CPSC 314 Homework 1

Term: Jan 2009, Instructor: Wolfgang Heidrich, heidrich@cs.ubc.ca, http://www.ugrad.cs.ubc.ca/˜cs314

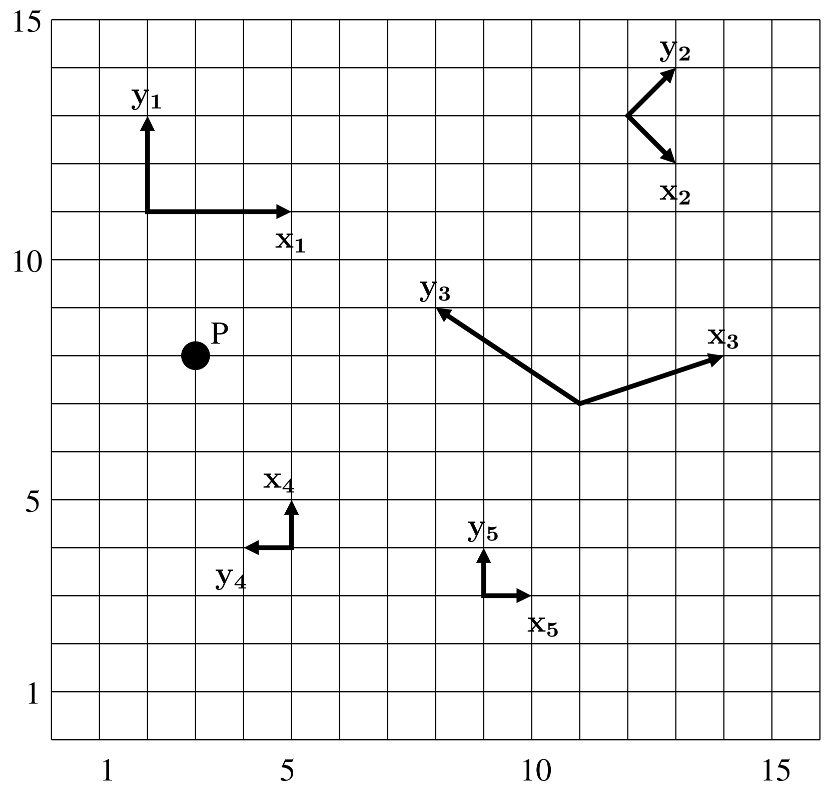
NOTE: These homework problems and the ones coming in future weeks are not graded. **However**, I strongly encourage you to take them seriously as a preparation for the quizzes and the final exam.

This set of homework problems is deals with coordinates and coordinate transformations. The solutions will be discussed in the week of January 19-23.

Problem 1: Transformations Between Different Coordinate Systems

Describe the point P in the following figure in terms of the coordinate systems given by the vectors $(\mathbf{x}_i, \mathbf{y_i}), i = 1 \ldots 5$. That is, determine a_i and b_i such that

$$P = a_i \cdot \mathbf{x}_i + b_i \cdot \mathbf{y}_i$$



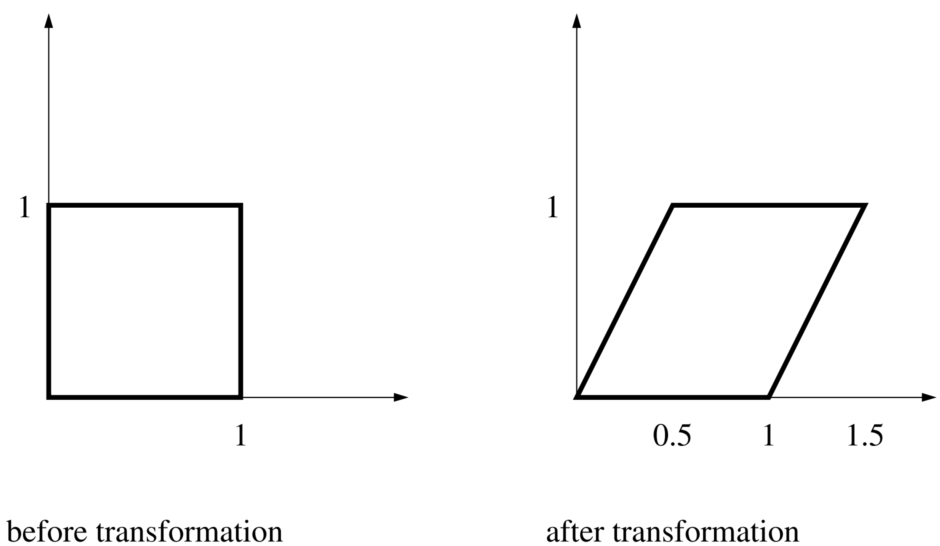
Problem 2: Linear Transformations

Derive the 2×2 matrices for the following linear transformations:

- a counter-clockwise rotation of an angle of ϕ.
- a scaling by a in x-direction and b in y-direction.
- a shearing operation as depicted in the figure below.

Note: we had the matrices in class, but you are supposed to derive them from the way they map points in one space onto points in another. Do not just write the result down, actually derive it!

HINT: remember that a linear transformation in 2D is uniquely defined by observing its effect on two distinct, linearly independent points.



Problem 3: Affine Transformations

Given that a 2D linear transformation is uniquely determined by its effect on 2 points, how many points do you need to uniquely determine a 2D affine transformation

$$\mathbf{v}' := \begin{pmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \end{pmatrix} \cdot \mathbf{v} + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}?$$

Problem 4: Properties of Affine Transformations

Let $T(x) = \mathbf{M} \cdot \mathbf{x} + \mathbf{t}$ be an affine transformation.

a) show that $T(x)$ preserves affine combinations, i.e. that

$$T\left(\sum_{i=1}^{n} a_i \mathbf{x}_i\right) = \sum_{i=1}^{N} a_i T(\mathbf{x}_i), \text{for} \sum_{i=1}^{N} a_i = 1$$

Hint: show the property for two points first, and then generalize, i.e. show that:

$$T((1-a)\mathbf{x}_1 + a\mathbf{x}_2) = (1-a) \cdot T(\mathbf{x}_1) + a \cdot T(\mathbf{x}_2)$$

b) show that $T(\mathbf{x})$ maps parallel lines to parallel lines.