incorporated)*

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Ray-Tracing

Basic Algorithm (Whithead):

```
for every pixel p_i {
    Generate ray r from camera position through pixel p_i
    p_i = background color
    for every object o in scene {
        if( r intersects o && intersection is closer than
            previously found intersections )
            Compute lighting at intersection point, using local
            normal and material properties; store result in p_i
    }
}
```

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Ray-Tracing

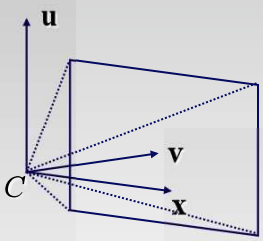
Issues:

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Efficient data structures so we don't have to test intersection with *every* object

Ray-Tracing – Generation of Rays

Camera Coordinate System

- Origin: C (camera position)
- Viewing direction: $\mathbf{v}$
- Up vector: $\mathbf{u}$
- x direction: $\mathbf{x} = \mathbf{v} \times \mathbf{u}$



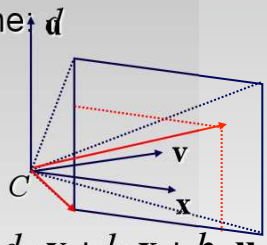
Note:

- Corresponds to viewing transformation in rendering pipeline!
- See gluLookAt…

Ray-Tracing – Generation of Rays

Other parameters:

- Distance of Camera from image plane: d
- Image resolution (in pixels): w, h
- Left, right, top, bottom boundaries in image plane: l, r, t, b



Then:

- Lower left corner of image: $O = C + d \cdot \mathbf{v} + l \cdot \mathbf{x} + b \cdot \mathbf{u}$
- Pixel at position i, j ($i=0..w-1, j=0..h-1$):

$$P_{i,j} = O + i \cdot \frac{r-l}{w-1} \cdot \mathbf{x} - j \cdot \frac{t-b}{h-1} \cdot \mathbf{u}$$

$$= O + i \cdot \Delta x \cdot \mathbf{x} - j \cdot \Delta y \cdot \mathbf{y}$$

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Ray-Tracing – Generation of Rays

Ray in 3D Space:

$$\mathrm{R}_{i,j}(t) = C + t \cdot (P_{i,j} - C) = C + t \cdot \mathbf{v}_{i,j}$$

where $t = 0 \ldots \infty$

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Ray-Tracing

Issues:

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Ray Intersections

Task:

- Given an object o, find ray parameter t, such that $\mathbf{R}_{i,j}(t)$ is a point on the object
  - *Such a value for t may not exist*
- Intersection test depends on geometric primitive

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Ray Intersections

Spheres at origin:

- Implicit function:

$$S(x,y,z): x^2 + y^2 + z^2 = r^2$$

- Ray equation:

$$\mathrm{R}_{i,j}(t) = C + t \cdot \mathbf{v}_{i,j} = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix} + t \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} c_x + t \cdot v_x \\ c_y + t \cdot v_y \\ c_z + t \cdot v_z \end{pmatrix}$$

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Ray Intersections

To determine intersection:

- Insert ray $\mathbf{R}_{i,j}(t)$ into $S(x,y,z)$:

$$(c_x + t \cdot v_x)^2 + (c_y + t \cdot v_y)^2 + (c_z + t \cdot v_z)^2 = r^2$$

- Solve for t (find roots)
  - *Simple quadratic equation*

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Ray Intersections

Other Primitives:

- Implicit functions:
  - *Spheres at arbitrary positions*
    - Same thing
  - *Conic sections (hyperboloids, ellipsoids, paraboloids, cones, cylinders)*
    - Same thing (all are quadratic functions!)
  - *Higher order functions (e.g. tori and other quartic functions)*
    - In principle the same
    - But root-finding difficult
    - Net to resolve to numerical methods

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Ray Intersections

Other Primitives (cont)

- Polygons:
  - *First intersect ray with plane*
    - linear implicit function
  - *Then test whether point is inside or outside of polygon (2D test)*
  - *For convex polygons*
    - Suffices to test whether point in on the right side of every boundary edge
    - Similar to computation of outcodes in line clipping

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Ray-Tracing

Issues:

- Generation of rays
- Intersection of rays with geometric primitives
- **Geometric transformations**
- Lighting and shading
- Efficient data structures so we don't have to test intersection with *every* object

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Ray-Tracing – Geometric Transformations

Geometric Transformations:

- Similar goal as in rendering pipeline:
  - *Modeling scenes convenient using different coordinate systems for individual objects*
- Problem:
  - *Not all object representations are easy to transform*
    - This problem is fixed in rendering pipeline by restriction to polygons (affine invariance!)

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Ray-Tracing – Geometric Transformations

Geometric Transformations:

- Similar goal as in rendering pipeline:
  - *Modeling scenes convenient using different coordinate systems for individual objects*
- Problem:
  - *Not all object representations are easy to transform*
    - This problem is fixed in rendering pipeline by restriction to polygons (affine invariance!)
  - *Ray-Tracing has different solution:*
    - The ray itself is always affine invariant!
    - Thus: transform ray into object coordinates!

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Ray-Tracing – Geometric Transformations

Ray Transformation:

- For intersection test, it is only important that ray is in same coordinate system as object representation
- Transform all rays into object coordinates
  - *Transform camera point and ray direction by inverse of model/view matrix*
- Shading has to be done in world coordinates (where light sources are given)
  - *Transform object space intersection point to world coordinates*
  - *Thus have to keep both world and object-space ray*

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Ray-Tracing

Issues:

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- **Lighting and shading**
- Efficient data structures so we don't have to test intersection with *every* object

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Ray-Tracing
Lighting and Shading

Local Effects:

- Local Lighting
  - *Any reflection model possible*
  - *Have to talk about light sources, normals…*
- Texture mapping
  - *Color textures*
  - *Bump maps*
  - *Environment maps*
  - *Shadow maps*

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Ray-Tracing Local Lighting

Light sources:

- For the moment: point and directional lights
- Later: are light sources
- More complex lights are possible
  - *Area lights*
  - *Global illumination*
    - Other objects in the scene reflect light
    - Everything is a light source!
    - Talk about this on Monday

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Ray-Tracing Local Lighting

Local surface information (normal…)

- For implicit surfaces $F(x,y,z)=0$: normal $\mathbf{n}(x,y,z)$ can be easily computed at every intersection point using the gradient

$$\mathbf{n}(x,y,z) = \begin{pmatrix} \partial F(x,y,z)/\partial x \\ \partial F(x,y,z)/\partial y \\ \partial F(x,y,z)/\partial z \end{pmatrix}$$

- Example: $F(x,y,z) = x^2 + y^2 + z^2 - r^2$

$$\mathbf{n}(x,y,z) = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

Needs to be normalized!

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Ray-Tracing Local Lighting

Local surface information

- Alternatively: can interpolate per-vertex information for triangles/meshes as in rendering pipeline
  - *Phong shading!*
  - *Same as discussed for rendering pipeline*
- Difference to rendering pipeline:
  - *Interpolation cannot be done incrementally*
  - *Have to compute Barycentric coordinates for every intersection point (e.g plane equation for triangles)*

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Ray-Tracing Texture Mapping

Approach:

- Works in principle like in rendering pipeline
  - *Given s, t parameter values, perform texture lookup*
  - *Magnification, minification just as discussed*
- Problem: how to get s, t
  - *Implicit surfaces often don't have parameterization*
  - *For special cases (spheres, other conic sections), can use parametric representation*
  - *Triangles/meshes: use interpolation from vertices*

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Ray-Tracing Lighting and Shading

Global Effects

- Shadows
- Reflections/refractions

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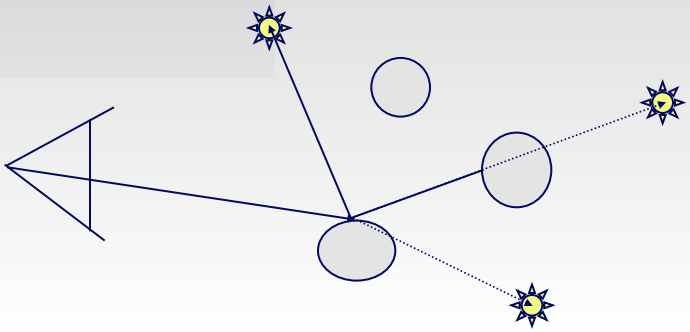
Ray-Tracing Shadows

Approach:

- To test whether point is in shadow, send out *shadow rays* to all light sources
  - *If ray hits another object, the point lies in shadow*



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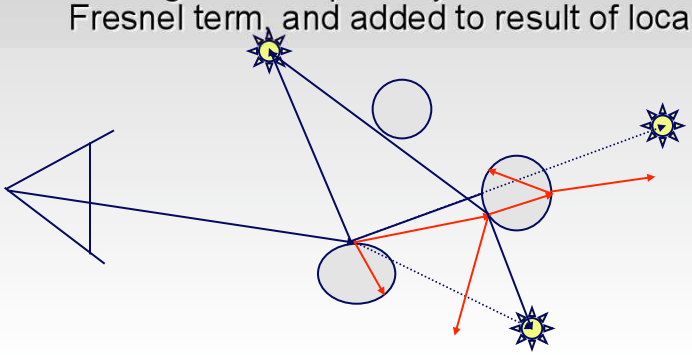
Ray-Tracing Reflections/Refractions

Approach:

- Send rays out in reflected and refracted direction to gather incoming light
- That light is multiplied by local surface color and Fresnel term, and added to result of local shading



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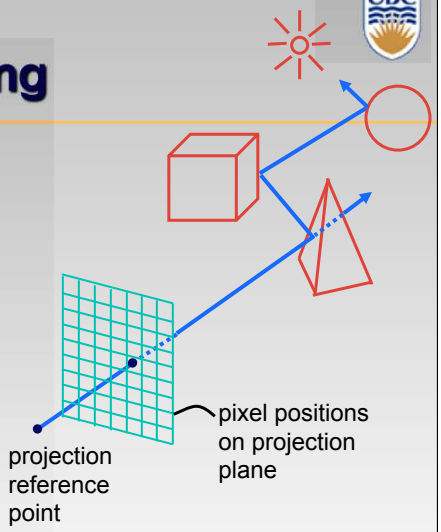
Recursive Ray Tracing

Ray tracing can handle

- Reflection (chrome)
- Refraction (glass)
- Shadows

Spawn secondary rays

- Reflection, refraction
  - *If another object is hit, recurse to find its color*
- Shadow
  - *Cast ray from intersection point to light source, check if intersects another object*



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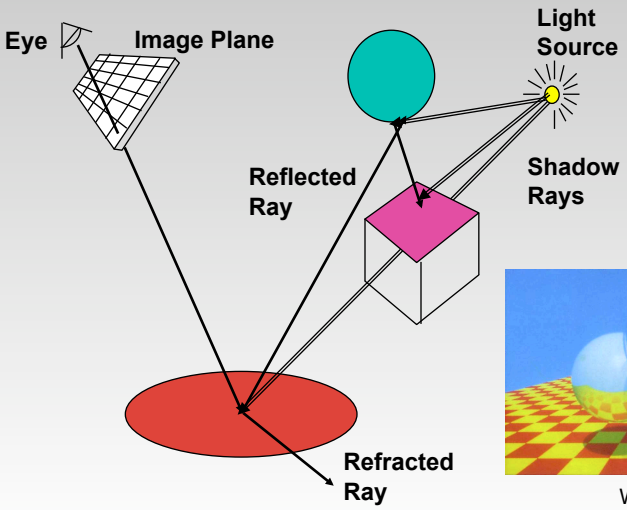
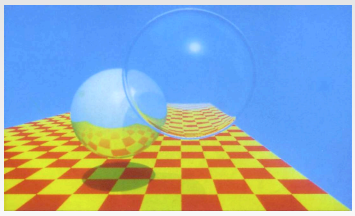
Recursive Ray-Tracing





Whitted, 1980

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Recursive Ray-Tracing Algorithm

```
RayTrace(r,scene)
obj := FirstIntersection(r,scene)
if (no obj)  return BackgroundColor;
else begin
   if ( Reflect(obj) ) then
      reflect_color := RayTrace(ReflectRay(r,obj));
   else
      reflect_color := Black;
  if ( Transparent(obj) ) then
      refract_color := RayTrace(RefractRay(r,obj));
   else
      refract_color := Black;
  return Shade(reflect_color,refract_color,obj);
end;
```

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Algorithm Termination Criteria

Termination criteria

- No intersection
- Reach maximal depth
  - *Number of bounces*
- Contribution of secondary ray attenuated below threshold
  - *Each reflection/refraction attenuates ray*

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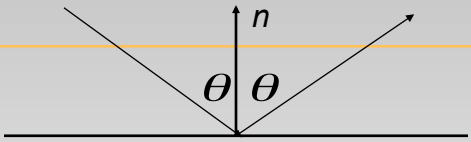
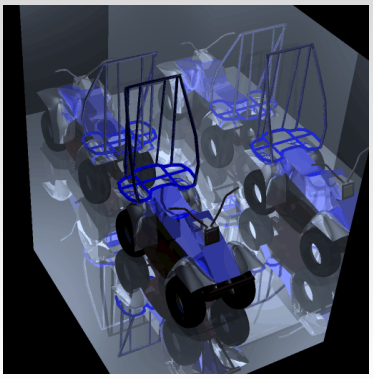
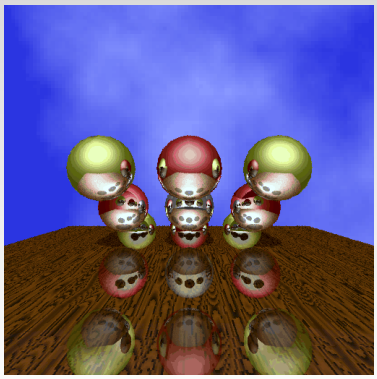
Reflection

Mirror effects

- Perfect specular reflection







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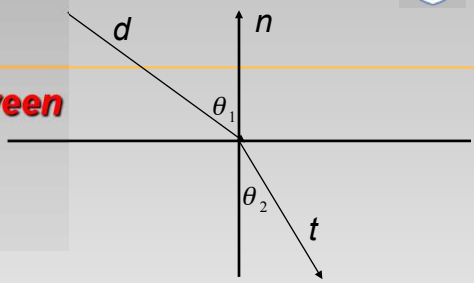
Refraction

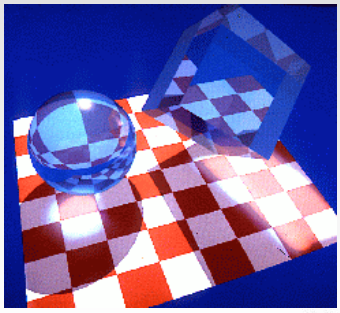


Happens at interface between transparent object and surrounding medium

- E.g. glass/air boundary

Snell's Law

- $c_1 \sin\theta_1 = c_2 \sin\theta_2$
- Light ray bends based on refractive indices c_1, c_2



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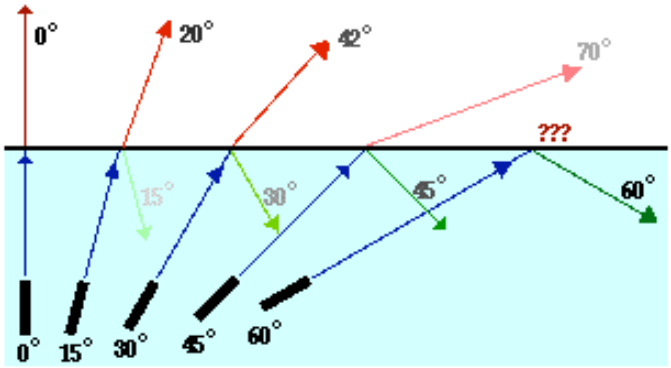
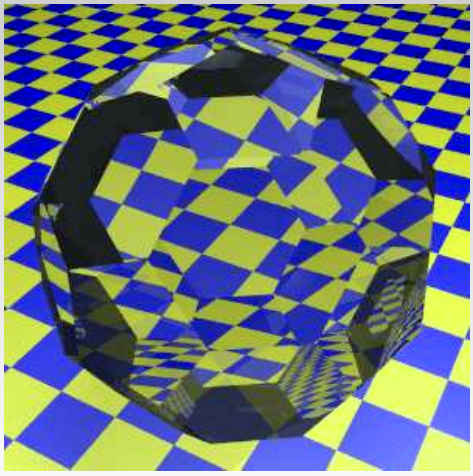
Total Internal Reflection

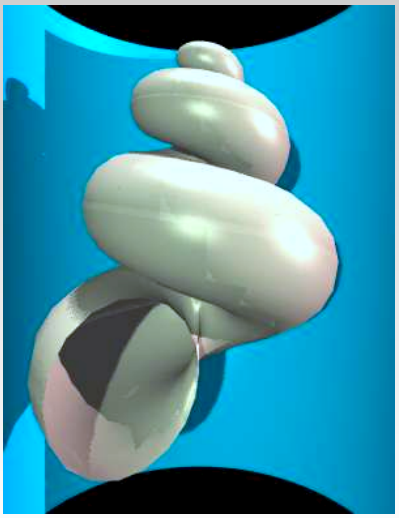
As the angle of incidence increases from 0 to greater angles ...



...the refracted ray becomes dimmer (there is less refraction)
...the reflected ray becomes brighter (there is more reflection)
...the angle of refraction approaches 90 degrees until finally a refracted ray can no longer be seen.

Ray-Tracing Example Images





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Ray-Tracing Terminology

Terminology:

- Primary ray: ray starting at camera
- Shadow ray
- Reflected/refracted ray
- Ray tree: all rays directly or indirectly spawned off by a single primary ray

Note:

- Need to limit maximum depth of ray tree to ensure termination of ray-tracing process!

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Ray-Tracing

Issues:

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- **Efficient data structures so we don't have to test intersection with *every* object**

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Ray Tracing

Data Structures

- Goal: reduce number of intersection tests per ray
- Lots of different approaches:
  - *(Hierarchical) bounding volumes*
  - *Hierarchical space subdivision*
    - Oct-tree, k-D tree, BSP tree

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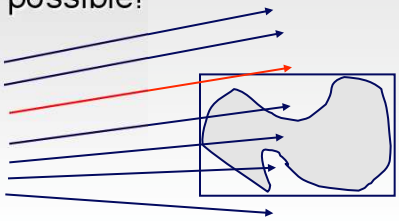
Bounding Volumes

Idea:

- Rather than testing every ray against a potentially very complex object (e.g. triangle mesh), do a quick *conservative* test first which eliminates most rays
  - *Surround complex object by simple, easy to test geometry (typically sphere or axis-aligned box)*
    - Want to make bounding volume as tight as possible!



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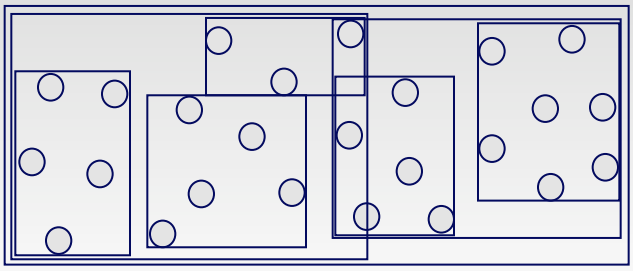
Hierarchical Bounding Volumes

Extension of previous idea:

- Use bounding volumes for groups of objects



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Spatial Subdivision Data Structures

Bounding Volumes:

- Find simple object completely enclosing complicated objects
  - *Boxes, spheres*
- Hierarchically combine into larger bounding volumes

Spatial subdivision data structure:

- Partition the whole space into cells
  - *Grids, oct-trees, (BSP trees)*
- Simplifies and accelerates traversal
- Performance less dependent on order in which objects are inserted

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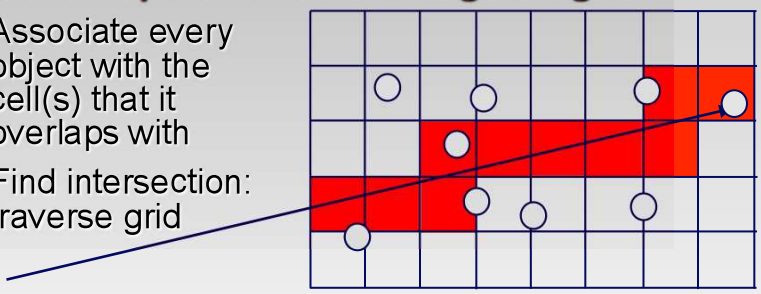
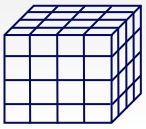
Regular Grid

Subdivide space into rectangular grid:

- Associate every object with the cell(s) that it overlaps with
- Find intersection: traverse grid



In 3D: regular grid of cubes (**voxels**):



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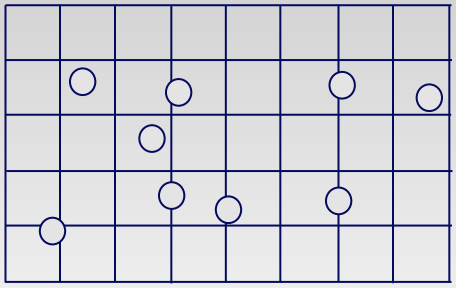
Creating a Regular Grid

Steps:

- Find bounding box of scene
- Choose grid resolution in x, y, z
- Insert objects
- Objects that overlap multiple cells get referenced by all cells they overlap



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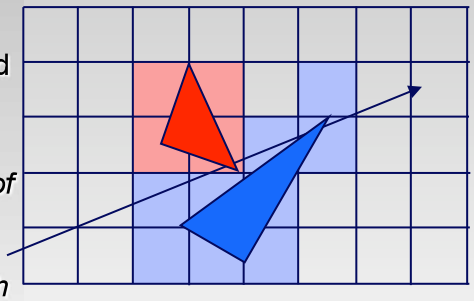
Grid Traversal

Traversal:

- Start at ray origin
- While no intersection found
  - *Go to next grid cell along ray*
  - *Compute intersection of ray with all objects in the cell*
  - *Determine closest such intersection*
  - ***Check if that intersection is inside the cell***
  - *If so, terminate search*



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Traversal

Note:

- This algorithm calls for computing the intersection points multiple times (once per grid cell)
- In practice: store intersections for a (ray, object) pair once computed, reuse for future cells

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Regular Grid Discussion

Advantages?

- Easy to construct
- Easy to traverse

Disadvantages?

- May be only sparsely filled
- Geometry may still be clumped

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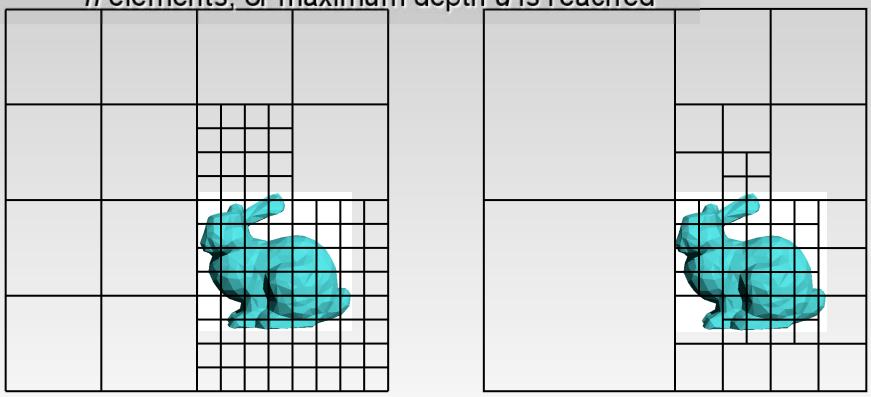
Adaptive Grids

- Subdivide until each cell contains no more than *n* elements, or maximum depth *d* is reached



Nested Grids Octree/(Quadtree)

- This slide and the next are curtsey of Fredo Durand at MIT

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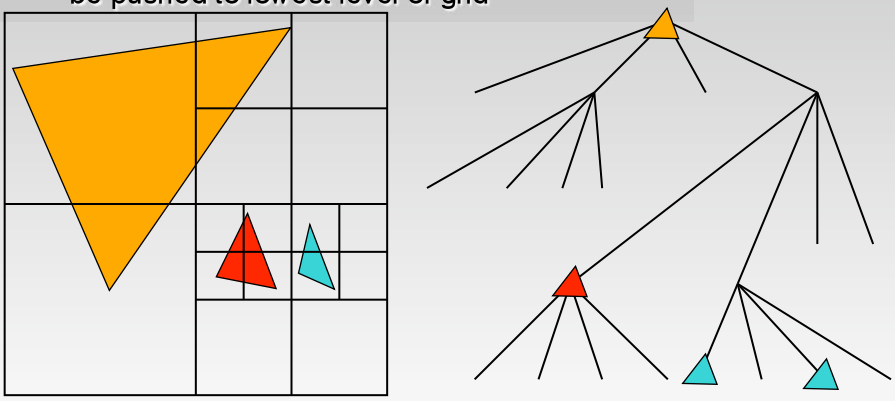
Primitives in an Adaptive Grid

- Can live at intermediate levels, or be pushed to lowest level of grid



Octree/(Quadtree)

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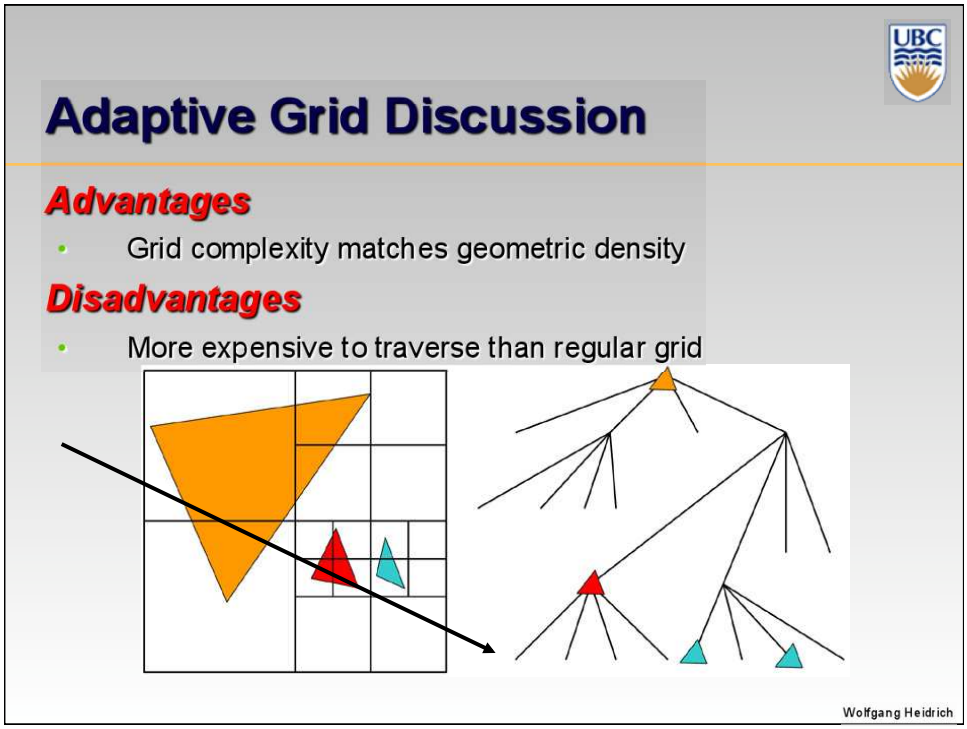
Adaptive Grid Discussion
Advantages
Grid complexity matches geometric density
Disadvantages
More expensive to traverse than regular grid
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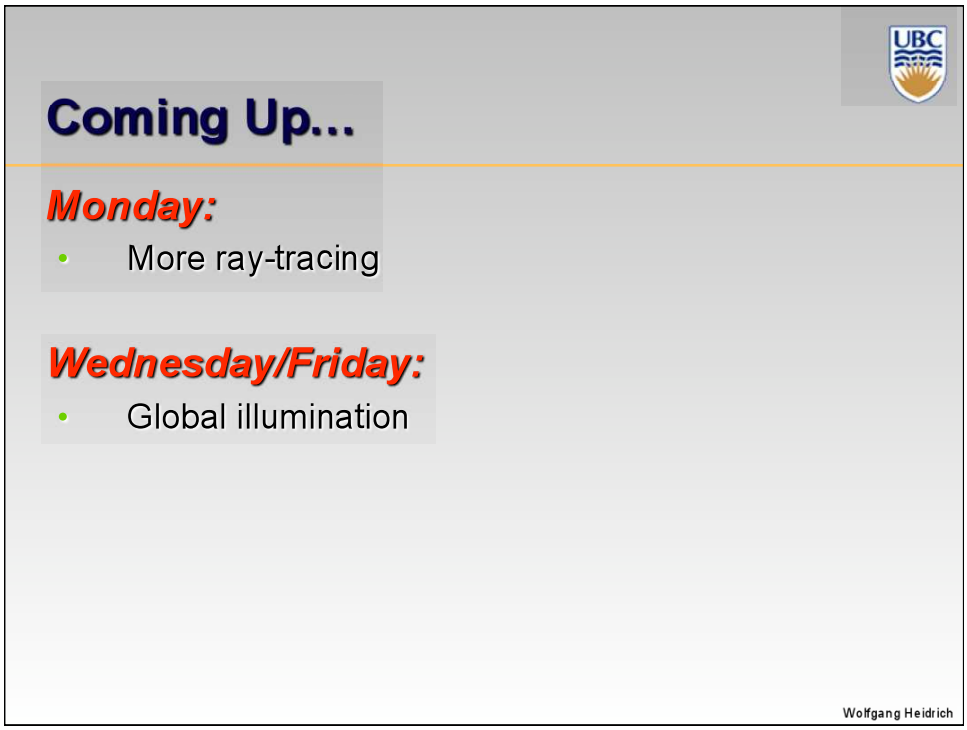
Coming Up...
Monday:
More ray-tracing
Wednesday/Friday:
Global illumination
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