Sampling & Reconstruction

Wolfgang Heidrich

Course News

Assignment 3 (project)

- Due April 1

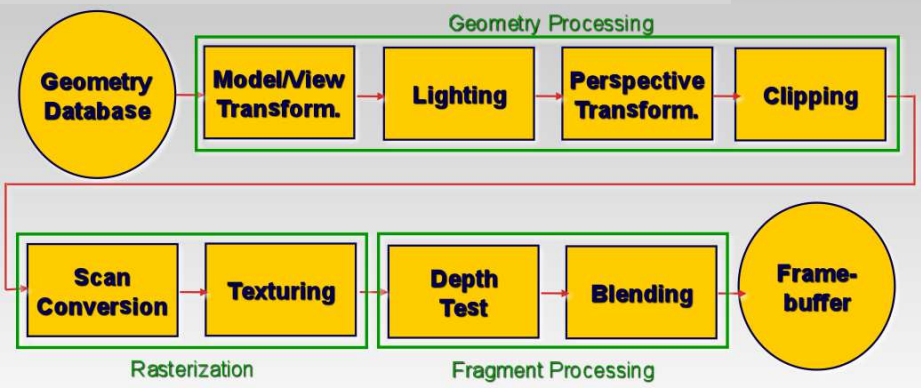
Homework

- This week: H6 – texture mapping
- Next week: no new homework (focus on quiz prep)

Quiz 2

- Friday, March 13
- Topics:
  - *Everything after transformations up to and including this lecture*
  - *Questions on rendering pipeline as a whole*

The Rendering Pipeline



Wolfgang Heidrich

Samples

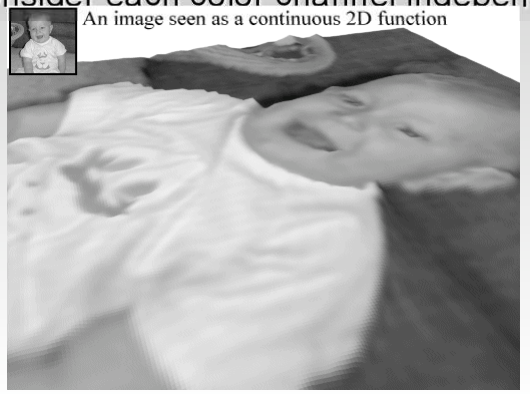
- Most things in the real world are continuous
- Everything in a computer is discrete
- The process of mapping a continuous function to a discrete one is called sampling
- The process of mapping a discrete function to a continuous one is called reconstruction
- The process of mapping a continuous variable to a discrete one is called quantization
- Rendering an image requires sampling and quantization
- Displaying an image involves reconstruction

Wolfgang Heidrich

Images

An image is a 2D function I(x, y)

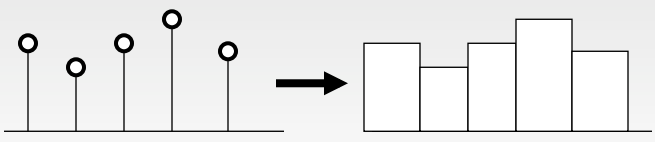
- Specifies intensity for each point (x, y)
- (we consider each color channel independently)



Wolfgang Heidrich

Image Sampling and Reconstruction

- Convert continuous image to discrete set of samples
- Display hardware reconstructs samples into continuous image
  - *Finite sized source of light for each pixel*



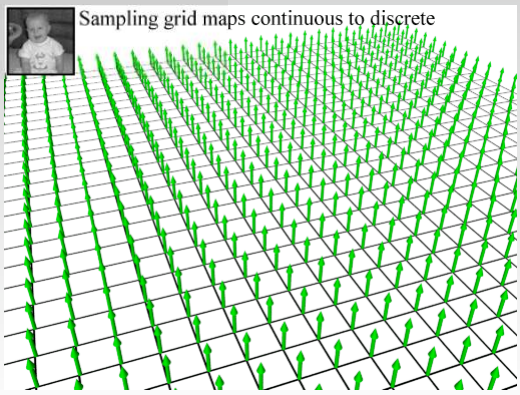
discrete input values

continuous light output

Wolfgang Heidrich

Point Sampling an Image

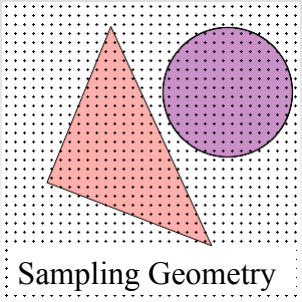
- Simplest sampling is on a grid
- Sample depends solely on value at grid points

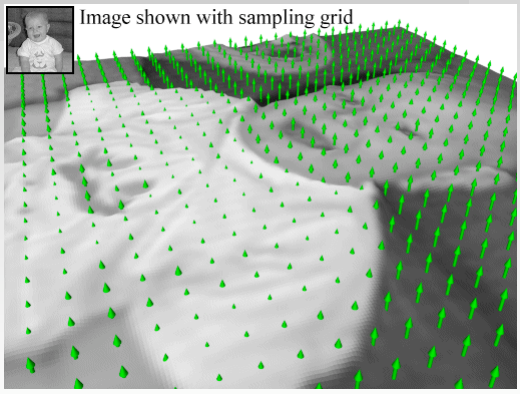


Wolfgang Heidrich

Point Sampling

Multiply sample grid by image intensity to obtain a discrete set of points, or samples.





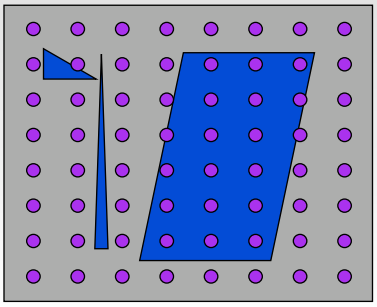
Wolfgang Heidrich

Sampling Errors

Some objects missed entirely, others poorly sampled

- Could try unweighted or weighted area sampling
- But how can we be sure we show everything?

Need to think about entire class of solutions!



Wolfgang Heidrich

Image As Signal

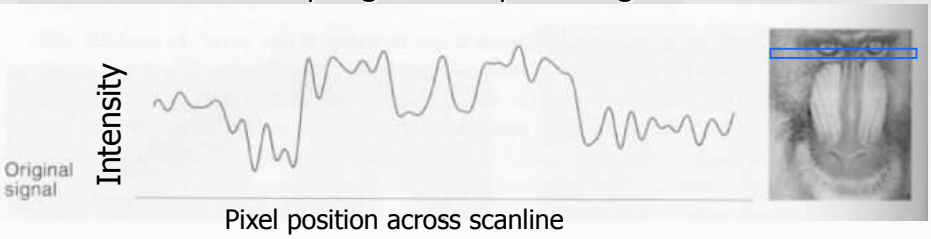
Image as spatial signal

2D raster image

- Discrete sampling of 2D spatial signal

1D slice of raster image

- Discrete sampling of 1D spatial signal



Examples from Foley, van Dam, Feiner, and Hughes

Wolfgang Heidrich

Sampling Theory

How would we generate a signal like this out of simple building blocks?

Theorem

- Any signal can be represented as an (infinite) sum of sine waves at different frequencies

Wolfgang Heidrich

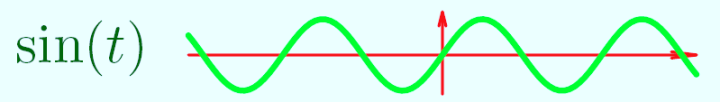
Sampling Theory in a Nutshell

Terminology

- Wavelength – length of repeated sequence on infinite signal
- Frequency – 1/wavelength (number of repeated sequences in unit length)

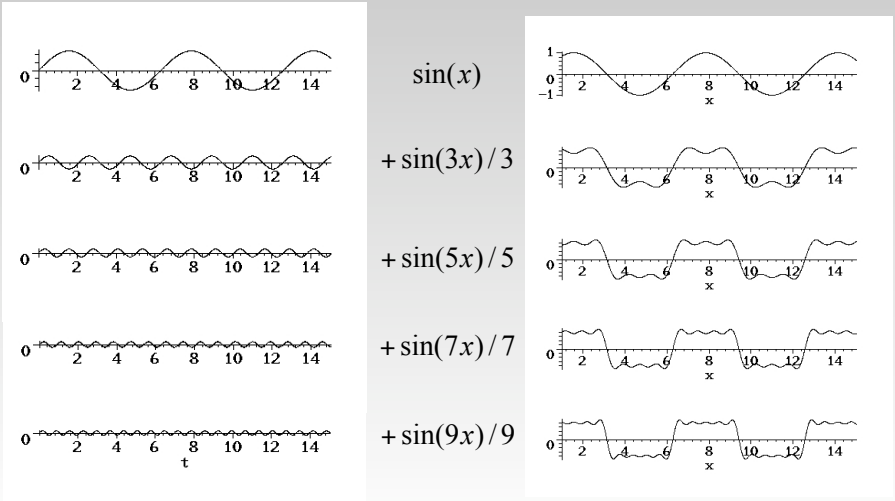
Example – sine wave

- Wavelength = 2π
- Frequency = $1/2\pi$



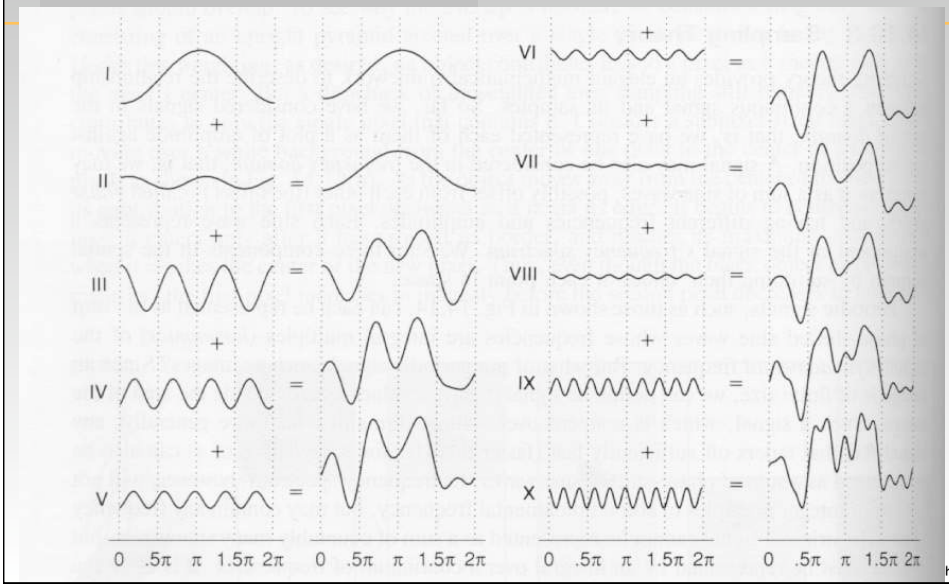
Wolfgang Heidrich

Fourier Transform – Summing Sinusoids

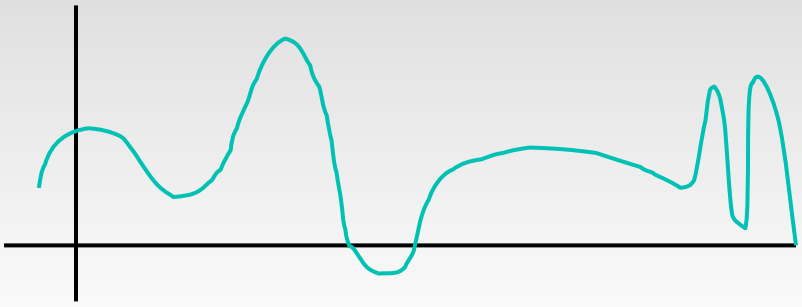


Wolfgang Heidrich

Fourier Transform

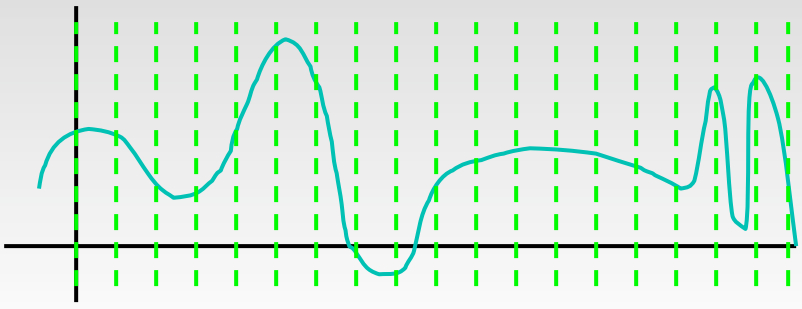


1D Sampling and Reconstruction



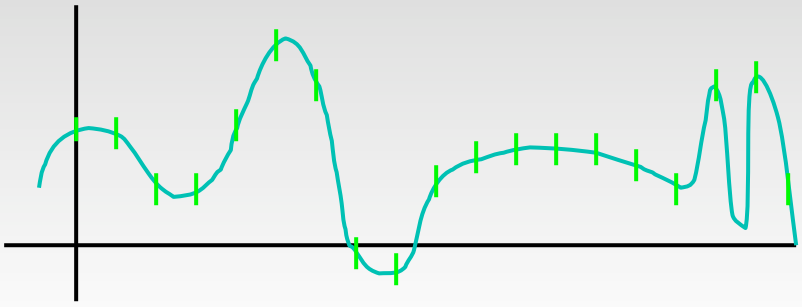
Wolfgang Heidrich

1D Sampling and Reconstruction



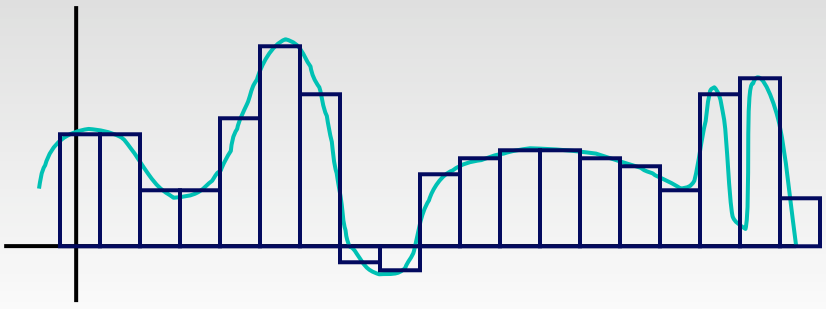
Wolfgang Heidrich

1D Sampling and Reconstruction



Wolfgang Heidrich

1D Sampling and Reconstruction

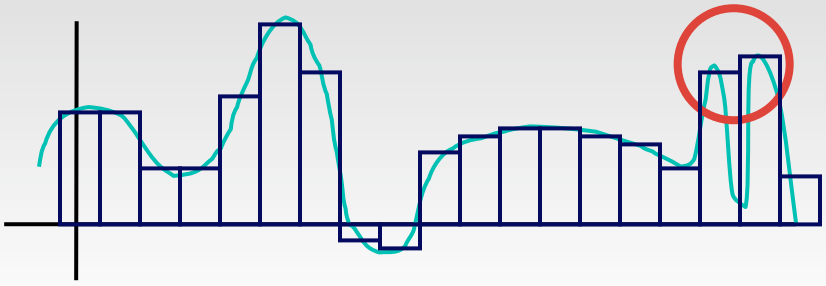


Wolfgang Heidrich

1D Sampling and Reconstruction

Problems

- Jaggies – abrupt changes
- Lose data

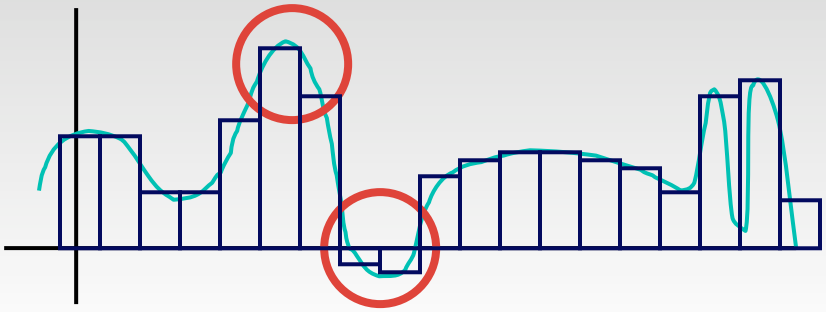


Wolfgang Heidrich

1D Sampling and Reconstruction

Problems

- Jaggies – abrupt changes



Wolfgang Heidrich

Sampling Theorem

- Continuous signal can be completely recovered from its samples

Iff

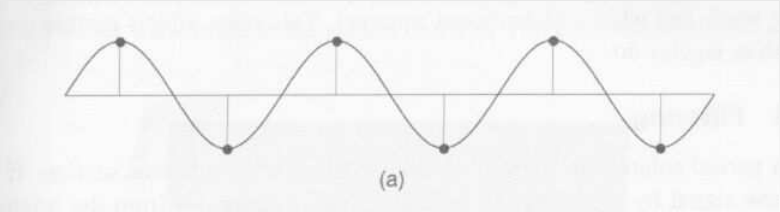
- Sampling rate greater than twice highest frequency present in signal

- Claude Shannon

Nyquist Rate

Lower bound on sampling rate

- Twice the highest frequency component in the image's spectrum



Falling Below Nyquist Rate

When sampling below Nyquist Rate, resulting signal looks like a lower-frequency one

- This is aliasing!

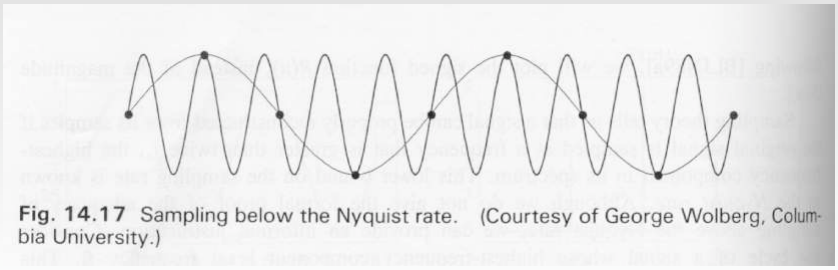
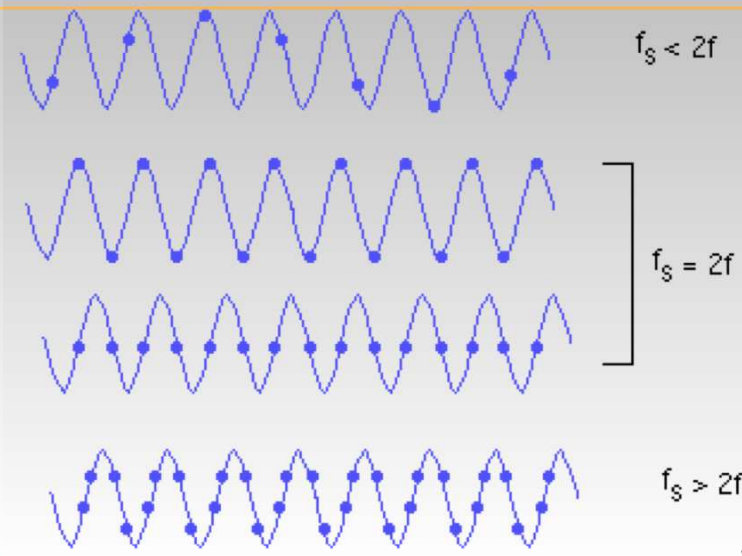


Fig. 14.17 Sampling below the Nyquist rate. (Courtesy of George Wolberg, Columbia University.)

Wolfgang Heidrich

Nyquist Rate



Wolfgang Heidrich

Aliasing

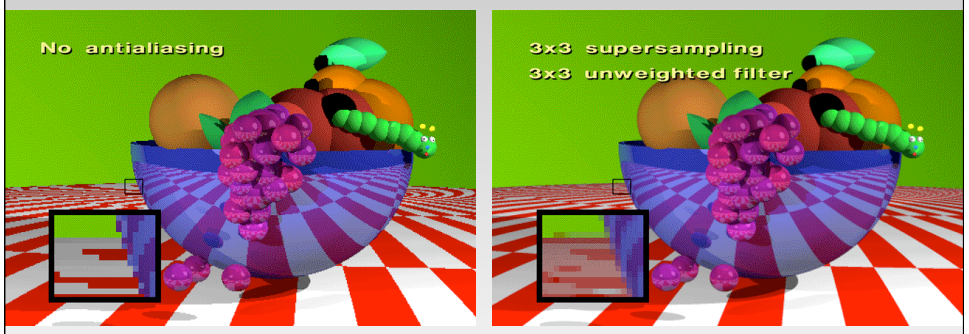
Incorrect appearance of high frequencies as low frequencies

To avoid: anti-aliasing

- Supersample
  - *Sample at higher frequency*
- Low pass filtering
  - *Remove high frequency function parts*
  - *Aka prefiltering, band-limiting*

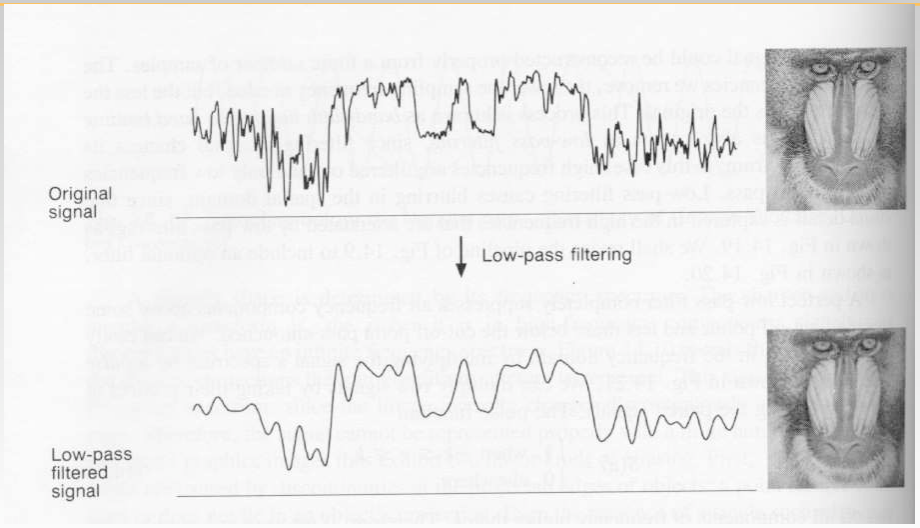
Wolfgang Heidrich

Supersampling



Wolfgang Heidrich

Low-Pass Filtering



Wolfgang Heidrich

Low-Pass Filtering

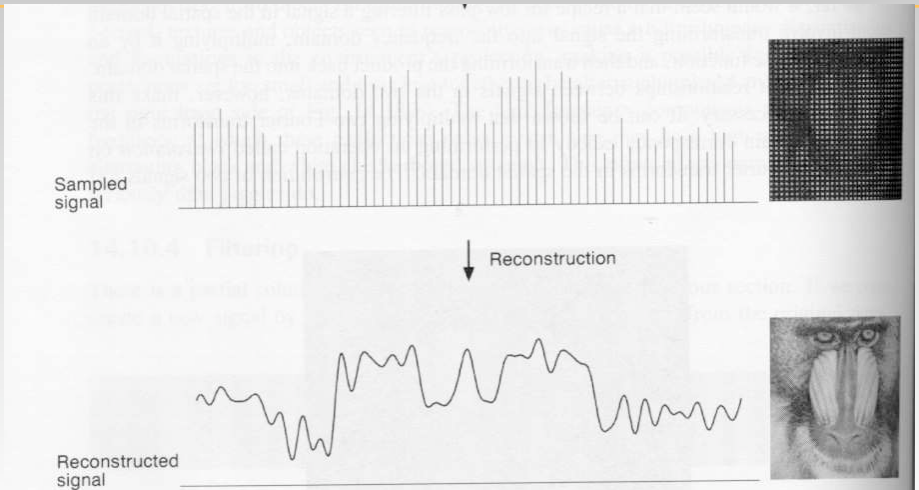
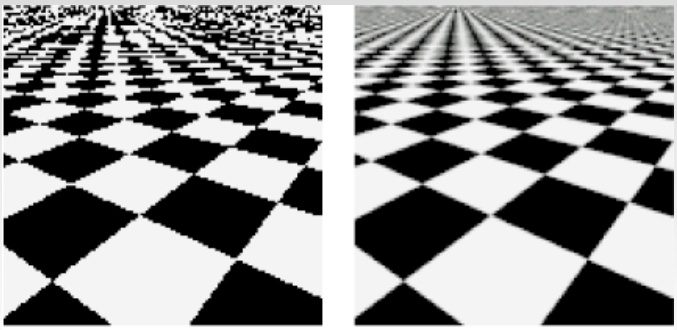


Fig. 14.20 The sampling pipeline with filtering. (Courtesy of George Wolberg, Columbia University.)

Previous Antialiasing Example

Texture mipmapping: low pass filter



(a) (b)

Wolfgang Heidrich

Discussion

Sampling & Reconstruction

- Fundamental issue in graphics, vision, and many other areas of computer science
  - *Whenever continuous signals need to be represented in a computer*
- Aliasing refers to the problem of reconstruction errors due to frequencies above the Nyquist limit
  - *These frequencies show up as erroneous low frequency content*

Wolfgang Heidrich

Discussion

Anti-Aliasing Approaches

- Low-pass filtering (before sampling!)
  - *Avoids aliasing*
  - *May not be practical in all settings*
  - *For images: artifacts around edges?!*
- Supersampling
  - *General algorithmic approach*
  - *Hoever: even the higher resolution image has a Nyquist limit!*
  - *Slow*

Wolfgang Heidrich

Coming Up:

Monday

- Programmable GPU architectures (Gordon Wetzstein)

Wednesday

- Shadows

Friday

- Quiz 2

Wolfgang Heidrich