Scan Conversion

Wolfgang Heidrich

Course News

Assignment 2

- Due March 2

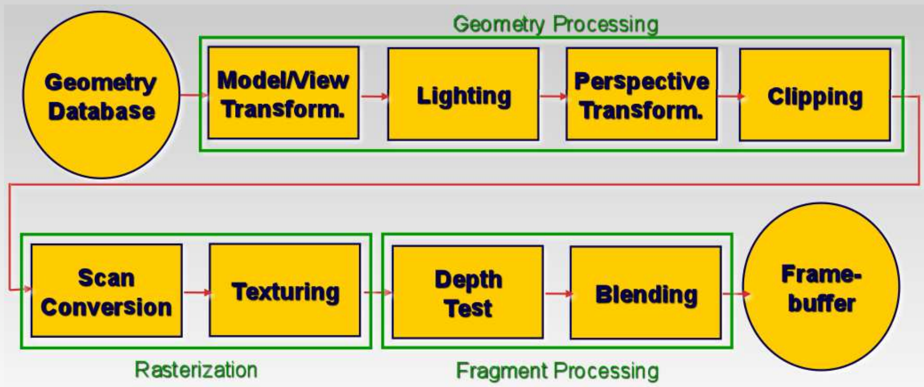
Homework 3

- Discussed in labs next week

Reading

- Chapter 3

The Rendering Pipeline



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Scan Conversion - Rasterization

Convert continuous rendering primitives into discrete fragments/pixels

- Lines
  - *Midpoint/Bresenham*
- Triangles
  - *Flood fill*
  - *Scanline*
  - *Implicit formulation*
- Interpolation

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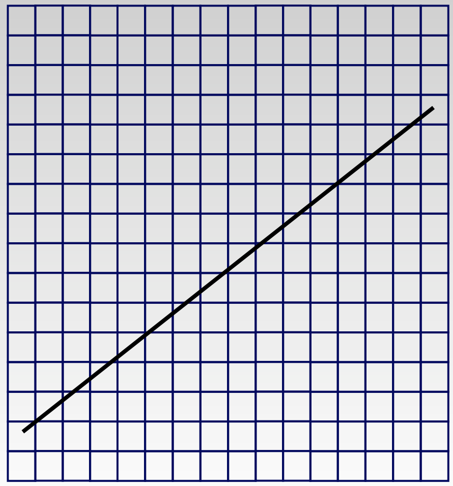
Scan Conversion - Lines



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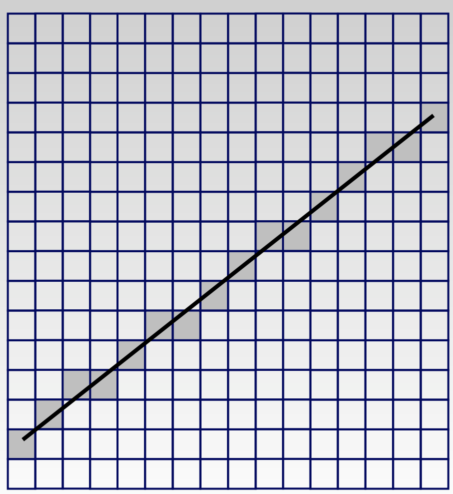
Scan Conversion - Lines



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Scan Conversion - Lines

First Attempt:

- Line (s,e) given in device coordinates
- Create the thinnest line that connects start point and end point without gap

Assumptions for now:

- Start point to the left of end point: xs< xe
- Slope of the line between 0 and 1 (I.e. elevation between 0 and 45 degrees:

$$0 \leq \frac{ye - ys}{xe - xs} \leq 1$$

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Scan Conversion of Lines - Digital Differential Analyzer

First Attempt:

```
dda( float xs, ys, xe, ye ) {
    // assume xs < xe, and slope m between 0 and 1
    float m= (ye-ys)/(xe-xs);
    float y= round( ys );
    for( int x= round( xs ) ; x<= xe ; x++ ) {
        drawPixel( x, round( y ) );
        y= y+m;
    }
}
```

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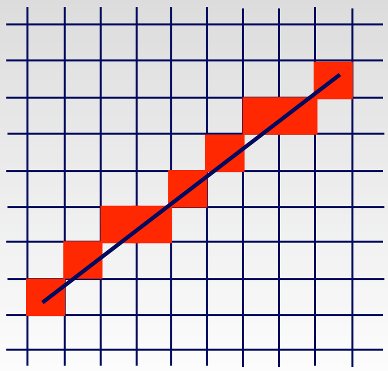
Scan Conversion of Lines

DDA:



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Scan Conversion of Lines
Midpoint Algorithm

Moving horizontally along x direction

- Draw at current y value, or move up vertically to y+1?
  - *Check if midpoint between two possible pixel centers above or below line*

Candidates

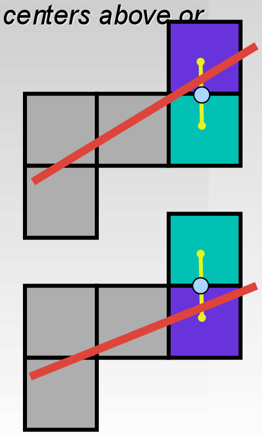
- Top pixel: (x+1,y+1)
- Bottom pixel: (x+1, y)

Midpoint: (x+1, y+.5)

Check if midpoint above or below line

- Below: top pixel
- Above: bottom pixel

Key idea behind Bresenham Alg.



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Scan Conversion of Lines

Idea: decision variable

```
dda( float xs, ys, xe, ye ) {
    float d= 0.0;
    float m= (ye-ys)/(xe-xs);
    int y= round( ys );
    for( int x= round( xs ) ; x<= xe ; x++ ) {
        drawPixel( x, y );
        d= d+m;
        if( d>= 0.5 ) { d= d-1.0; y++; }
    }
}
```

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Scan Conversion of Lines
Bresenham Algorithm ('63)

- Use decision variable to generate purely integer algorithm
- Explicit line equation:

$$y = \frac{(y_e - y_s)}{(x_e - x_s)}(x - x_s) + y_s$$

- Implicit version:

$$L(x, y) = \frac{(y_e - y_s)}{(x_e - x_s)}(x - x_s) - (y - y_s) = 0$$

- In particular for specific x, y, we have
  - *L(x,y)>0 if (x,y) below the line, and*
  - *L(x,y)<0 if (x,y) above the line*

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Scan Conversion of Lines
Bresenham Algorithm

- Decision variable: after drawing point (x,y) decide whether to draw
  - $(x+1,y)$: *case E (for "east")*
  - $(x+1,y+1)$: *case NE (for "north-east")*
- Check whether $(x+1,y+1/2)$ is above or below line

$$d = L(x+1, y+\frac{1}{2})$$

- Point above line if and only if $d<0$

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Scan Conversion of Lines

Bresenham Algorithm

- Problem: how to update d?
- Case E (point above line, $d<=0$)
  - *x= x+1;*
  - $d= L(x+2, y+1/2)= d+ (y_e-y_s)/(x_e-x_s)$
- Case NE (point below line, $d>0$)
  - *x= x+1; y= y+1;*
  - $d= L(x+2, y+3/2)= d+ (y_e-y_s)/(x_e-x_s) -1$
- Initialization:
  - $d= L(x_s+1, y_s+1/2)= (y_e-y_s)/(x_e-x_s) -1/2$

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Scan Conversion of Lines

Bresenham Algorithm

- This is still floating point
- But: only sign of d matters
- Thus: can multiply everything by $2(x_e-x_s)$

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Scan Conversion of Lines

Bresenham Algorithm

```
Bresenham( int xs, ys, xe, ye ) {
    int y= ys;
    incrE= 2(ye - ys);
    incrNE= 2((ye - ys) – (xe-xs));
    for( int x= xs ; x<= xe ; x++ ) {
        drawPixel( x, y );
        if( d<= 0 ) d+= incrE;
        else { d+= incrNE; y++; }
    }
}
```

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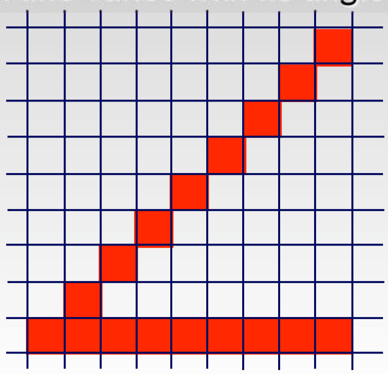
Scan Conversion of Lines

Discussion

- Bresenham sets same pixels as DDA
- Intensity of line varies with its angle!



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Scan Conversion of Lines

Discussion

- Bresenham
  - *Good for hardware implementations (integer!)*
- DDA
  - *May be faster for software (depends on system)!*
  - *Floating point ops higher parallelized (pipelined)*
    - E.g. RISC CPUs from MIPS, SUN
  - *No if statements in inner loop*
    - More efficient use of processor pipelining

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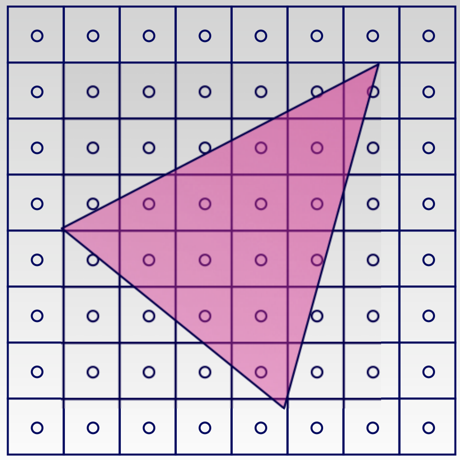
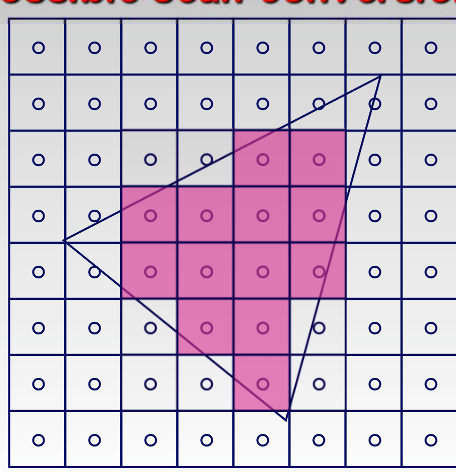
Scan Conversion of Polygons



Scan Conversion of Polygons

One possible scan conversion



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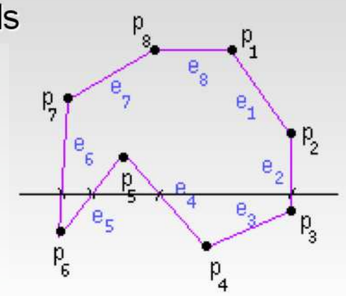
Scan Conversion of Polygons

A General Algorithm

- Intersect each scanline with all edges
- Sort intersections in x
- Calculate parity to determine in/out
- Fill the 'in' pixels



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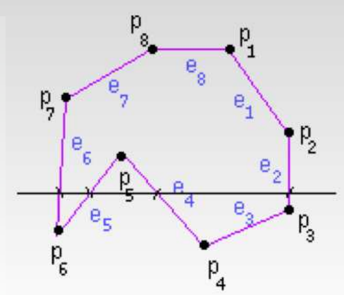
Scan Conversion of Polygons

- Works for arbitrary polygons
- Efficiency improvement:
  - *Exploit row-to-row coherence using "edge table"*



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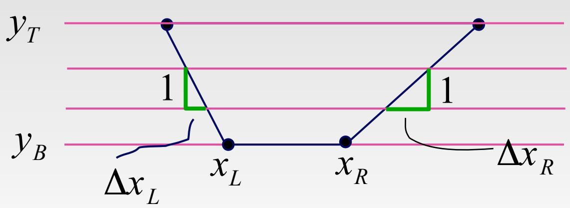
Edge Walking

Past graphics hardware

- Exploit continuous L and R edges on trapezoid

scanTrapezoid($x_L, x_R, y_B, y_T, \Delta x_L, \Delta x_R$**)**



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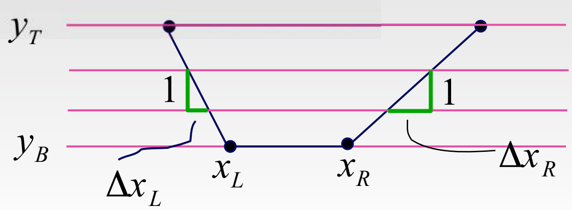
Edge Walking

```
for (y=yB; y<=yT; y++) {
    for (x=xL; x<=xR; x++)
     setPixel(x,y);
    xL += DxL;
    xR += DxR;
}
```



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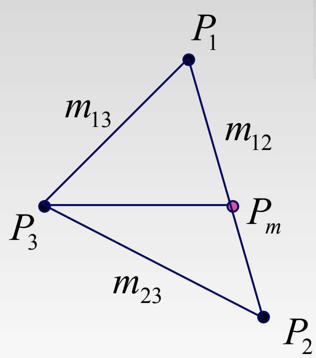
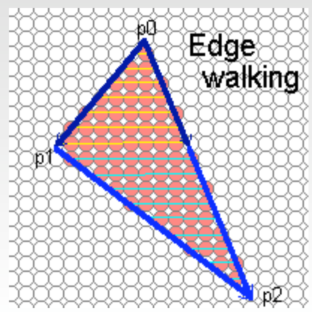
Edge Walking Triangles

- Split triangles into two regions with continuous left and right edges

scanTrapezoid($x_3, x_m, y_3, y_1, \frac{1}{m_{13}}, \frac{1}{m_{12}}$)

scanTrapezoid($x_2, x_2, y_2, y_3, \frac{1}{m_{23}}, \frac{1}{m_{12}}$)





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Edge Walking Triangles

Issues

- Many applications have small triangles
  - *Setup cost is non-trivial*
- Clipping triangles produces non-triangles
  - *This can be avoided through re-triangulation, as discussed*

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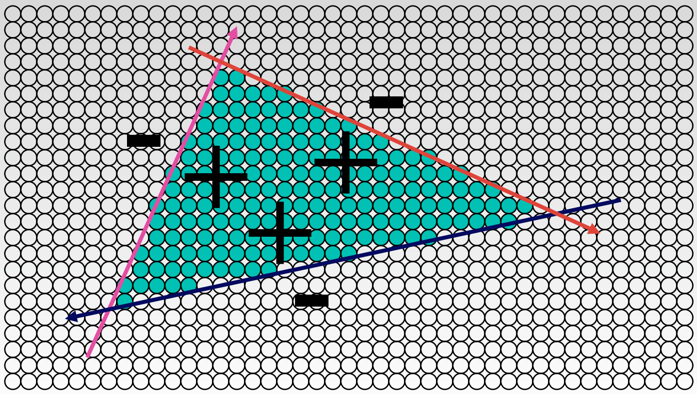
Modern Rasterization: Edge Equations

Define a triangle as follows:



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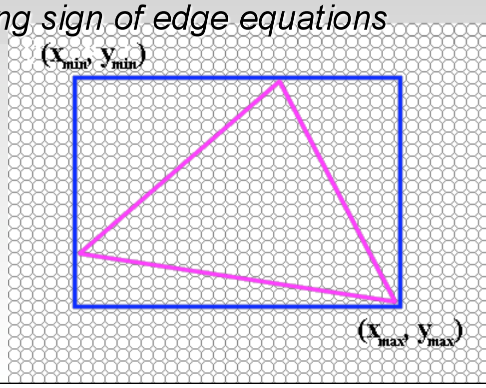
Using Edge Equations

Usage:

- Go over each pixel in bounding rectangle
- Check if pixel is inside/outside of triangle
  - *Using sign of edge equations*



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Computing Edge Equations

Implicit equation of a triangle edge:

$$L(x,y) = \frac{(y_e - y_s)}{(x_e - x_s)}(x - x_s) - (y - y_s) = 0$$

(see Bresenham algorithm)

- L(x,y) positive on one side of edge, negative on the other

Question:

- What happens for vertical lines?

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Edge Equations

Multiply with denominator

$$L(x,y) = (y_e - y_s)(x - x_s) - (y - y_s)(x_e - x_s) = 0$$

- Avoids singularity
- Works with vertical lines

What about the sign?

- Which side is in, which is out?

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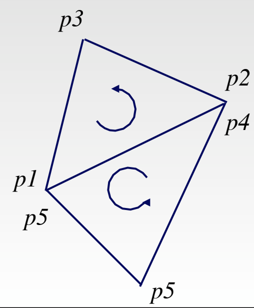
Edge Equations

Determining the sign

- Which side is “in” and which is “out” depends on order of start/end vertices…
- Convention: specify vertices in counter-clockwise order



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Edge Equations

Counter-Clockwise Triangles

- The equation L(x,y) as specified above is *negative inside, positive outside*
  - *Flip sign:*

$$L(x,y) = -(y_e - y_s)(x - x_s) + (y - y_s)(x_e - x_s) = 0$$

Clockwise triangles

- *Use original formula*

$$L(x,y) = (y_e - y_s)(x - x_s) - (y - y_s)(x_e - x_s) = 0$$

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Discussion of Polygon Scan Conversion Algorithms

On old hardware:

- Use first scan-conversion algorithm
  - *Scan-convert edges, then fill in scanlines*
  - *Compute interpolated values by interpolating along edges, then scanlines*
- Requires clipping of polygons against viewing volume
- Faster if you have a few, large polygons
- Possibly faster in software

Discussion of Polygon Scan Conversion Algorithms

Modern GPUs:

- Use edge equations
  - *And plane equations for attribute interpolation*
  - *No clipping of primitives required*
- Faster with many small triangles

Additional advantage:

- Can control the order in which pixels are processed
- Allows for more memory-coherent traversal orders
  - *E.g. tiles or space-filling curve rather than scanlines*

Triangle Rasterization Issues (Independent of Algorithm)

Exactly which pixels should be lit?

- A: Those pixels inside the triangle edge (of course)

But what about pixels exactly on the edge?

- Draw them: order of triangles matters (it shouldn't)
- Don't draw them: gaps possible between triangles

We need a consistent (if arbitrary) rule

- Example: draw pixels on left or top edge, but not on right or bottom edge

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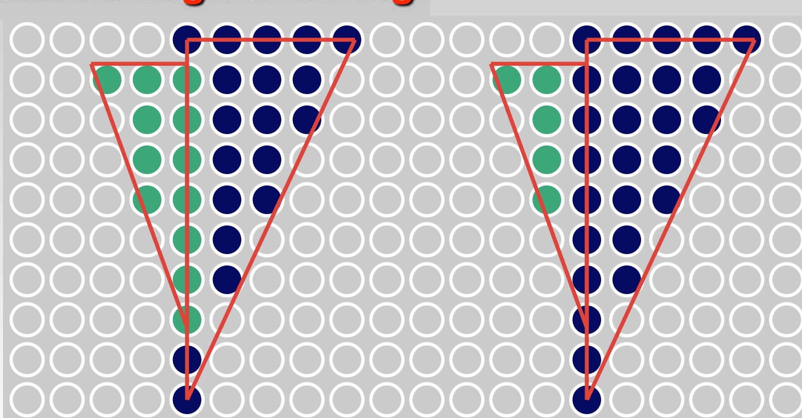
Triangle Rasterization Issues

Shared Edge Ordering



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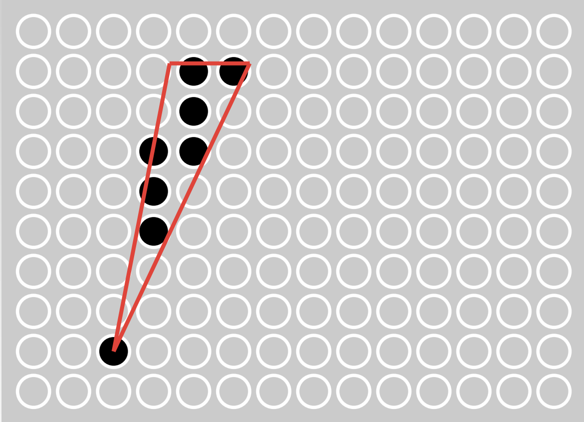
Triangle Rasterization Issues

Sliver



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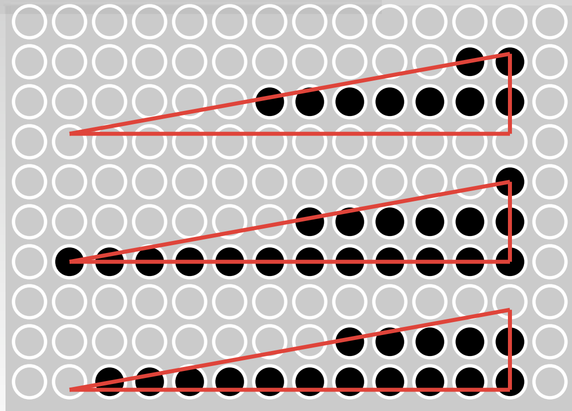
Triangle Rasterization Issues

Moving Slivers



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Triangle Rasterization Issues

These are ALIASING Problems

- Problems associated with representing continuous functions (triangles) with finite resolution (pixels)
- More on this problem when we talk about sampling…

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Shading

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Interpolation During Scan Conversion

Need to propagate vertex attributes to pixels

- Interpolate between vertices:
  - *z (depth)*
  - *r,g,b color components*
  - N_x, N_y, N_z *surface normals*
  - *u,v texture coordinates (talk about these later)*
- Three equivalent ways of viewing this (for triangles)
  1. *Bilinear interpolation*
  2. *Barycentric coordinates*
  3. *Plane Equation*

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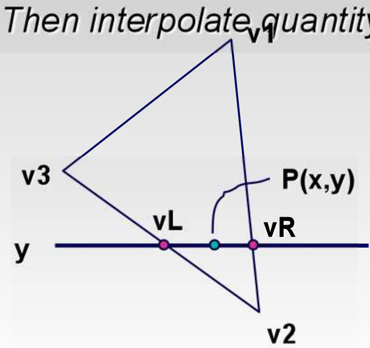
1. Bilinear Interpolation

We've seen this before:

- Interpolate quantity along LH and RH edges, as a function of y
  - *Then interpolate quantity as a function of x*



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2. Barycentric Coordinates

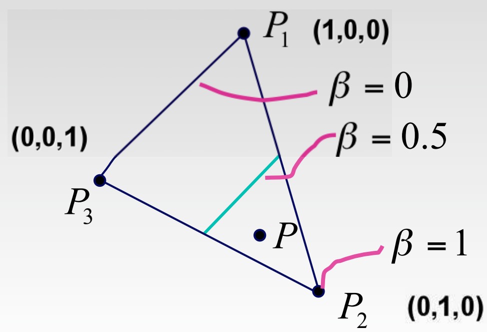
This too:

- Barycentric Coordinates: weighted combination of vertices

$$P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3$$

$$\alpha + \beta + \gamma = 1$$

$$0 \le \alpha, \beta, \gamma \le 1$$



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3. Plane Equation

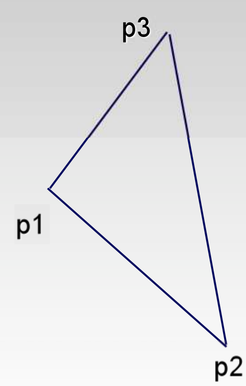
Observatiop: Quantities vary linearly across image plane

- E.g.: r = Ax + By + C
  - *r= red channel of the color*
  - *Same for g, b, Nx, Ny, Nz, z…*
- From info at vertices we know:

$$r_1 = Ax_1 + By_1 + C$$

$$r_2 = Ax_2 + By_2 + C$$

$$r_3 = Ax_3 + By_3 + C$$



  - *Solve for A, B, C*
  - *One-time set-up cost per triangle and interpolated quantity*

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Flat Shading

- Simplest approach calculates illumination at a single point for each polygon

- Obviously inaccurate for smooth surfaces

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Flat Shading Approximations

If an object really <u>is</u> faceted, is this accurate?

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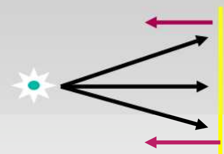
Flat Shading Approximations

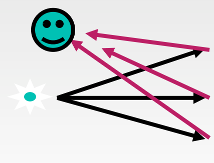
If an object really <u>is</u> faceted, is this accurate?

no!

- For point sources, the direction to light varies across the facet



- For specular reflectance, direction to eye varies across the facet



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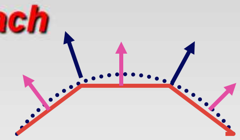
Improving Flat Shading

What if evaluate Phong lighting model at each pixel of the polygon?

- Better, but result still clearly faceted

For smoother-looking surfaces we introduce vertex normals at each vertex



- Usually different from facet normal
- Used ***only*** for shading
- Think of as a better approximation of the ***real*** surface that the polygons approximate

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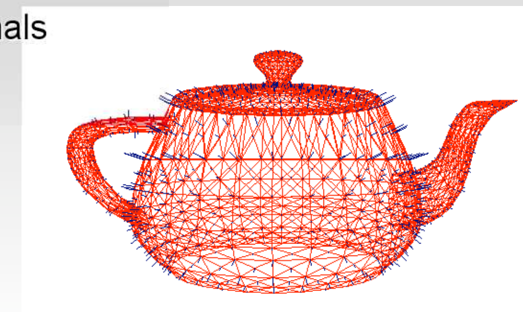
Vertex Normals

Vertex normals may be

- Provided with the model
- Computed from first principles
- Approximated by averaging the normals of the facets that share the vertex



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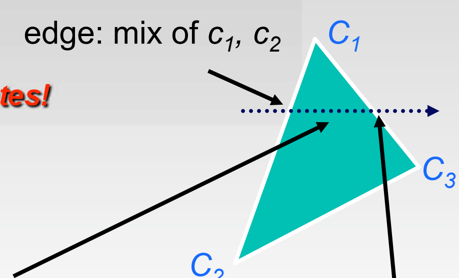
Gouraud Shading

Most common approach, and what OpenGL does

- Perform Phong lighting at the vertices
- Linearly interpolate the resulting colors over faces
  - *Along edges*
  - *Along scanlines*

Same as Barycentric Coordinates!



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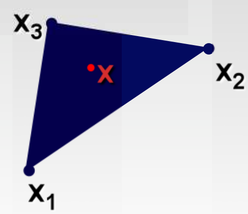
Barycentric Coordinates

- Convex combination of 3 points

$$\mathbf{x} = \alpha \cdot \mathbf{x}_1 + \beta \cdot \mathbf{x}_2 + \gamma \cdot \mathbf{x}_3$$
$$\text{with } \alpha + \beta + \gamma = 1,\ 0 \le \alpha, \beta, \gamma \le 1$$

- α, β, and γ are called *barycentric coordinates*



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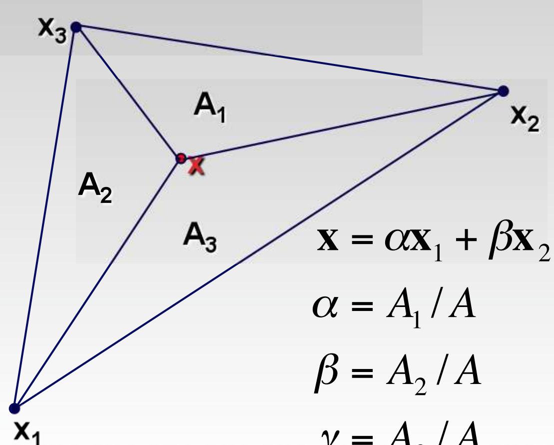
Barycentric Coordinates

One way to compute them:



$$\mathbf{x} = \alpha \mathbf{x}_1 + \beta \mathbf{x}_2 + \gamma \mathbf{x}_3 \text{ with}$$
$$\alpha = A_1 / A$$
$$\beta = A_2 / A$$
$$\gamma = A_3 / A$$

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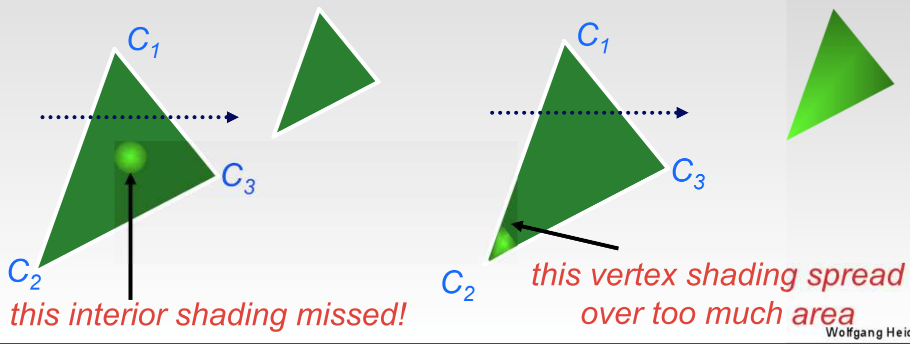
Gouraud Shading Artifacts

often appears dull, chalky

lacks accurate specular component

- if included, will be averaged over entire polygon



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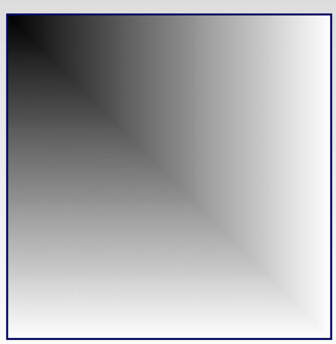
Gouraud Shading Artifacts

Mach bands

- Eye enhances discontinuity in first derivative
- Very disturbing, especially for highlights



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Phong Shading

linearly interpolating surface normal across the facet, applying Phong lighting model at every pixel

- Same input as Gouraud shading
- Pro: much smoother results
- Con: considerably more expensive

Not the same as Phong lighting

- Common confusion
- **Phong lighting:** empirical model to calculate illumination at a point on a surface

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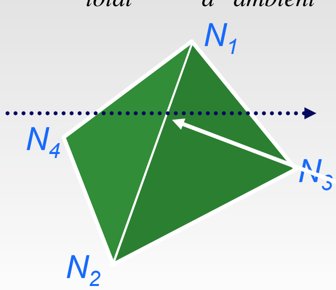
Phong Shading

Linearly interpolate the vertex normals

- Compute lighting equations at each pixel
- Can use specular component

$$I_{total} = k_a I_{ambient} + \sum_{i=1}^{\#lights} I_i \left(k_d (\mathbf{n} \cdot \mathbf{l_i}) + k_s (\mathbf{v} \cdot \mathbf{r_i})^{n_{shiny}} \right)$$



remember: normals used in diffuse and specular terms

discontinuity in normal's rate of change harder to detect

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Phong Shading Difficulties

Computationally expensive

- Per-pixel vector normalization and lighting computation!
- Floating point operations required

Lighting after perspective projection

- Messes up the angles between vectors
- Have to keep eye-space vectors around

No direct support in hardware

- But can be simulated with texture mapping

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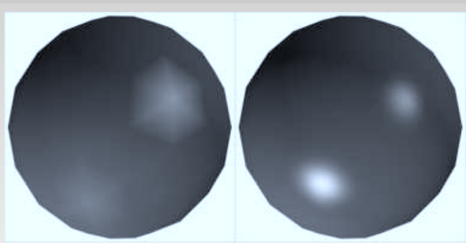
Shading Artifacts: Silhouettes

Polygonal silhouettes remain



Gouraud *Phong*

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Coming Up:

Friday

- Scan conversion / shading

Next week

- Clipping, hidden surface removal

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