Perspective Projection (cont.)

Wolfgang Heidrich

Summer Internship

Wanted:

- Undergraduate student for summer research on graphics (mostly 2D imaging, digital photography)
- NSERC undergraduate research fellowship

Prerequisites:

- Strong programming skills, ideally C++
- Not afraid to learn new math & algorithms

Interested?

- Talk to me after lecture, or send email…

Course News

Assignment 1

- Due February 2

Homework 1

- Discussed in labs this week

Homework 2

- Exercise problems for perspective
- Discussed in labs next week

Quiz 1

- One week from today (Wed, Jan 28)

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Course News (cont.)

Reading list

- Previously published chapters numbers were from an old book version…

Reading for Quiz (new book version):

- Math prereq: Chapter 2.1-2.4, 4
- Intro: Chapter 1
- Affine transformations: Ch. 6 (was: Ch. 5, old book)
- Perspective: Ch 7 (was: Ch. 6, old book)
  - *Also reading for this week…*

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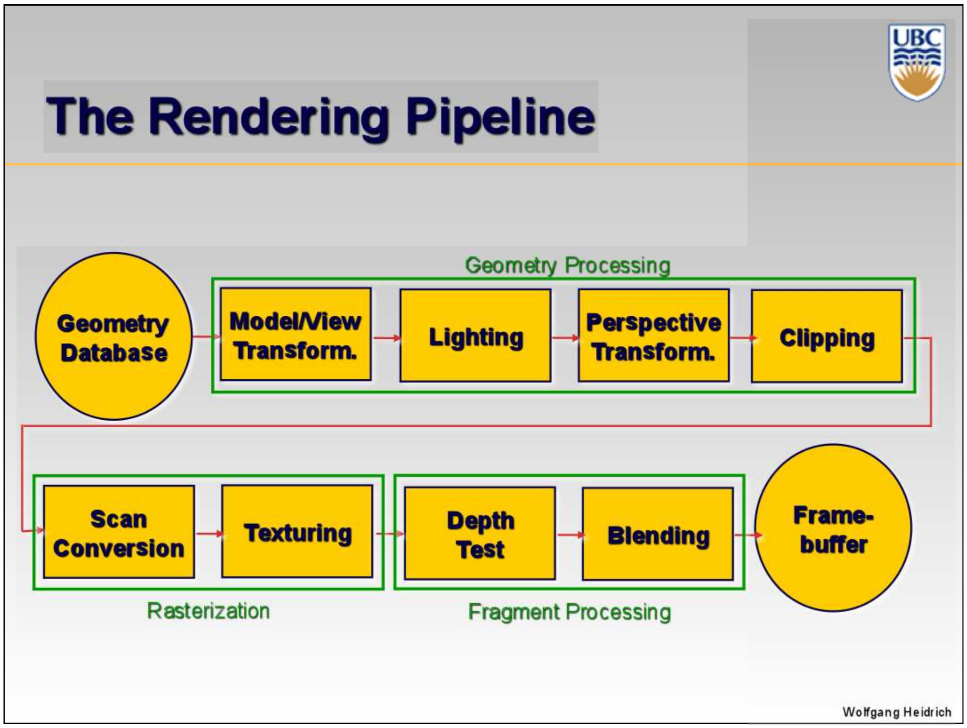
The Rendering Pipeline
Geometry Processing
Geometry Database
Model/View Transform.
Lighting
Perspective Transform.
Clipping
Scan Conversion
Texturing
Depth Test
Blending
Frame-buffer
Rasterization
Fragment Processing
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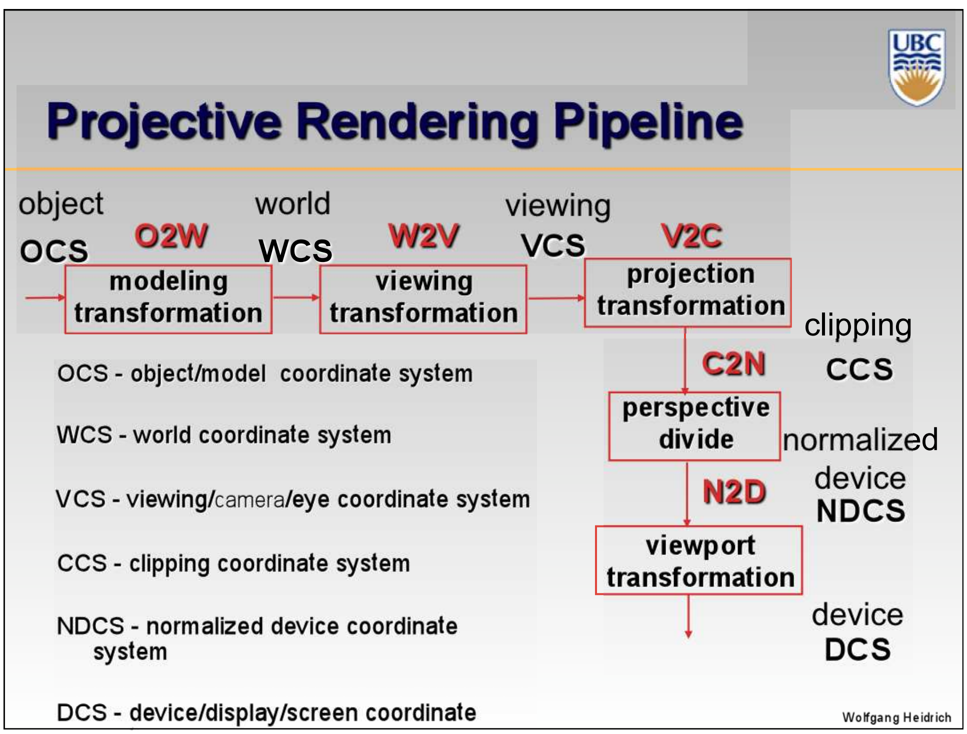
Projective Rendering Pipeline
object
OCS
O2W
modeling transformation
world
WCS
W2V
viewing transformation
viewing
VCS
V2C
projection transformation
clipping
CCS
C2N
perspective divide
normalized device
NDCS
N2D
viewport transformation
device
DCS
OCS - object/model coordinate system
WCS - world coordinate system
VCS - viewing/camera/eye coordinate system
CCS - clipping coordinate system
NDCS - normalized device coordinate system
DCS - device/display/screen coordinate
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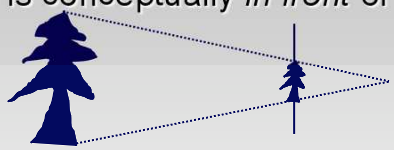
Perspective Transformation

In computer graphics:

- Image plane is conceptually *in front* of the center of projection



- Perspective transformations belong to a class of operations that are called *projective transformations*
- Linear and affine transformations also belong to this class
- *All* projective transformations can be expressed as *4x4* matrix operations

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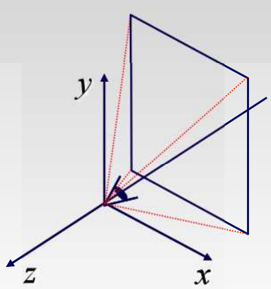
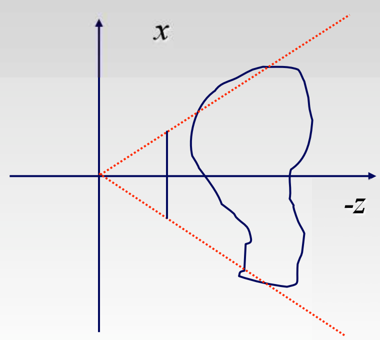
Perspective Projection

Synopsis:

- Project all geometry through a common <u>*center of projection*</u> (<u>*eye point*</u>) onto an <u>*image plane*</u>





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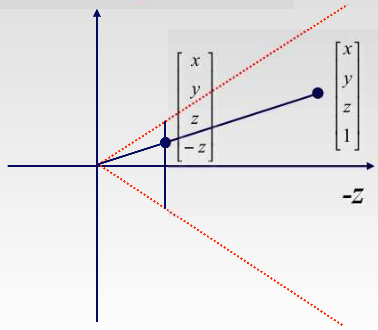
Perspective Projection

Example:

- Assume image plane at $z=-1$
- A point $[x,y,z,1]^T$ projects to
  $[-x/z,-y/z,-z/z,1]^T \equiv [x,y,z,-z]^T$



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Perspective Projection

Analysis:

- This is a special case of a general family of transformations called *projective transformations*
- These can be expressed as 4x4 homogeneous matrices!
  – *E.g. in the example:*

$$T\left(\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ -z \end{bmatrix} \equiv \begin{bmatrix} -x/z \\ -y/z \\ -1 \\ 1 \end{bmatrix}$$

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Demos

Tuebingen applets from Frank Hanisch

- http://www.gris.uni-tuebingen.de/edu/projects/grdev/doc/html/

  (this is the English version)

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Projective Rendering Pipeline

object **OCS** → **O2W** modeling transformation → world **WCS** → **W2V** viewing transformation → viewing **VCS** → **V2C** projection transformation → clipping **CCS** → **C2N** perspective divide → normalized device **NDCS** → **N2D** viewport transformation → device **DCS**

OCS - object/model coordinate system

WCS - world coordinate system

VCS - viewing/camera/eye coordinate system

CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device/display/screen coordinate

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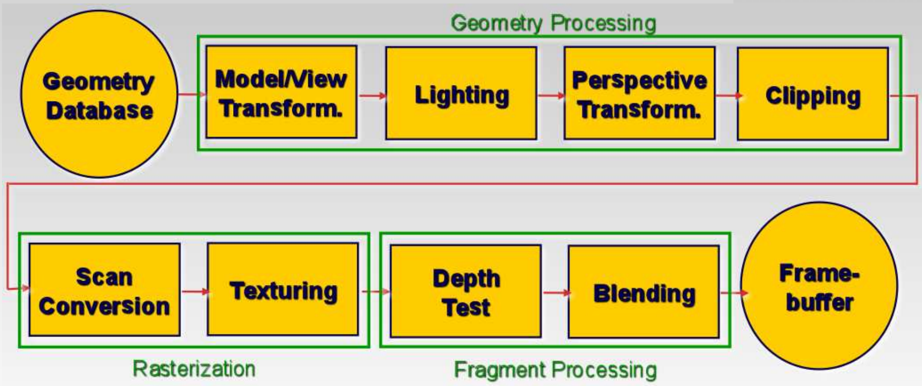
The Rendering Pipeline



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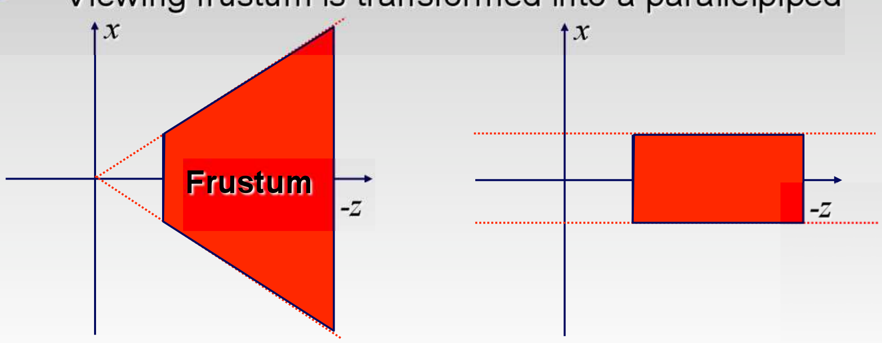
Projective Transformations

Transformation of space:

- Center of projection moves to infinity
- Viewing frustum is transformed into a parallelpiped



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Projective Transformations

Convention:

- Viewing frustum is mapped to a specific parallelpiped
  - *Normalized Device Coordinates (NDC)*
- Only objects inside the parallelpiped get rendered
- Which parallelpied is used depends on the rendering system

OpenGL:

- Left and right image boundary are mapped to x=-1 and x=+1
- Top and bottom are mapped to y=-1 and y=+1
- Near and far plane are mapped to -1 and 1

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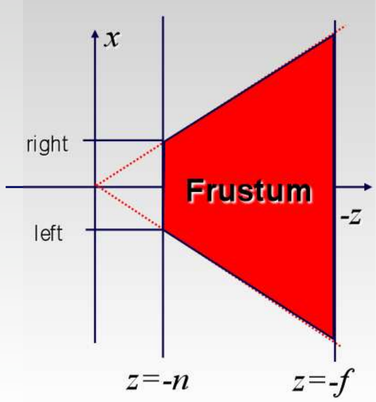
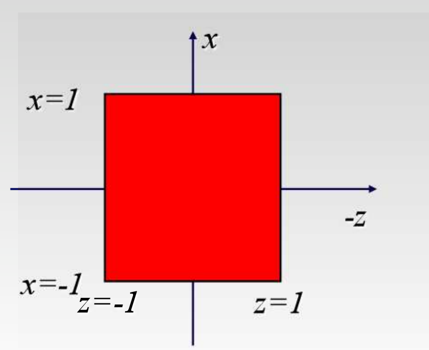
Projective Transformations

OpenGL Convention

Camera coordinates



Clipping Coordinates



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Projective Transformations

Why near and far plane?

- Near plane:
  - *Avoid singularity (division by zero, or very small numbers)*
- Far plane:
  - *Store depth in fixed-point representation (integer), thus have to have fixed range of values (0…1)*
  - *Avoid/reduce numerical precision artifacts for distant objects*

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Projective Transformations

Determining the matrix representation

- Need to observe 5 points in general position, e.g.
  - $[left,0,0,1]^T \rightarrow [1,0,0,1]^T$
  - $[0,top,0,1]^T \rightarrow [0,1,0,1]^T$
  - $[0,0,-f,1]^T \rightarrow [0,0,1,1]^T$
  - $[0,0,-n,1]^T \rightarrow [0,0,0,1]^T$
  - $[left*f/n,top*f/n,-f,1]^T \rightarrow [1,1,1,1]^T$
- Solve resulting equation system to obtain matrix

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Perspective Derivation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = Ex + Az$$
$$y' = Fy + Bz$$
$$z' = Cz + D$$
$$w' = -z$$

$$x = left \rightarrow x'/w' = 1$$
$$x = right \rightarrow x'/w' = -1$$
$$y = top \rightarrow y'/w' = 1$$
$$y = bottom \rightarrow y'/w' = -1$$
$$z = -near \rightarrow z'/w' = 1$$
$$z = -far \rightarrow z'/w' = -1$$

$$y' = Fy + Bz, \quad \frac{y'}{w'} = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{-z},$$

$$1 = F\frac{y}{-z} + B\frac{z}{-z}, \quad 1 = F\frac{y}{-z} - B, \quad 1 = F\frac{top}{-(-near)} - B,$$

$$1 = F\frac{top}{near} - B$$

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Perspective Derivation

similarly for other 5 planes

6 planes, 6 unknowns

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

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Perspective Example

view volume
- *left = -1, right = 1*
- *bot = -1, top = 1*
- *near = 1, far = 4*

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/3 & -8/3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

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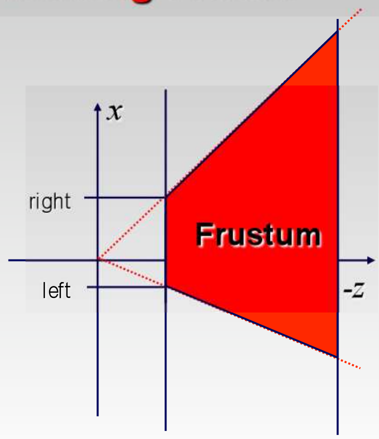
Projective Transformations

Asymmetric Viewing Frusta



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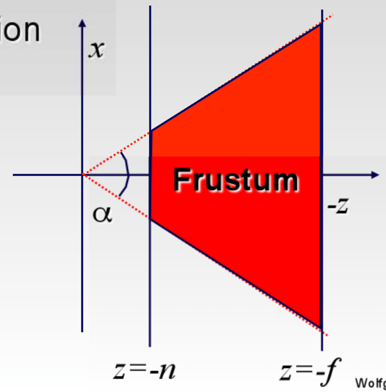
Projective Transformations

Alternative specification of symmetric frusta

- Field-of-view (fov) α
- Fov/2
- Field-of-view in y-direction (fovy) + aspect ratio



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Perspective Matrices in OpenGL

Perspective Matrices:

- glFrustum(left, right, bottom, top, near, far)
  - *Specifies perspective transform (near, far are always positive)*

Convenience Function:

- gluPerspective(fovy, aspect, near, far)
  - *Another way to do perspective*

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Projective Transformations

Properties:

- All transformations that can be expressed as homogeneous 4x4 matrices (in 3D)
- 16 matrix entries, but multiples of the same matrix all describe the same transformation
  - *15 degrees of freedom*
  - *The mapping of 5 points uniquely determines the transformation*

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Projective Transformations

Properties

- Lines are mapped to lines and triangles to triangles
- Parallel lines do NOT remain parallel
  - *E.g. rails vanishing at infinity*
- Affine combinations are NOT preserved
  - *E.g. center of a line does not map to center of projected line (perspective foreshortening)*

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Orthographic Camera Projection

- Camera's back plane parallel to lens
- Infinite focal length
- No perspective convergence
- Just throw away z values

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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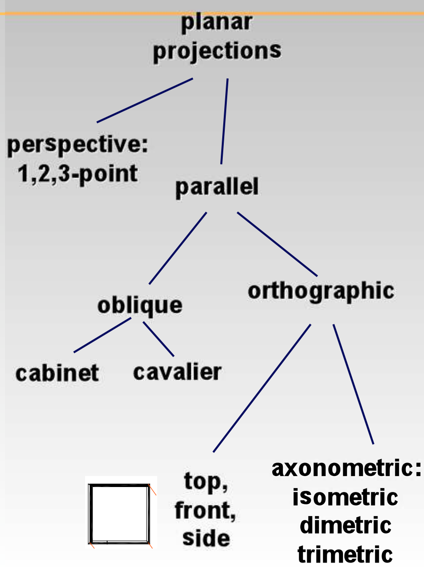
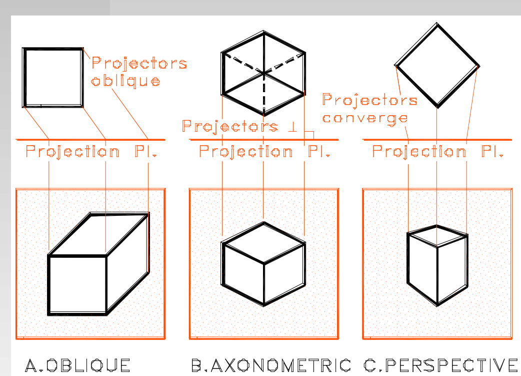
Projection Taxonomy





http://ceprofs.tamu.edu/tkramer/ENGR%20111/5.1/20

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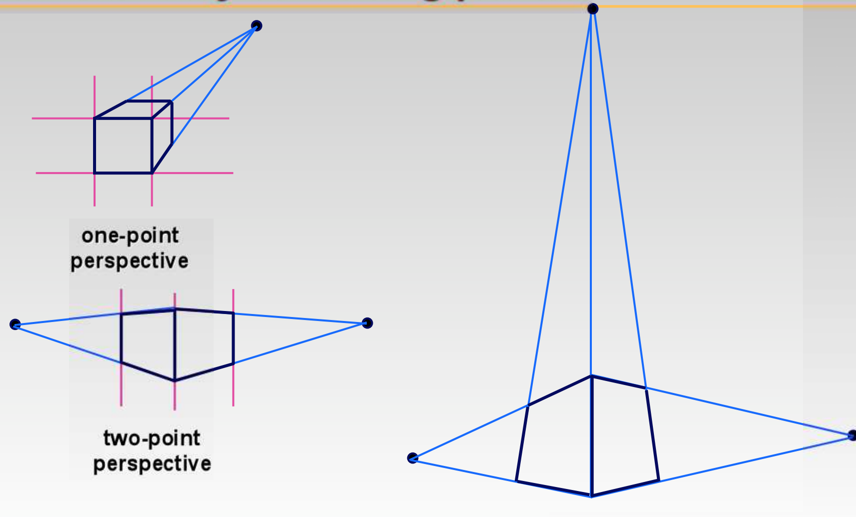
Perspective Projections

classified by vanishing points



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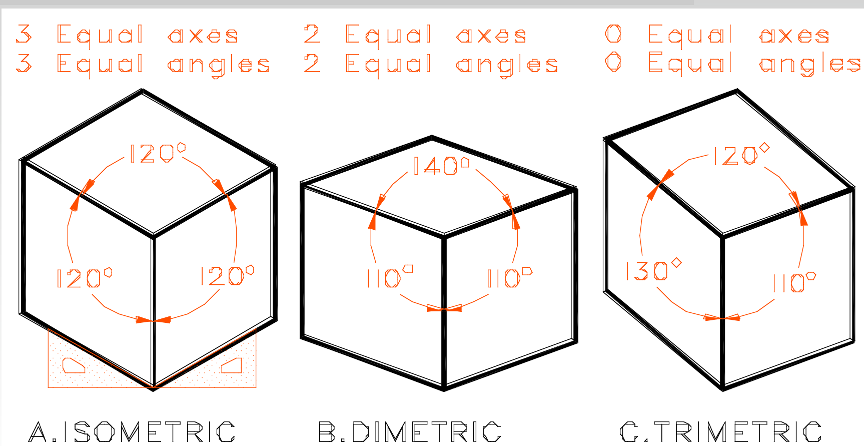
Axonometric Projections

- projectors perpendicular to image plane



http://ceprofs.tamu.edu/tkramer/ENGR%20111/5.1/20

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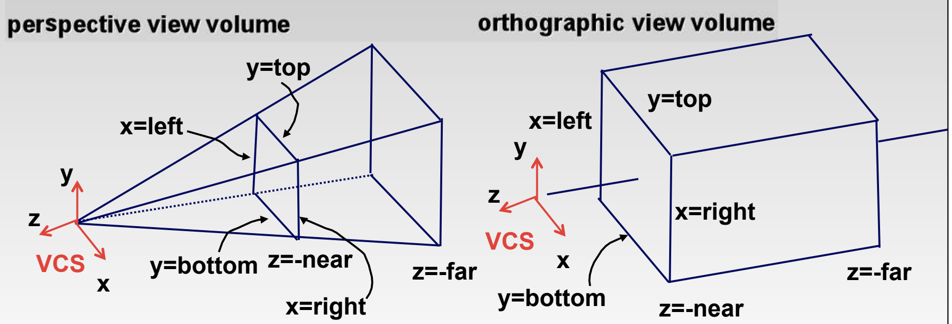
View Volumes

- specifies field-of-view, used for clipping
- restricts domain of ***z*** stored for visibility test



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View Volume

Convention

- Viewing frustum mapped to specific parallelepiped
  - *Normalized Device Coordinates (NDC)*
  - *Same as clipping coords*
- Only objects inside the parallelepiped get rendered
- Which parallelepiped?
  - *Depends on rendering system*

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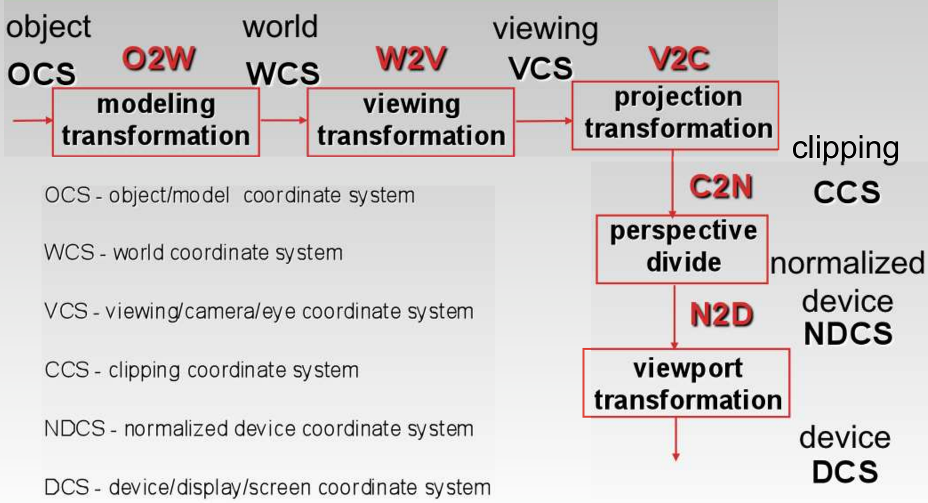
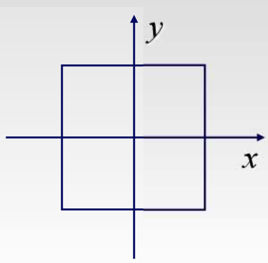
Projective Rendering Pipeline

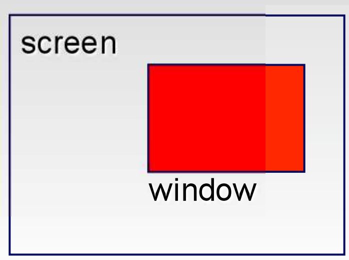


Window-To-Viewport Transformation

Generate pixel coordinates

- Map x, y from range –1…1 (*normalized device coordinates*) to pixel coordinates on the screen
- Map z from -1…1 to 0…1 (used later for visibility)
- Involves 2D scaling and translation





Coming Up:

Friday:

- Transformations of planes and normals

Friday/Next Week

- Lighting/shading

Don't forget the quiz…!

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