Color

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Course News

Assignment 3 (project)
• Due April 1

Reading (this week)
• Chapter 20 (color)

Reading (this week & next)
• Chapter 10 (ray tracing)
Course Topics for the Rest of the Term

**Color**
- Monday, Today

**Ray-tracing & Global Illumination**
- Friday, next week

**Parametric Curves/Surfaces**
- March 30/April 1
- Taught by Robert Bridson - I will be at a conference

**Overview of current research**
- April 3/6 (Ivo Ihrke – I am still at conference)

**April 8 – Final Q&A (I will be back for that)**

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Electromagnetic Spectrum

**THE ELECTROMAGNETIC SPECTRUM**

- Wavelength (meters)
- Frequency (waves per second)
- Energy of light photon (electron-volts)
- Sources (radio, TV, radar, light, X-rays, gamma rays)
- Common name of waves (radio, infrared, ultraviolet, X-rays, gamma rays)
- Map of a wavelength

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Light Sources

Common light sources differ in the kind of spectrum they emit:
- Continuous spectrum
  - Energy is emitted at all wavelengths
    - Blackbody radiation
    - Tungsten light bulbs
    - Certain fluorescent lights
    - Sunlight
    - Electrical arcs
- Line spectrum
  - Energy is emitted at certain discrete frequencies

Blackbody Radiation

Black body
- Dark material, so that reflection can be neglected
- Spectrum of emitted light changes with temperature
  - This is the origin of the term “color temperature”
    - E.g. when setting a white point for your monitor
  - Cold: mostly infrared
  - Hot: redish
  - Very hot: bluish
- Demo:
  http://www.mhhe.com/physsci/astronomy/applets/Blackbody/frame.html
Line Spectrum

**Examples:**
- Ionized gases
- Lasers
- Some fluorescent lamps

Physiology of Vision

**The retina**
- Rods
  - B/w, edges
- Cones
  - Color!
Physiology of Vision

*Center of retina is densely packed region called the fovea.*

- Cones much denser here than the *periphery*

1.35 mm from retina center

4 μm

8 mm from retina center

Color/Lightness Constancy

*Do they match?*

Image courtesy of John McGinn

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Color/Lightness Constancy

Color Constancy

- Automatic “white balance” from change in illumination
- Vast amount of processing behind the scenes!
- Colorimetry vs. perception

From Color Appearance Models, fig 8-1
Tristimulus Theory of Color Vision

• Although light sources can have extremely complex spectra, it was empirically determined that colors could be described by only 3 **primaries**

• Colors that look the same but have different spectra are called **metamers**

• Metamer demo:
  http://www.cs.brown.edu/exploratories/freeSoftware/catalogs/color_theory.html

Color Matching Experiments

**Performed in the 1930s**

**Idea: perceptually based measurement**

• shine given wavelength (λ) on a screen

• User must control three pure lights producing three other wavelengths (say R=700 nm, G=546 nm, and B=438 nm)

• Adjust intensity of RGB until colors are identical
Color Matching Experiment

Results
- It was found that any color $S(\lambda)$ could be matched with three suitable primaries $A(\lambda)$, $B(\lambda)$, and $C(\lambda)$
  - Used monochromatic light at 438, 546, and 700 nanometers
- Also found the space is linear, i.e. if $R(\lambda) \equiv S(\lambda)$
  then $R(\lambda) + M(\lambda) \equiv S(\lambda) + M(\lambda)$
  and $k \cdot R(\lambda) \equiv k \cdot S(\lambda)$

Negative Lobes

Actually:
- Exact target match possible sometimes requires “negative light”
- Some red has to be added to target color to permit exact match using “knobs” on RGB intensity output
- Equivalent mathematically to removing red from RGB output
**Notation**

**Don’t confuse:**
- Primaries: the spectra of the three different light sources: **R, G, B**
  - For the matching experiments, these were **monochromatic** (i.e. single wavelength) light!
  - Display primaries usually have a wider spectrum
- Coefficients **R, G, B**
  - Specify how much of **R, G, B** is in a given color
- Color matching functions: r(λ), g(λ), b(λ)
  - Specify how much of **R, G, B** is needed to produce a color that is a metamer for pure monochromatic light of wavelength λ

**Determine Matching for Arbitrary Spectra**

**Given**
- Some light spectrum s(λ)

**How do we find R, G, B?**
- Coefficients to describe color of s(λ) in RGB space
  - i.e. as mixtures of the specific monochromatic colors mentioned!
Determine Matching for Arbitrary Spectra

**Given**
- Some light spectrum \(s(\lambda)\)

**How do we find \(R, G, B\)?**
- Coefficients to describe color of \(s(\lambda)\) in RGB space

**A: Integrate with color matching functions**
- Treat spectra as vector space
- Dot product of \(s_1, s_2\) defined as
  \[
  \int s_1(\lambda)s_2(\lambda)d\lambda
  \]
  \[
  R = \int s(\lambda)r(\lambda)d\lambda
  \]
  \[
  G = \int s(\lambda)g(\lambda)d\lambda
  \]
  \[
  B = \int s(\lambda)b(\lambda)d\lambda
  \]

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**Negative Lobes**

**So:**
- Can't generate all other wavelengths with any set of three positive monochromatic lights!

**Solution:**
- Convert to new synthetic "primaries" to make the color matching easy
  \[
  \begin{pmatrix}
  X \\
  Y \\
  Z
  \end{pmatrix} = \begin{pmatrix}
  2.36460 & -0.89653 & -0.46807 \\
  -0.51515 & 1.42640 & 0.08875 \\
  0.00520 & -0.01441 & 1.00921
  \end{pmatrix}
  \begin{pmatrix}
  R \\
  G \\
  B
  \end{pmatrix}
  \]

**Note:**
- \(R, G, B\) are the same monochromatic primaries as before
- The corresponding matching functions \(x(\lambda), y(\lambda), z(\lambda)\) are now positive everywhere
- But the primaries contain "negative" light contributions, and are therefore not physically realizable

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Negative Lobes

In general:
- It is not possible to find three color primaries (monochromatic or continuous spectrum) that can produce all visible colors with positive weights.

Q: How can this be?
- We only have 3 types of cones, after all?

A: the spectral sensitivity curves of cones overlap
- i.e. the cones span a linear color (vector) space, but this space is not orthonormal!
- Orthonormalization introduces negative weights…
Matching Functions - CIE Color Space

- CIE defined three “imaginary” lights X, Y, and Z, any wavelength \( \lambda \), can be matched perceptually by positive combinations

![Graph showing X, Y, and Z functions]

Matching Functions - Measured vs. CIE Color Spaces

**Measured basis**
- Monochromatic lights
- Physical observations
- Negative lobes

**Transformed basis**
- “Imaginary” lights
- All positive, unit area matching functions
- Y is luminance, no hue
- X, Z no luminance
Notation

Don’t confuse:
- Synthetic primaries $X, Y, Z$
  - Contain negative frequencies
  - Do not correspond to visible colors
- Color matching functions $x(\lambda), y(\lambda), z(\lambda)$
  - Are non-negative everywhere
- Coefficients $X, Y, Z$
- Normalized chromaticity values

$$x = \frac{X}{X + Y + Z}, \quad y = \frac{Y}{X + Y + Z}, \quad z = \frac{Z}{X + Y + Z}$$

CIE Gamut and $\lambda$ Chromaticity Diagram

3D gamut

Chromaticity diagram
- Hue only, no intensity
Facts about the CIE “Horseshoe” Diagram

- All visible colors lie inside the horseshoe
  - *Result from color matching experiments*
- Spectral (monochromatic) colors lie around the border
  - *The straight line between blue and red contains the purple tones*
- Colors combine linearly (i.e. along lines), since the xy-plane is a plane from a linear space

Facts about the CIE “Horseshoe” Diagram (cont.)

*A point C can be chosen as a white point corresponding to an illuminant*

- Usually this point is of the curve swept out by the black body radiation spectra for different temperatures
- Relative to C, two colors are called complementary if they are located along a line segment through C, but on opposite sides (i.e. C is an affine combination of the two colors)
- The dominant wavelength of the color is found by extending the line from C through the color to the edge of the diagram
- Some colors (i.e. purples) do not have a dominant wavelength, but their complementary color does
CIE Diagram

- Blackbody curve
- Illumination:
  - Candle 2000K
  - Light bulb 3000K (A)
  - Sunset/sunrise 3200K
  - Day light 6500K (D)
  - Overcast day 7000K
  - Lightning >20,000K

Color Interpolation, Dominant & Opponent Wavelength

Complementary wavelength
RGB Color Space (Color Cube)

- Define colors with \((r, g, b)\) amounts of red, green, and blue
  - Used by OpenGL
  - Hardware-centric
  - Describes the colors that can be generated with specific RGB light sources

RGB color cube sits within CIE color space
- Subset of perceivable colors
- Scaled, rotated, sheared cube

Device Color Gamuts

- Use CIE chromaticity diagram to compare the gamuts of various devices
  - X, Y, and Z are hypothetical light sources, not used in practice as device primaries
Gamut Mapping

Where does this color go?

Additive vs. Subtractive Colors

**Additive: light**
- Monitors, LCDs
- RGB model

\[
\begin{bmatrix}
C \\
M \\
Y
\end{bmatrix} = \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix} - \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

**Subtractive: pigment**
- Printers
- CMY(K) model
HSV Color Space

More intuitive color space for people

- H = Hue
- S = Saturation
- V = Value
  - Or brightness B
  - Or intensity I

Monitors

Monitors have nonlinear response to input

- Characterize by **gamma**
  - displayedIntensity = a\(^\gamma\) (maxIntensity)

**Gamma correction**

- displayedIntensity = \((a^{1/\gamma})^\gamma\) (maxIntensity)
  = a (maxIntensity)

**Gamma for CRTs:**

- Around 2.4
Coming Up...

*Friday, next week:*

- Ray-tracing