Clipping

Wolfgang Heidrich

Course News

Assignment 2

- Due March 2

Homework 4

- Out today

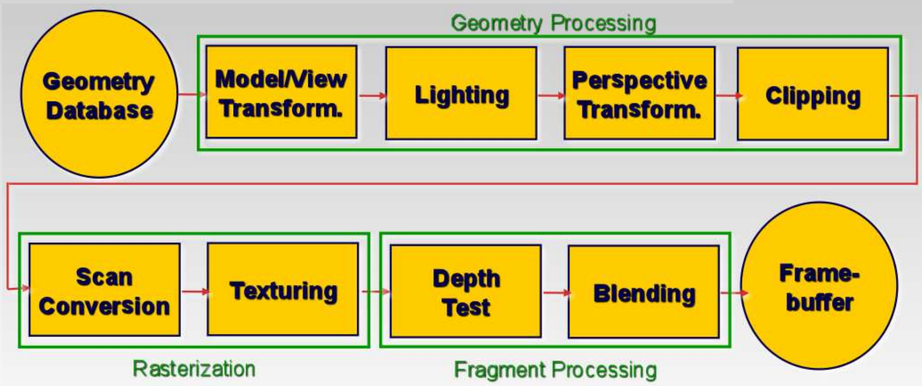
Reading (this week)

- Chapter 8

Reading (reading week)

- Rehearsal: Ch. 2, 3 (except 3.8), 6, 7, 8
- New: remainder of Ch. 2, Ch. 12

The Rendering Pipeline

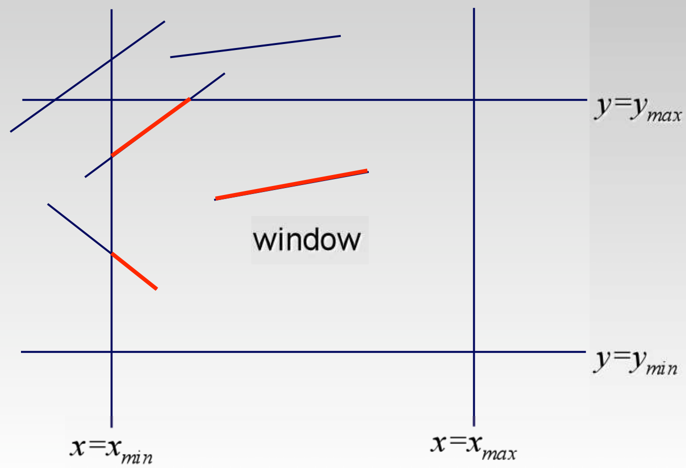


Line Clipping

Purpose

- Originally: 2D
  - *Determine portion of line inside an axis-aligned rectangle (screen or window)*
- 3D
  - *Determine portion of line inside axis-ligned parallelpiped (viewing frustum in NDC)*
  - *Simple extension to the 2D algorithms*

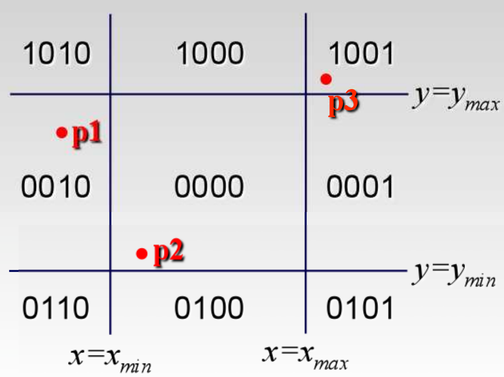
Line Clipping



Line Clipping

Outcodes (Cohen, Sutherland '74)

- 4 flags encoding position of a point relative to top, bottom, left, and right boundary
- E.g.:
  - OC(**p1**)=0010
  - OC(**p2**)=0000
  - OC(**p3**)=1001



Line Clipping

Line segment:

- (**p1**,**p2**)

Trivial cases:

- OC(**p1**)== 0 && OC(**p2**)==0
  - *Both points inside window, thus line segment completely visible (trivial accept)*
- (OC(**p1**) & OC(**p2**))!= 0 (i.e. **bitwise** "and"!)
  - *There is (at least) one boundary for which both points are outside (same flag set in both outcodes)*
  - *Thus line segment completely outside window (trivial reject)*

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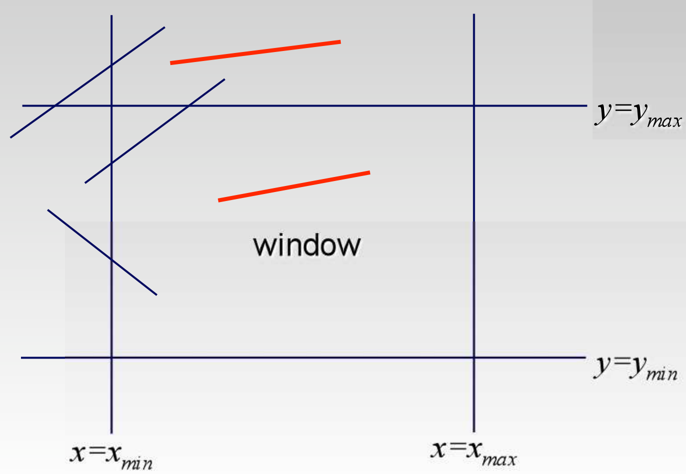
Line Clipping



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Line Clipping

α-Clipping

- Handling of all the non-trivial cases
- Improvement of earlier algorithms (Cohen/ Sutherland, Cyrus/Beck, Liang/Barsky)
- Define *window-edge-coordinates* of a point $\mathbf{p}=(x,y)^T$
  - $WEC_L(\mathbf{p}) = x - x_{min}$
  - $WEC_R(\mathbf{p}) = x_{max} - x$
  - $WEC_B(\mathbf{p}) = y - y_{min}$
  - $WEC_T(\mathbf{p}) = y_{max} - y$

Negative if outside!

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Line Clipping

α-Clipping

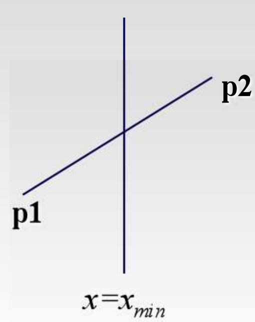
- Line segment defined as: $p1 + \alpha(p2 - p1)$
- Intersection point with one of the borders (say, left):

$$x_1 + \alpha(x_2 - x_1) = x_{min} \Leftrightarrow$$

$$\alpha = \frac{x_{min} - x_1}{x_2 - x_1}$$

$$= \frac{x_{min} - x_1}{(x_2 - x_{min}) - (x_1 - x_{min})}$$

$$= \frac{\mathrm{WEC}_L(x_1)}{\mathrm{WEC}_L(x_1) - \mathrm{WEC}_L(x_2)}$$



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Line Clipping

α-Clipping: algorithm

```
alphaClip( p1, p2, window ) {
    Determine window-edge-coordinates of p1, p2
    Determine outcodes OC(p1), OC(p2)

    Handle trivial accept and reject

    α1= 0; // line parameter for first point
    α2= 1; // line parameter for second point
    …
```

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Line Clipping

α-Clipping: algorithm (cont.)

```
    …
    // now clip point p1 against all edges
    if( OC(p1) & LEFT_FLAG ) {
        α= WEC_L(p1)/(WEC_L (p1) - WEC_L (p2));
        α1= max(α1, α );
    }

    Similarly clip p1 against other edges
    …
```

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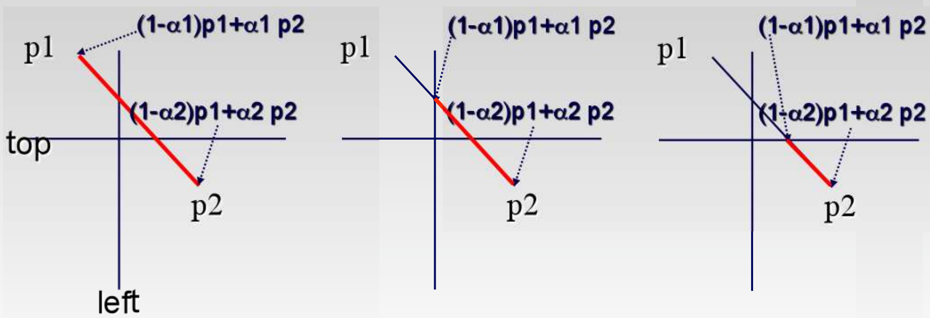
Line Clipping

α-Clipping: example for clipping p1



Start configuration | After clipping to left | After clipping to top

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Line Clipping

α-Clipping: algorithm (cont.)

```
…
// now clip point p2 against all edges
if( OC(p2) & LEFT_FLAG ) {
    α= WEC_L(p2)/(WEC_L (p1) - WEC_L (p2));
    α2= min(α2, α );
}
```

Similarly clip p1 against other edges

…

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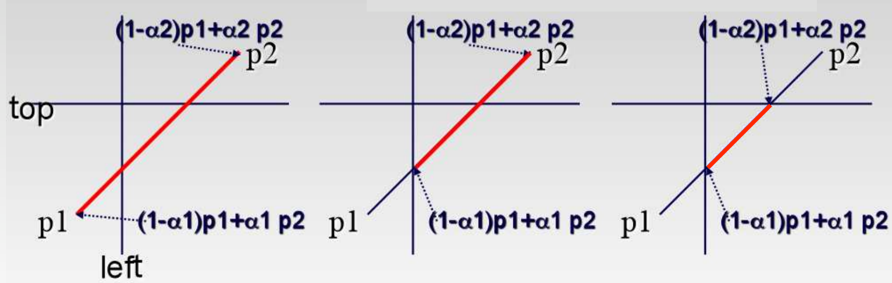
α-Clipping: algorithm (cont.)

```
...
// wrap-up
if(α1 > α2 )
    no output;
else
    output line from p1+α1(p2-p1) to p1+α2(p2-p1)
} // end of algorithm
```


Line Clipping

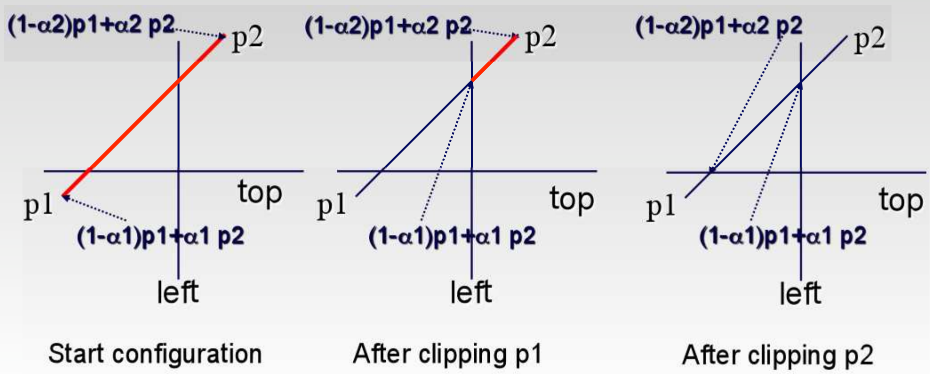
Example



Start configuration After clipping p1 After clipping p2

Line Clipping

Another Example



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Line Clipping in 3D

Approach:

- Clip against parallelpiped in NDC (*after* perspective transform)
- Means that the clipping volume is always the same!
  - OpenGL: $x_{min}=y_{min}= -1,\ x_{max}=y_{max}= 1$
- Boundary lines become boundary planes
  - *But outcodes and WECs still work the same way*
  - *Additional front and back clipping plane*
    - $z_{min}=-1,\ z_{max}=1$ in OpenGL

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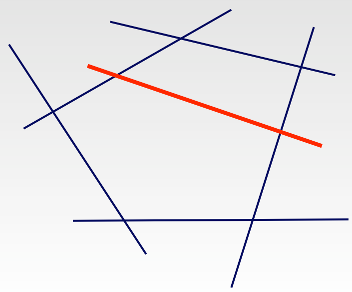
Line Clipping

Extensions

- Algorithm can be extended to clipping lines against
  - *Arbitrary convex polygons (2D)*
  - *Arbitrary convex polytopes (3D)*



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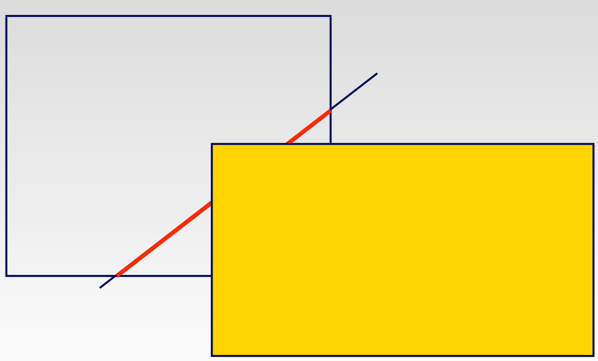
Line Clipping

Non-convex clipping regions

- E.g.: windows in a window system!



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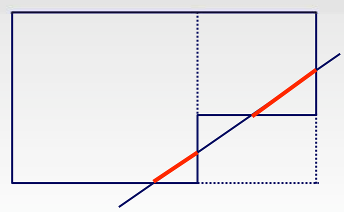
Line Clipping

Non-convex clipping regions

- Problem: arbitrary number of visible line segments
- Different approaches:
  - *Break down polygon into convex parts*
  - *Scan convert for full window, and discard hidden pixels*



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Polygon Clipping

Objective

- 2D: clip polygon against rectangular window
  - *Or general convex polygons*
  - *Extensions for non-convex or general polygons*
- 3D: clip polygon against parallelpiped
  - *Left, right, top, bottom, near, far planes*

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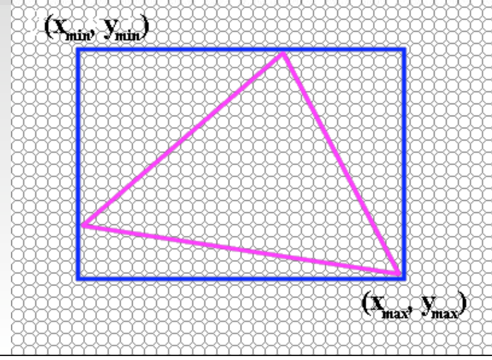
Polygon Clipping

Triangles Scan-Converted with Edge Equations:

- Go over each pixel in bounding rectangle
- Check if pixel is inside/outside of triangle



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Triangle Clipping (w/ Edge Equation Scan Conversion)

Note:

- Once we use edge equations, we no longer really have to clip the geometry against window boundary!
- Instead: clip bounding rectangle against window
  - *Only evaluate edge equations for pixels inside the window!*
- Near/far clipping: when interpolating depth values, detect whether point is closer than near or farther than far
  - *If so, don't draw it*

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General Polygon Clipping

Task:

- Clipping of general polygons
- Convex and concave
- Works with other scan conversion algorithms
  - *Independent of edge equations*

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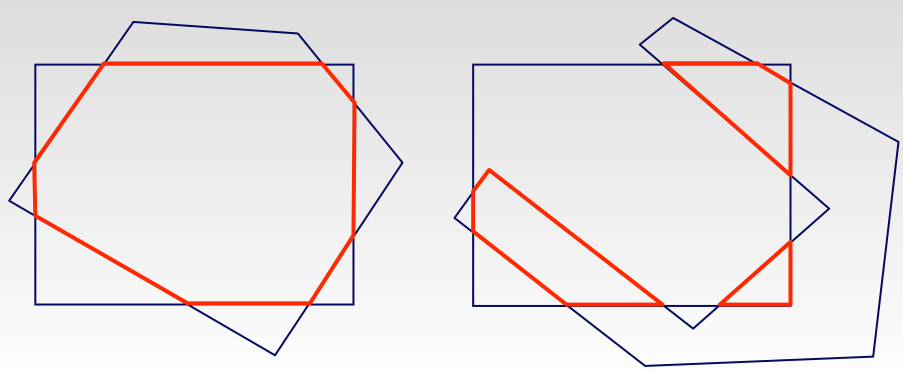
Polygon Clipping

Not just clipping all boundary lines

- May have to introduce new line segments



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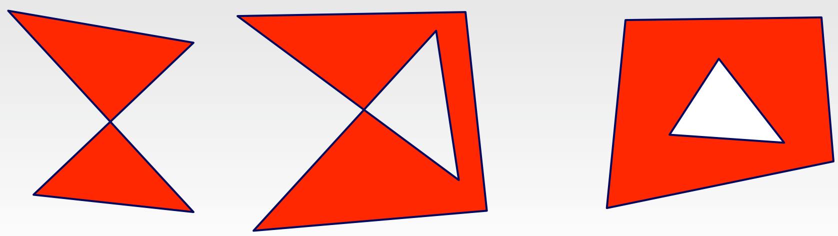
Polygon Clipping

Classes of Polygons

- Triangles
- Convex
- Concave
- Holes and self-intersection



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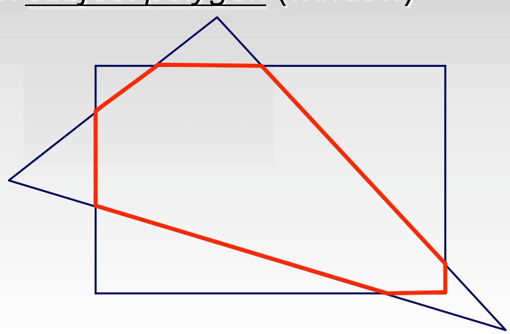
Polygon Clipping

Sutherland/Hodgeman Algorithm ('74)

- Arbitrary convex or concave object polygon
  - *Restriction to triangles does not simplify things*
- Convex subject polygon (window)



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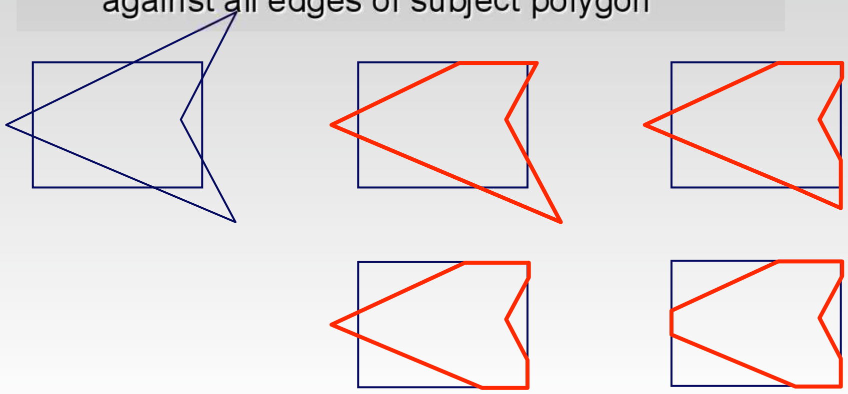
Polygon Clipping

Sutherland/Hodgeman Algorithm ('74)

- Approach: clip object polygon independently against all edges of subject polygon



Polygon Clipping

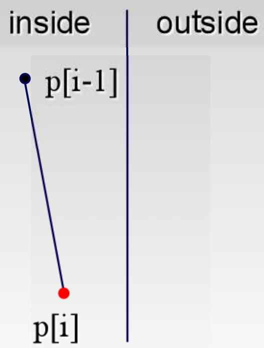
Clipping against one edge:

```
clipPolygonToEdge( p[n], edge ) {
    for( i= 0 ; i< n ; i++ ) {
        if( p[i] inside edge ) {
          if( p[i-1] inside edge ) // p[-1]= p[n-1]
             output p[i];
          else {
            p= intersect( p[i-1], p[i], edge );
            output p, p[i];
          }
        } else...
```

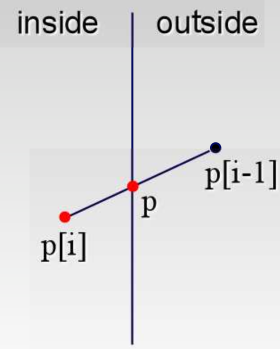
Polygon Clipping

Clipping against one edge (cont)

- p[i] inside: 2 cases



Output: p[i]



Output: p, p[i]

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Polygon Clipping

Clipping against one edge (cont)

```
…
else {            // p[i] is outside edge
    if( p[i-1] inside edge ) {
        p= intersect(p[i-1], p[I], edge );
        output p;
    }
} // end of algorithm
```

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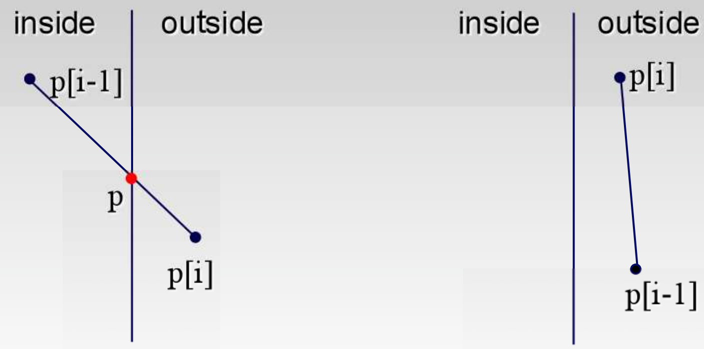
Polygon Clipping

Clipping against one edge (cont)

- p[i] outside: 2 cases



Output: p

Output: nothing

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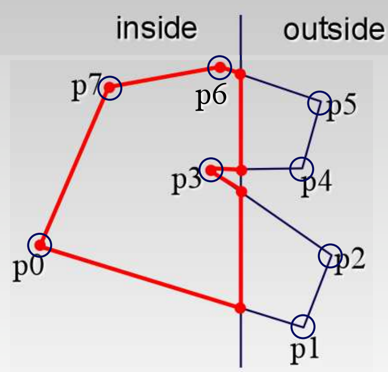
Polygon Clipping

Example



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Polygon Clipping

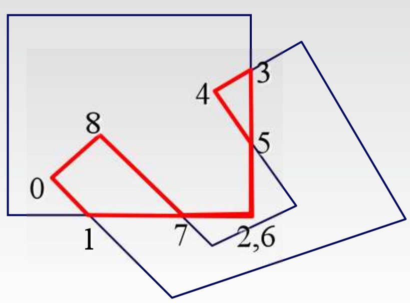
Sutherland/Hodgeman Algorithm

- Inside/outside tests: outcodes
- Intersection of line segment with edge: window-edge coordinates
- Similar to Cohen/Sutherland algorithm for line clipping

Polygon Clipping

Sutherland/Hodgeman Algorithm

- Discussion:
  - *Works for concave polygons*
  - *But generates degenerate cases*



Polygon Clipping

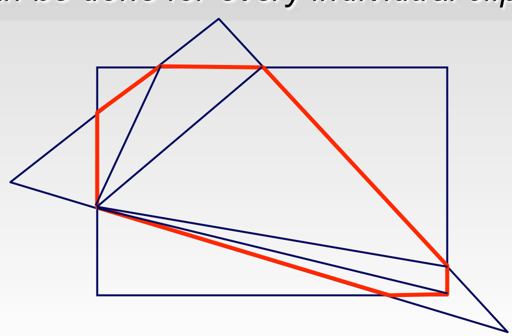
Sutherland/Hodgeman Algorithm

- Discussion:
  - *Clipping against individual edges independent*
    - Great for hardware (pipelining)
  - *All vertices required in memory at the same time*
    - Not so good, but unavoidable
    - Another reason for using triangles only in hardware rendering

Polygon Clipping

Sutherland/Hodgeman Algorithm

- For Rendering Pipeline:
  - *Re-triangulate resulting polygon (can be done for every individual clipping edge)*



Polygon Clipping

Other Polygon Clipping Algorithms

- Weiler/Aetherton '77:
  - *Arbitrary concave polygons with holes both as subject and as object polygon*
- Vatti '92:
  - *Self intersection allowed as well*
- … many more
  - *Improved handling of degenerate cases*
  - *But not often used in practice due to high complexity*

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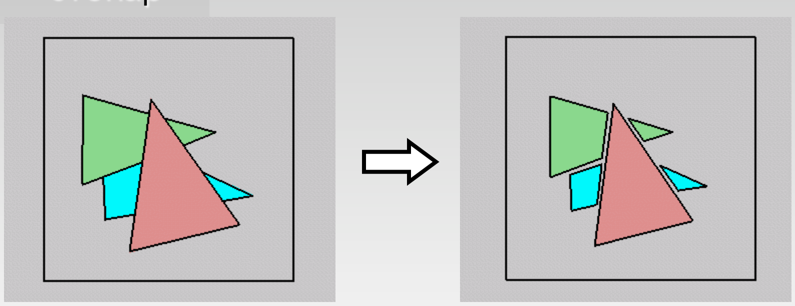
Occlusion

- For most interesting scenes, some polygons overlap



- To render the correct image, we need to determine which polygons occlude which

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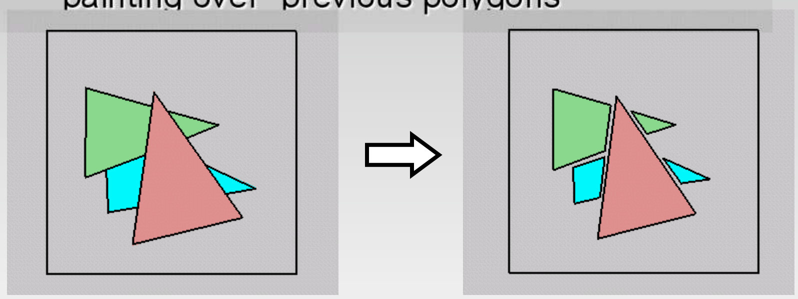
Painter's Algorithm

- Simple: render the polygons from back to front, "painting over" previous polygons



- Draw cyan, then green, then red

will this work in the general case?

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Painter's Algorithm: Problems

- *Intersecting polygons* present a problem
- Even non-intersecting polygons can form a cycle with no valid visibility order:

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Hidden Surface Removal

Object Space Methods:

- Work in 3D before scan conversion
  - *E.g. Painter's algorithm*
- Usually independent of resolution
  - *Important to maintain independence of output device (screen/printer etc.)*

Image Space Methods:

- Work on per-pixel/per fragment basis after scan conversion
- Z-Buffer/Depth Buffer
- Much faster, but resolution dependent

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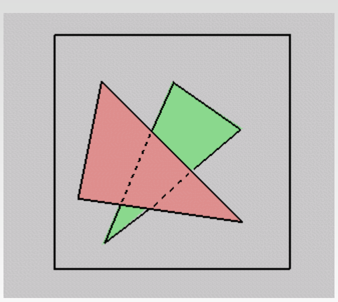
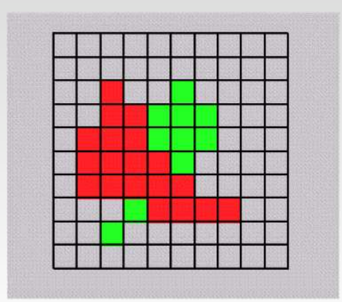
The Z-Buffer Algorithm

- What happens if multiple primitives occupy the same pixel on the screen?
- Which is allowed to paint the pixel?





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The Z-Buffer Algorithm

Idea: retain depth after projection transform

- Each vertex maintains z coordinate
  - *Relative to eye point*
- Can do this with canonical viewing volumes

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The Z-Buffer Algorithm

Augment color framebuffer with Z-buffer

- Also called depth buffer
- Stores z value at each pixel
- At frame beginning, initialize all pixel depths to ∞
- When scan converting: interpolate depth (z) across polygon
- Check z-buffer before storing pixel color in framebuffer and storing depth in z-buffer
- don't write pixel if its z value is more distant than the z value already stored there

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Z-Buffer

Store (r,g,b,z) for each pixel

- typically 8+8+8+24 bits, can be more

```
for all i,j {
 Depth[i,j] = MAX_DEPTH
 Image[i,j] = BACKGROUND_COLOUR
}
for all polygons P {
  for all pixels in P {
    if (Z_pixel < Depth[i,j]) {
      Image[i,j] = C_pixel
      Depth[i,j] = Z_pixel
    }
  }
}
```

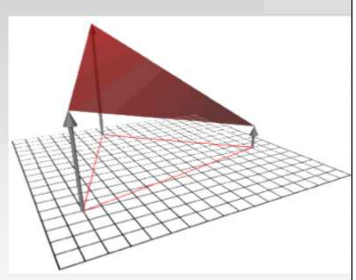

Interpolating Z

Edge walking

- Just interpolate Z along edges and across spans

Barycentric coordinates

- Interpolate z like other parameters
- E.g. color



The Z-Buffer Algorithm (mid-70's)

History:

- Object space algorithms were proposed when memory was expensive
- First 512x512 framebuffer was >$50,000!

Radical new approach at the time

- The big idea:
  - *Resolve visibility* ***independently at each pixel***

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Depth Test Precision

- Reminder: projective transformation maps eye-space *z* to generic *z*-range (NDC)
- Simple example:

$$T\left(\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Thus:

$$z_{NDC} = \frac{a \cdot z_{eye} + b}{z_{eye}} = a + \frac{b}{z_{eye}}$$

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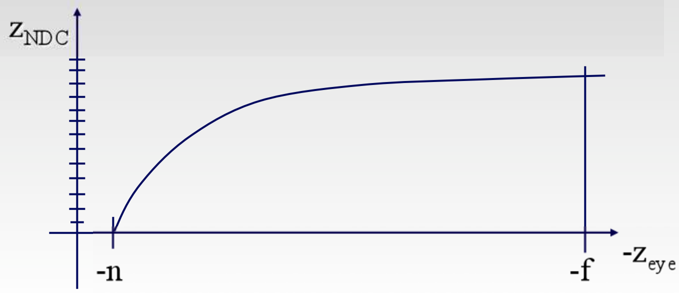
Depth Test Precision

- Therefore, depth-buffer essentially stores 1/z, rather than z!
- Issue with integer depth buffers
  - *High precision for near objects*
  - *Low precision for far objects*



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Depth Test Precision

- Low precision can lead to depth fighting for far objects
  - *Two different depths in eye space get mapped to same depth in framebuffer*
  - *Which object "wins" depends on drawing order and scan-conversion*
- Gets worse for larger ratios $f{:}n$
  - Rule of thumb: $f{:}n < 1000$ for 24 bit depth buffer
- With 16 bits cannot discern millimeter differences in objects at 1 km distance

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Z-Buffer Algorithm Questions

- How much memory does the Z-buffer use?
- Does the image rendered depend on the drawing order?
- Does the time to render the image depend on the drawing order?
- How does Z-buffer load scale with visible polygons? with framebuffer resolution?

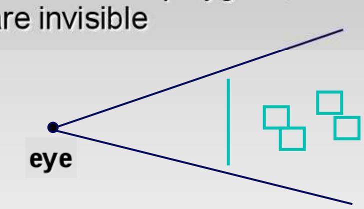
Z-Buffer Pros

- Simple!!!
- Easy to implement in hardware
  - *Hardware support in all graphics cards today*
- Polygons can be processed in arbitrary order
- Easily handles polygon interpenetration

Z-Buffer Cons

Poor for scenes with high depth complexity

- Need to render all polygons, even if most are invisible



Shared edges are handled inconsistently

- *Ordering dependent*

Z-Buffer Cons

Requires lots of memory

- (e.g. 1280x1024x32 bits)

Requires fast memory

- Read-Modify-Write in inner loop

Hard to simulate transparent polygons

- We throw away color of polygons behind closest one
- Works if polygons ordered back-to-front
  - *Extra work throws away much of the speed advantage*

Object Space Algorithms

Determine visibility on object or polygon level

- Using camera coordinates

Resolution independent

- Explicitly compute visible portions of polygons

Early in pipeline

- After clipping

Requires depth-sorting

- Painter's algorithm
- BSP trees

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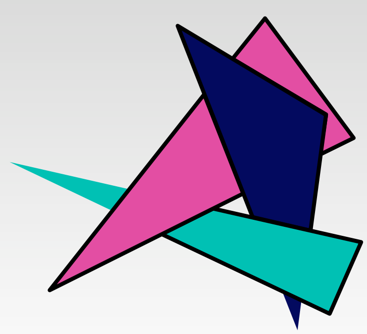
Object Space Visibility Algorithms

- Early visibility algorithms computed the set of visible ***polygon fragments*** directly, then rendered the fragments to a display:



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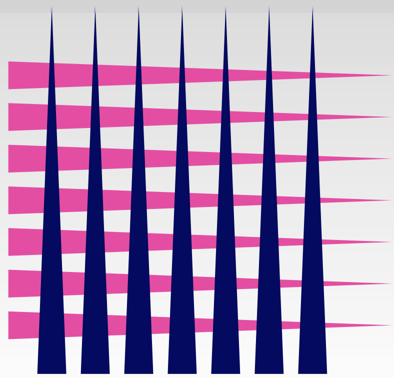
Object Space Visibility Algorithms

What is the minimum worst-case cost of computing the fragments for a scene composed of *n* polygons?

Answer:

$O(n^2)$



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Object Space Visibility Algorithms

- So, for about a decade (late 60s to late 70s) there was intense interest in finding efficient algorithms for hidden surface removal
- We'll talk about one:
  - Binary Space Partition (BSP) Trees
  - *Still in use today for ray-tracing, and in combination with z-buffer*

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Binary Space Partition Trees (1979)

BSP Tree: partition space with binary tree of planes

- Idea: divide space recursively into half-spaces by choosing splitting planes that separate objects in scene
- Preprocessing: create binary tree of planes
- Runtime: correctly traversing this tree enumerates objects from back to front

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Creating BSP Trees: Objects

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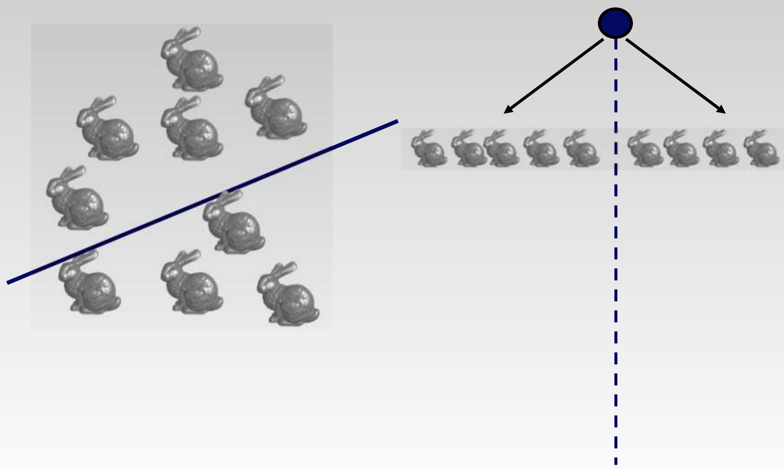
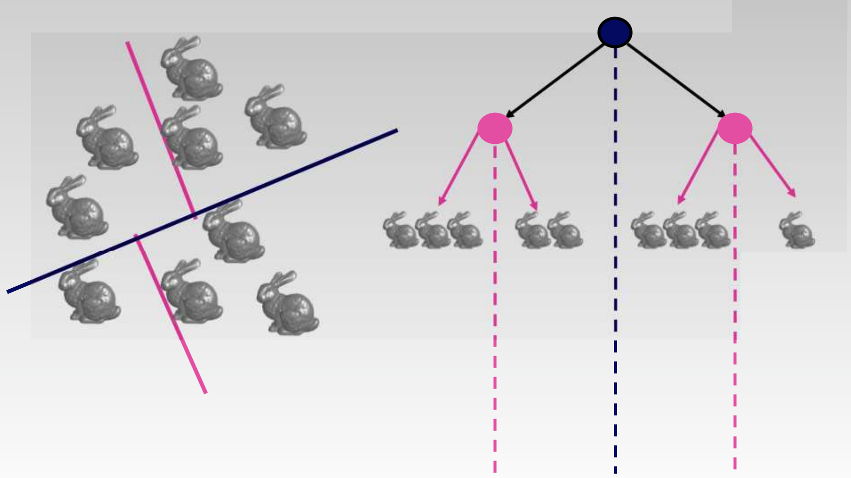
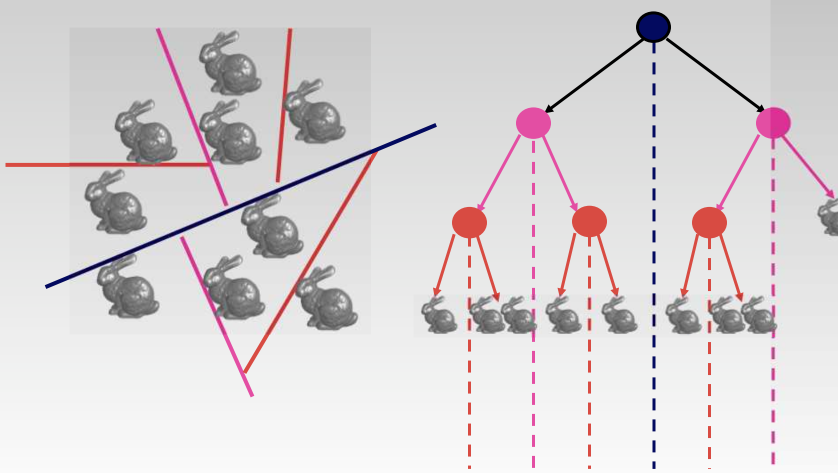
Creating BSP Trees: Objects



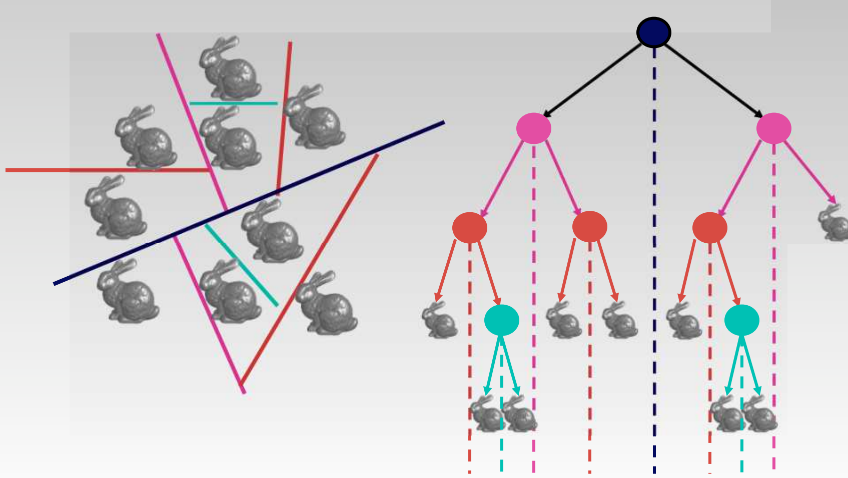
Creating BSP Trees: Objects



Creating BSP Trees: Objects



Creating BSP Trees: Objects

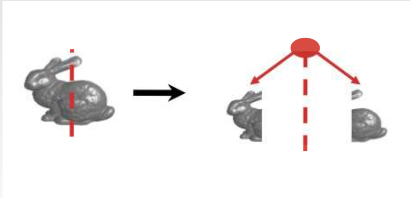


Splitting Objects

No bunnies were harmed in previous example

But what if a splitting plane passes through an object?

- Split the object; give half to each node



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Traversing BSP Trees

Tree creation independent of viewpoint

- Preprocessing step

Tree traversal uses viewpoint

- Runtime, happens for many different viewpoints

Each plane divides world into near and far

- For given viewpoint, decide which side is near and which is far
  - *Check which side of plane viewpoint is on independently for each tree vertex*
  - *Tree traversal differs depending on viewpoint!*
- Recursive algorithm
  - *Recurse on far side*
  - *Draw object*
  - *Recurse on near side*

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Traversing BSP Trees

```
renderBSP(BSPtree *T)
    BSPtree *near, *far;
    if (eye on left side of T->plane)
     near = T->left; far = T->right;
    else
     near = T->right; far = T->left;
    renderBSP(far);
    if (T is a leaf node)
     renderObject(T)
  renderBSP(near);
```

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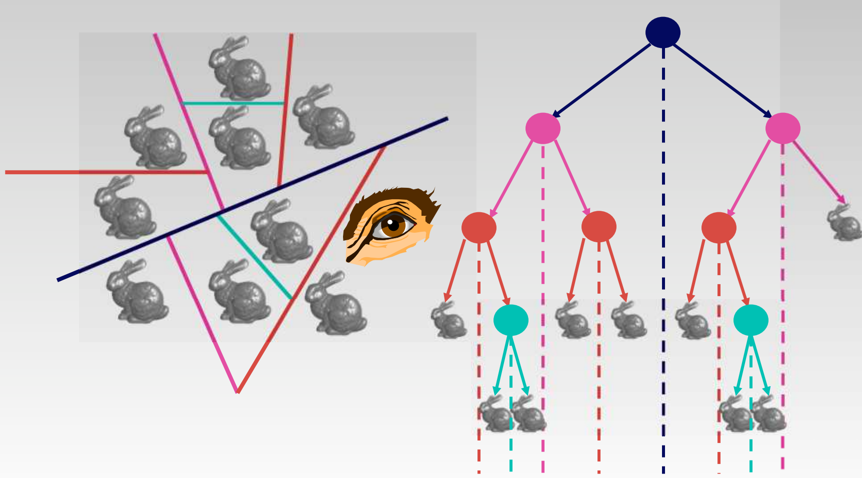
BSP Trees : Viewpoint A



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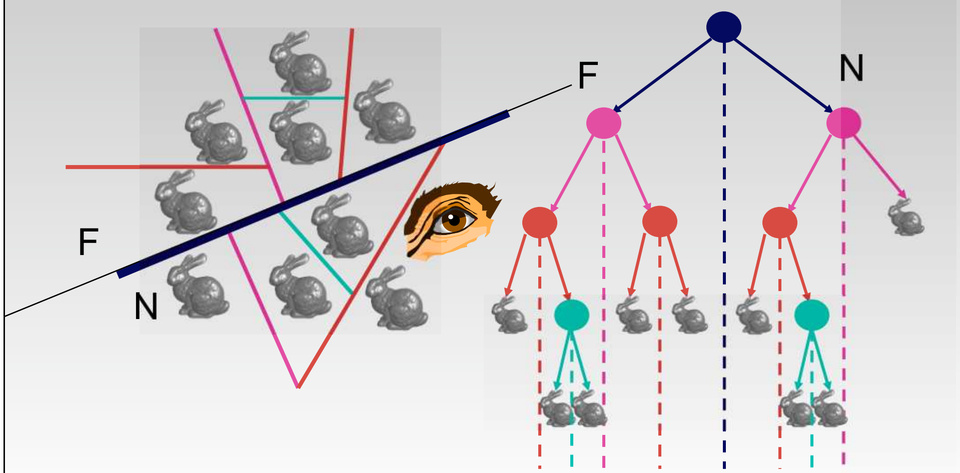
BSP Trees : Viewpoint A



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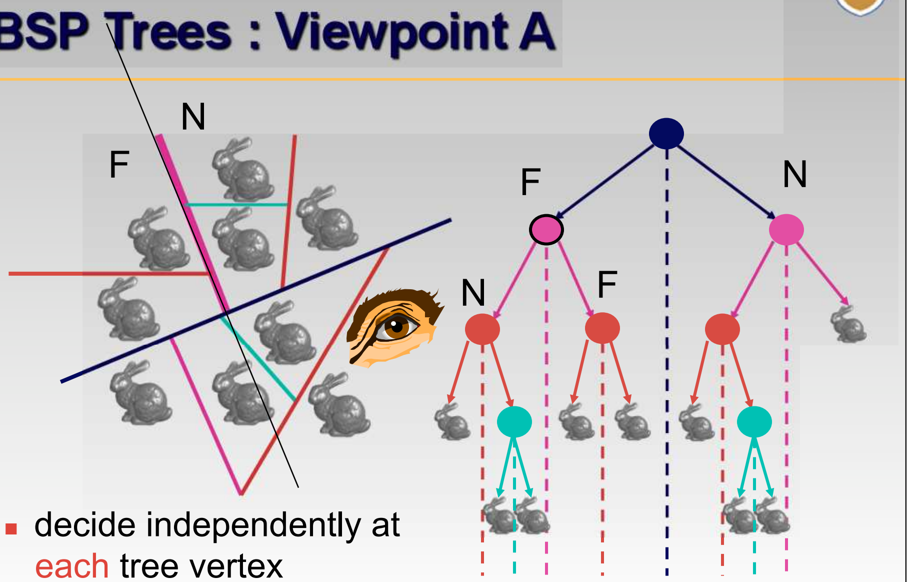
BSP Trees : Viewpoint A



- decide independently at each tree vertex
  - not just left or right child!

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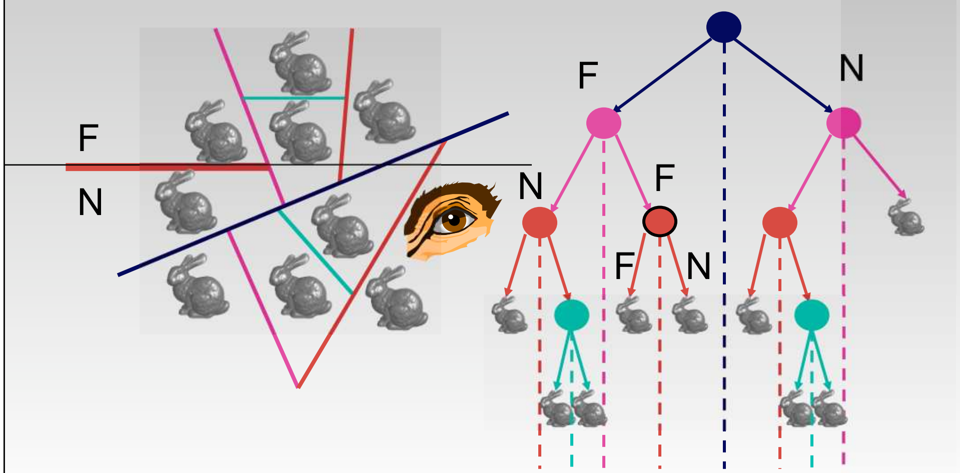
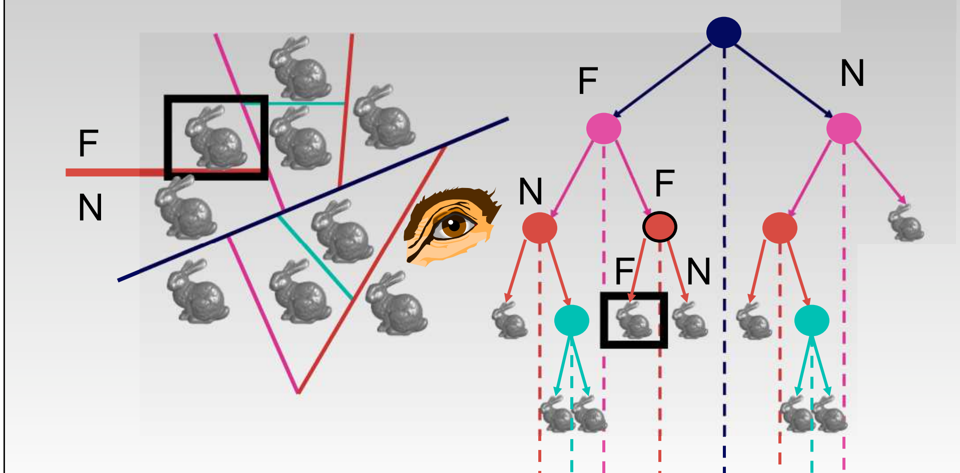
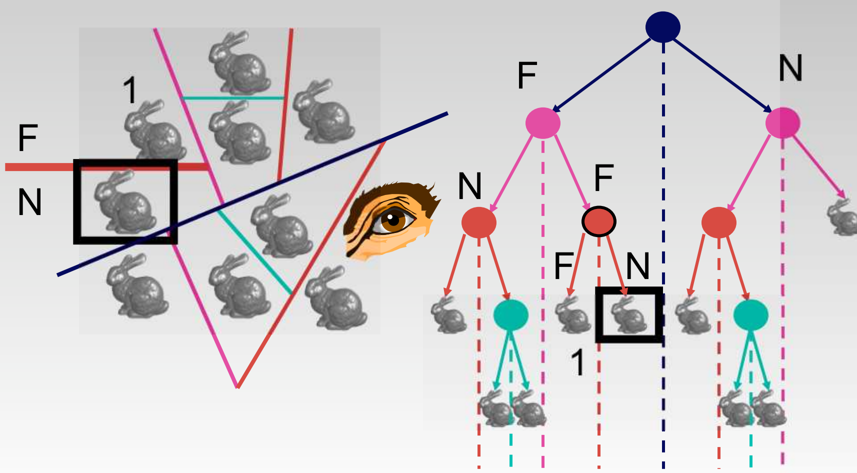
BSP Trees : Viewpoint A



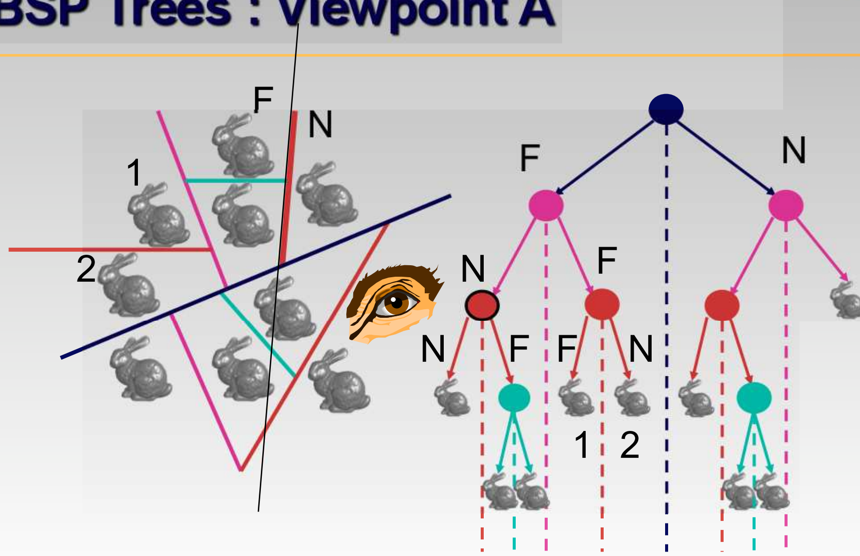
BSP Trees : Viewpoint A



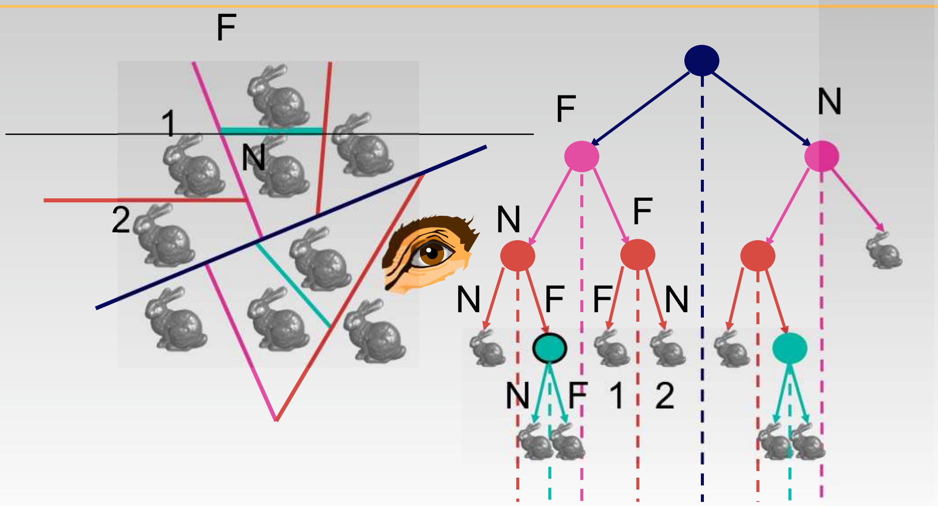
BSP Trees : Viewpoint A



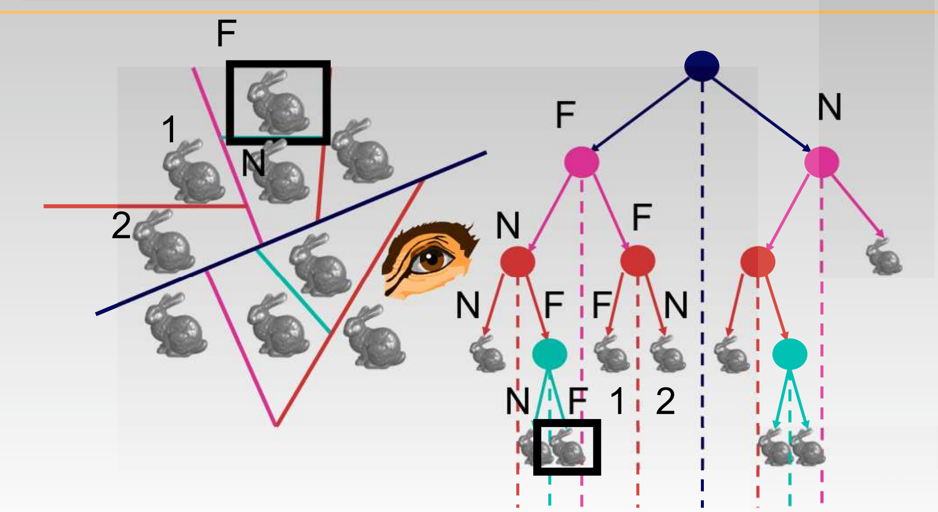
BSP Trees : Viewpoint A



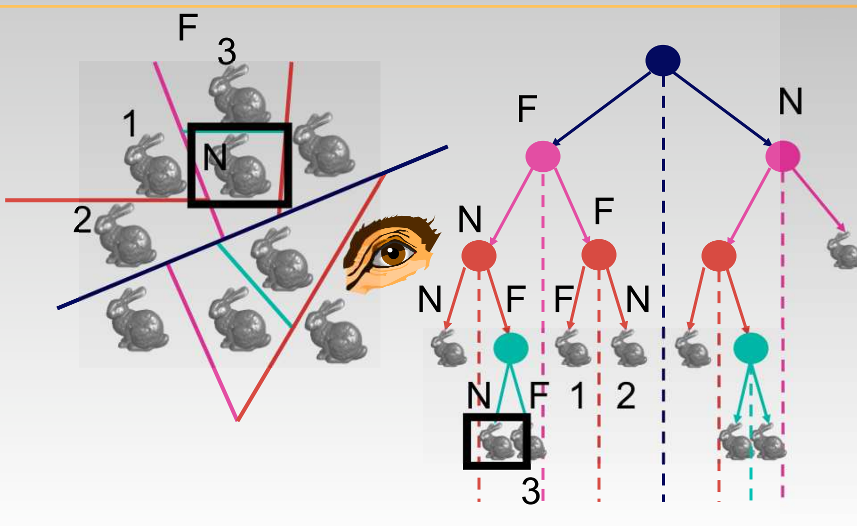
BSP Trees : Viewpoint A



BSP Trees : Viewpoint A



BSP Trees : Viewpoint A



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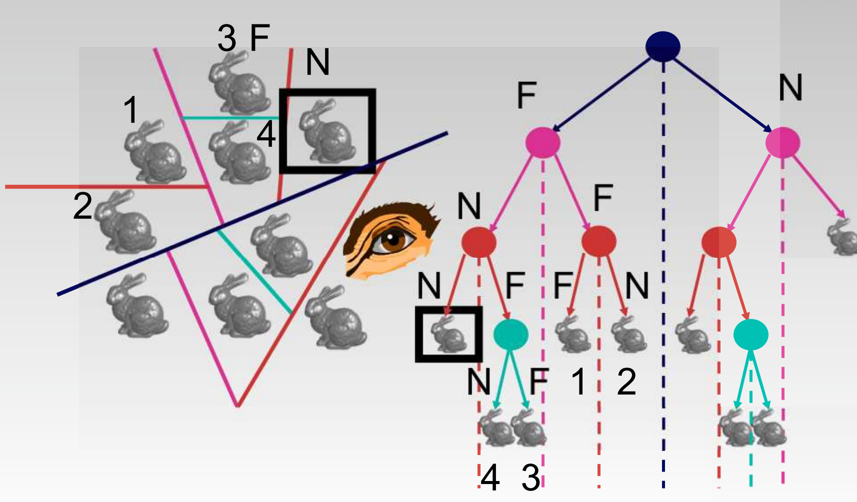
BSP Trees : Viewpoint A



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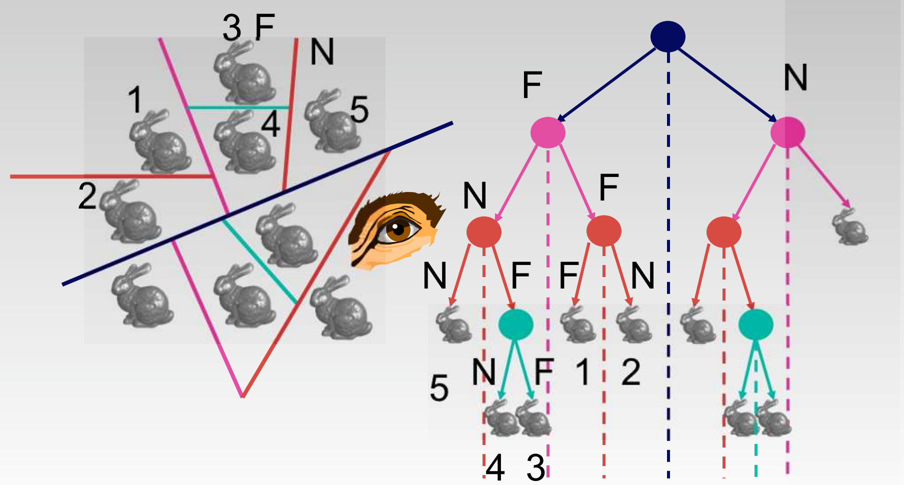
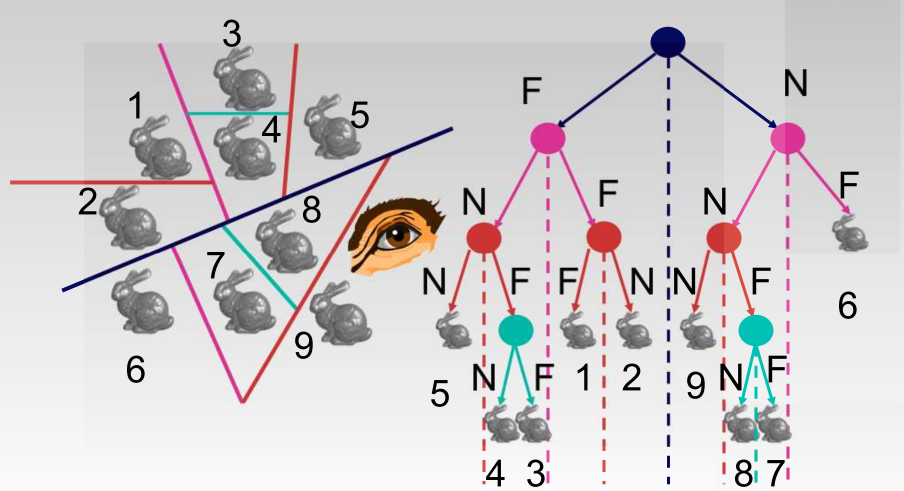
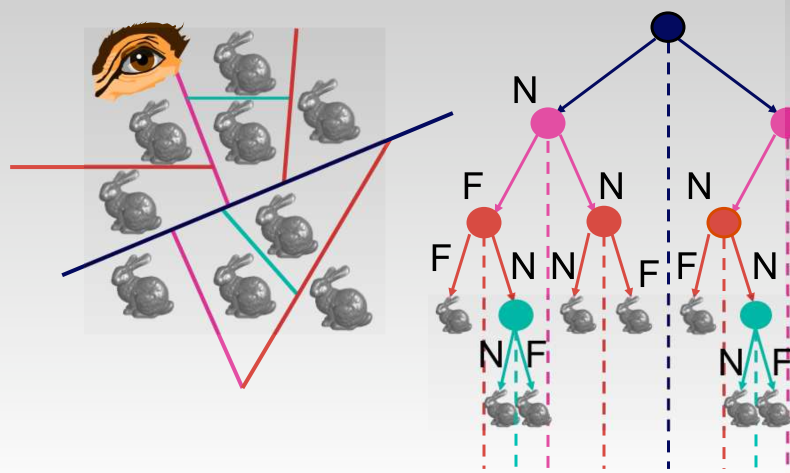
BSP Trees : Viewpoint A



BSP Trees : Viewpoint A



BSP Trees : Viewpoint B



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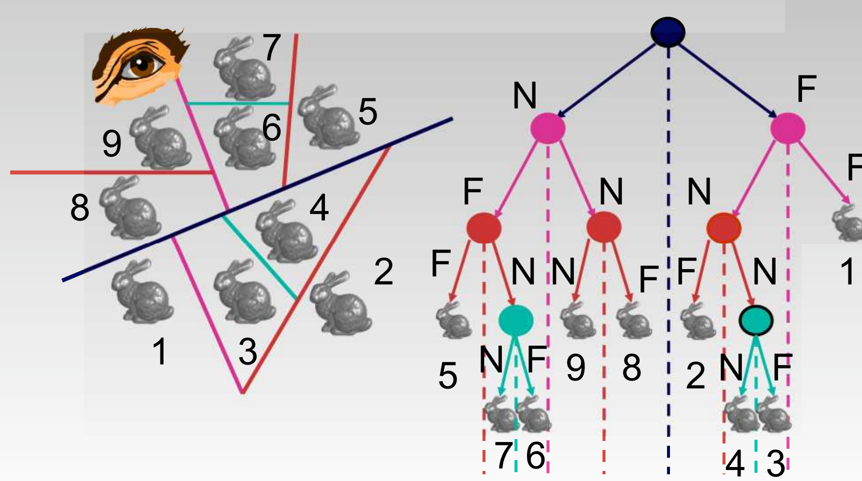
BSP Trees : Viewpoint B



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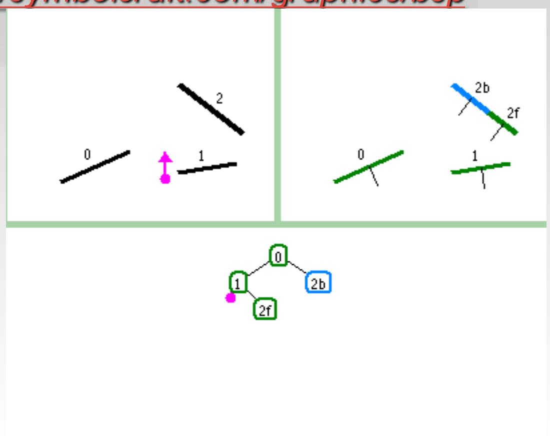
BSP Tree Traversal: Polygons

- Split along the plane defined by any polygon from scene
- Classify all polygons into positive or negative half-space of the plane
  - *If a polygon intersects plane, split polygon into two and classify them both*
- Recurse down the negative half-space
- Recurse down the positive half-space

BSP Demo

Useful demo:

http://symbolcraft.com/graphics/bsp



Summary: BSP Trees

Pros:

- Simple, elegant scheme
- Correct version of painter's algorithm back-to-front rendering approach
- Still very popular for video games (but getting less so)

Cons:

- Slow(ish) to construct tree: O(n log n) to split, sort
- Splitting increases polygon count: $O(n^2)$ worst-case
- Computationally intense preprocessing stage restricts algorithm to static scenes

Coming Up:

After Reading Week

- More hidden surface removal
- Blending
- Texture mapping