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Scan Conversion

Projective Rendering Pipeline

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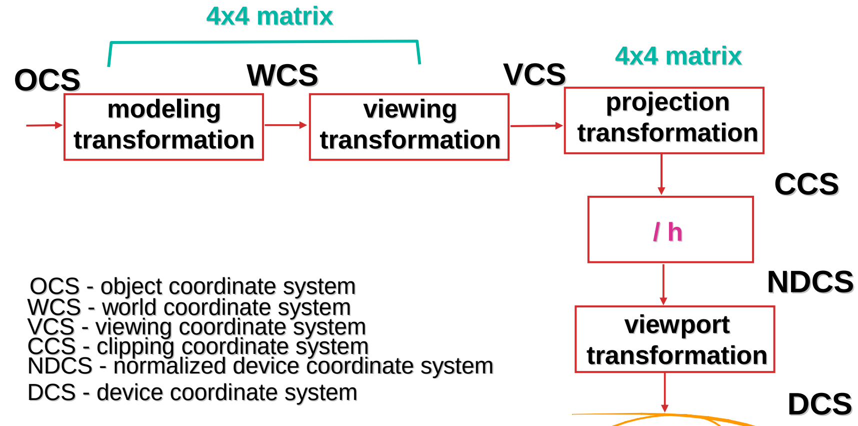
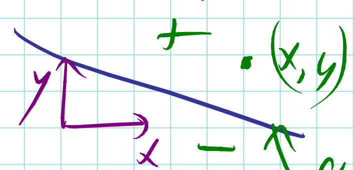


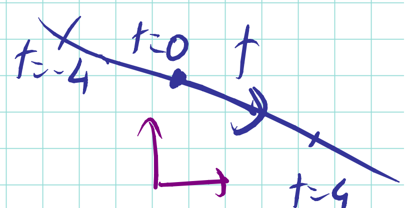
OCS - object coordinate system
WCS - world coordinate system
VCS - viewing coordinate system
CCS - clipping coordinate system
NDCS - normalized device coordinate system
DCS - device coordinate system

① Implicit ② Explicit ③ Parametric

Line in 2D space

$y = f(x)$



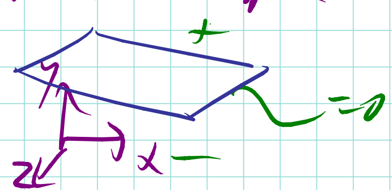


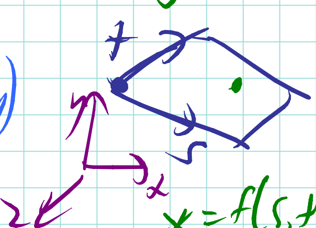
$f(x,y)$

$x = f(t)$
$y = g(t)$

Plane in 3D space

$z = f(x,y)$





$f(x,y,z)$

$x = f(s,t)$
$y = g(s,t)$
$z = h(s,t)$

$0 = -y + mx + b$

Lines and Curves

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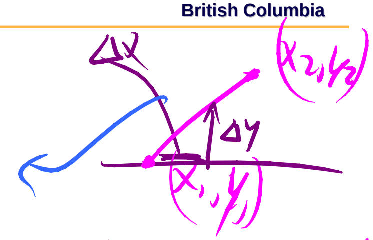
Explicit

line $y = mx + b$

$$y = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) + y_1$$

circle $y = \pm\sqrt{r^2 - x^2}$



$y = y_1 + \Delta y$

$\Delta y = \frac{(y_2 - y_1)}{(x_2 - x_1)} (x - x_1)$

+

plane $z = Ex + Fy + G$

sphere $z = \pm\sqrt{r^2 - x^2 - y^2}$

text p.30-41

Lines and Curves

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Parametric

line

$$x(t) = x_0 + t(x_1 - x_0)$$
$$y(t) = y_0 + t(y_1 - y_0)$$
$$t \in [0,1]$$
$$P(t) = P_0 + t(P_1 - P_0)$$
$$P(t) = (1-t)P_0 + tP_1$$

P_0, P_1, P_2, t, s

$t=0$, $t=1$, P_0, P_1

$1-t$, t

0 1 t

basis functions

circle

$$x(\theta) = r\cos(\theta)$$
$$y(\theta) = r\sin(\theta)$$
$$\theta \in [0, 2\pi]$$

plane $P(s,t) = P_0 + s(P_1 - P_0) + t(P_2 - P_0)$

Lines and Curves

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$y = mx + b \quad 0 = -y + mx + b$

Implicit

$dy\,x + (dx)\,b + \begin{matrix}-x_0 dy \\ +y_0 dx\end{matrix}$

$= ax + by + c$

$P_0(x_0, y_0)$, dx, dy

line $F(x,y) = (x - x_0)dy - (y - y_0)dx$

$F(x,y) = 0$ **(x,y) is on line**

$F(x,y) > 0$ **(x,y) is below line**

$F(x,y) < 0$ **(x,y) is above line**

$r^2 = x^2 + y^2$

circle $F(x,y) = x^2 + y^2 - r^2$

$F(x,y) = 0$ **(x,y) is on circle**

$F(x,y) > 0$ **(x,y) is outside**

$F(x,y) < 0$ **(x,y) is inside**

$0 = -r^2 + x^2 + y^2$

$0 = r^2 - x^2 - y^2$

$+$

plane: $F(x,y,z) = Ax + By + Cz + D = N \cdot P + D$

plane: $F(x,y,z) = Ax + By + Cz + D$

$= N \cdot P + D$

$\langle A, B, C \rangle \quad \langle x, y, z \rangle$

N: normal to the plane

Polygons

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Interactive graphics uses Polygons

- Can represent any surface *with arbitrary accuracy*
  - *Splines, mathematical functions, ...*
- simple, regular rendering algorithms
  - *embed well in hardware*

Polygons

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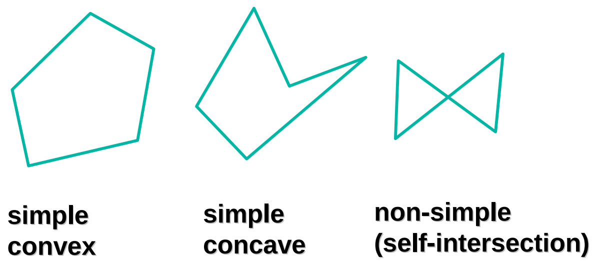
Basic Types



From Polygons to Triangles

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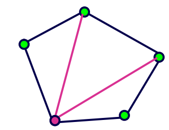
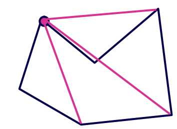
- why?   triangles are planar and convex
- simple convex polygons
  - *break into triangles, trivial*
  - `glBegin(GL_POLYGON) ... glEnd()`



- concave or non-simple polygons
  - *break into triangles, more effort*
  - `gluNewTess(), gluTessCallback(), ...`



What is Scan Conversion? (a.k.a. Rasterization)

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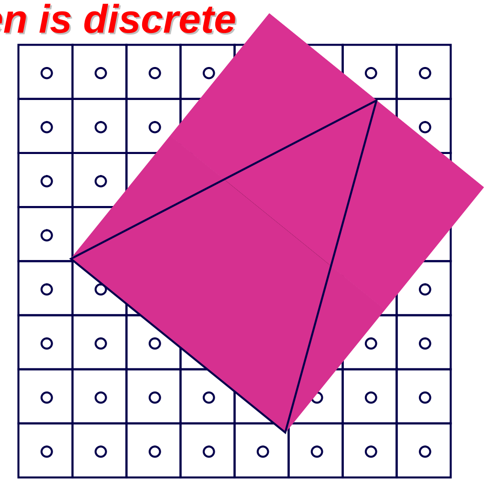
screen is discrete



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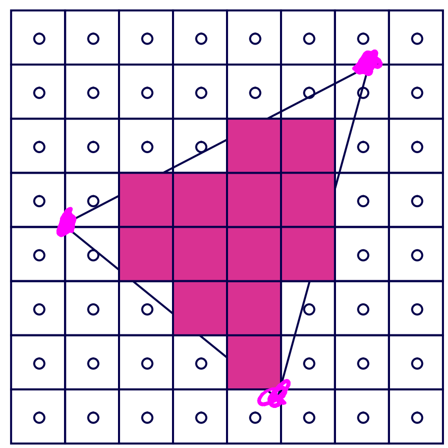
one possible scan conversion



Scan Conversion

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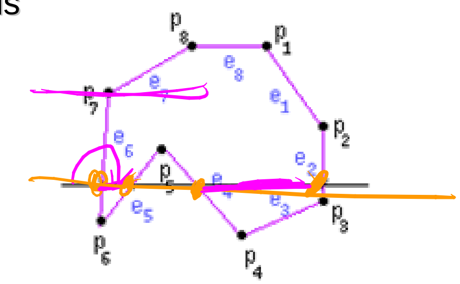
A General Algorithm

- intersect each scanline with all edges
- sort intersections in x
- calculate parity to determine in/out
- fill the 'in' pixels



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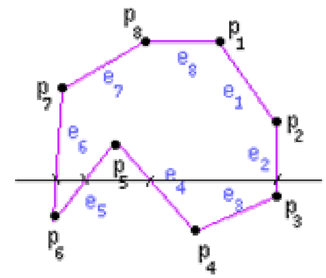
- works for arbitrary polygons
- efficiency improvement:
  - *exploit row-to-row coherence using "edge table"*



Edge Walking

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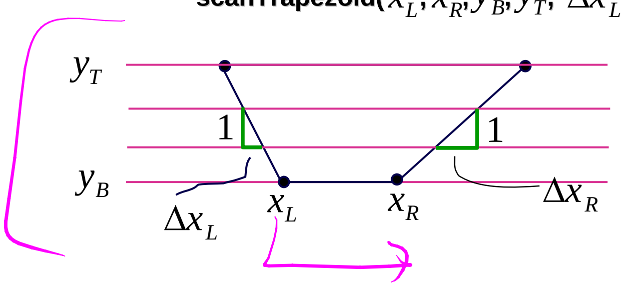
past graphics hardware

- exploit continuous L and R edges on trapezoid

scanTrapezoid($x_L, x_R, y_B, y_T, \Delta x_L, \Delta x_R$)



$\Delta x_L \equiv \frac{1}{m_L}$

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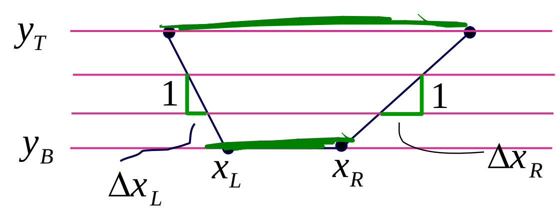
```
for (y=yB; y<=yT; y++) {
  for (x=xL; x<=xR; x++)
    setPixel(x,y);
  xL += DxL;
  xR += DxR;
}
```



Edge Walking Triangles

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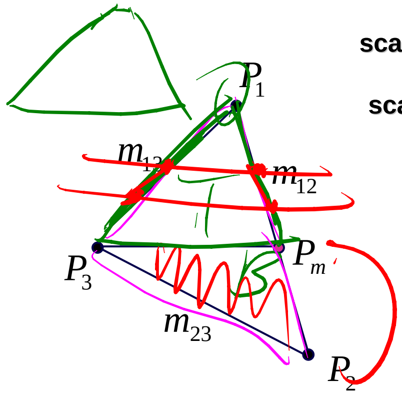
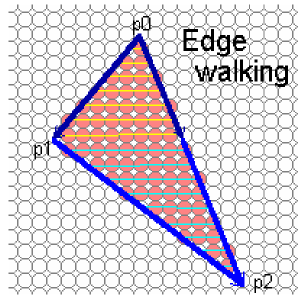
- split triangles into two regions with continuous left and right edges



scanTrapezoid($x_3, x_m, y_3, y_1, \frac{1}{m_{13}}, \frac{1}{m_{12}}$ **)**

scanTrapezoid($x_2, x_2, y_2, y_3, \frac{1}{m_{23}}, \frac{1}{m_{12}}$ **)**



Edge Walking Triangles

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Issues

- many applications have small triangles
  - *setup cost is non-trivial*
- clipping triangles produces non-triangles

Modern Rasterization

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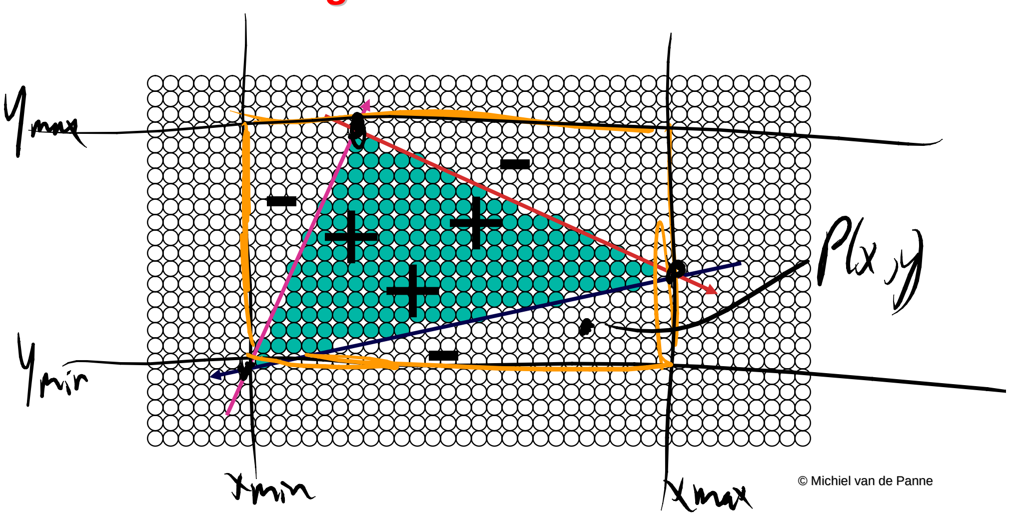
Define a triangle as follows:



Using Edge Equations

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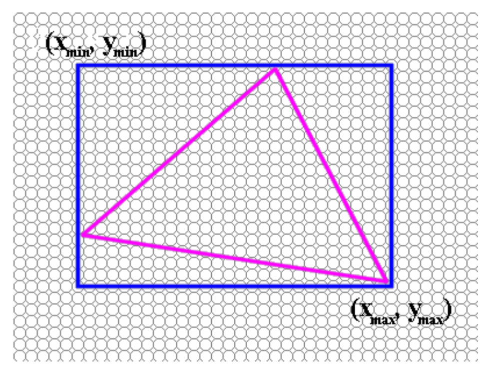

Computing Edge Equations

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Computing A,B,C from (x_1, y_1), (x_2, y_2)

$Ax_1 + By_1 + C = 0$

$Ax_2 + By_2 + C = 0$

- two equations, three unknowns
- solve for A & B in terms of C

Computing Edge Equations

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$$\begin{bmatrix} x_0 & y_0 \\ x_1 & y_1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = -C \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{-C}{x_0 y_1 - x_1 y_0} \begin{bmatrix} y_1 - y_0 \\ x_1 - x_0 \end{bmatrix}$$

- choose $C = x_0 y_1 - x_1 y_0$ for convenience
- Then $A = y_0 - y_1$ and $B = x_0 - x_1$

Edge Equations

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- So…we can find edge equation from two verts.
- Given P_0, P_1, P_2, what are our three edges?

  How do we make sure the half-spaces defined by the edge equations all share the same sign on the interior of the triangle?
- A: Be consistent (Ex: $[P_0 P_1]$, $[P_1 P_2]$, $[P_2 P_0]$)

  How do we make sure that sign is positive?
- A: Test, and flip if needed ($A = -A$, $B = -B$, $C = -C$)

Edge Equations: Code

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Basic structure of code:

- Setup: compute edge equations, bounding box
- (Outer loop) For each scanline in bounding box...
- (Inner loop) …check each pixel on scanline, evaluating edge equations and drawing the pixel if all three are positive

Edge Equations: Code

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```
findBoundingBox(&xmin, &xmax, &ymin, &ymax);
setupEdges (&a0,&b0,&c0,&a1,&b1,&c1,&a2,&b2,&c2);

for (int y = yMin; y <= yMax; y++) {
      for (int x = xMin; x <= xMax; x++) {
            float e0 = a0*x + b0*y + c0;
            float e1 = a1*x + b1*y + c1;
            float e2 = a2*x + b2*y + c2;
            if (e0 > 0 && e1 > 0 && e2 > 0)
                  Image[x][y] = TriangleColor;
      }
}
```


Edge Equations: Code

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```
  // more efficient inner loop
for (int y = yMin; y <= yMax; y++) {
      float e0 = a0*xMin + b0*y + c0;
      float e1 = a1*xMin + b1*y + c1;
      float e2 = a2*xMin + b2*y + c2;
      for (int x = xMin; x <= xMax; x++) {
            if (e0 > 0 && e1 > 0 && e2 > 0)
                  Image[x][y] = TriangleColor;
            e0 += a0;   e1+= a1;    e2 += a2;
      }
}
```


Triangle Rasterization Issues

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Exactly which pixels should be lit?

A: Those pixels inside the triangle edges

What about pixels exactly on the edge?

- Draw them: order of triangles matters (it shouldn't)
- Don't draw them: gaps possible between triangles

We need a consistent (if arbitrary) rule

- Example: draw pixels on left or top edge, but not on right or bottom edge

Triangle Rasterization Issues

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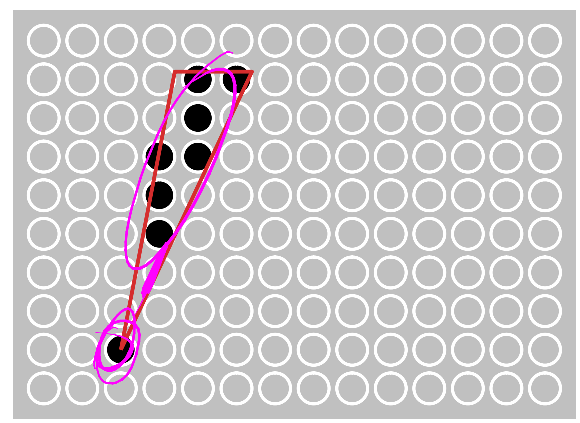
Sliver



Triangle Rasterization Issues

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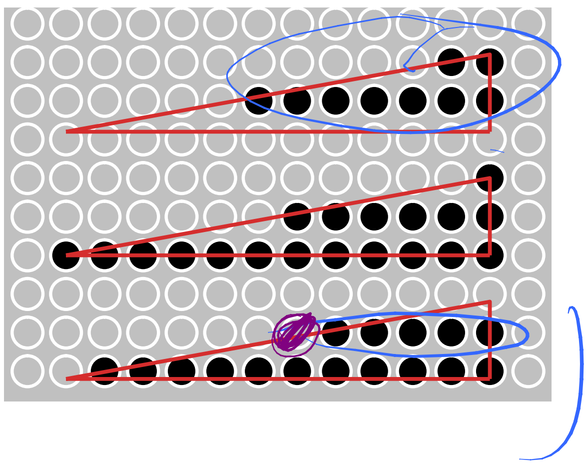
Moving Slivers



Triangle Rasterization Issues

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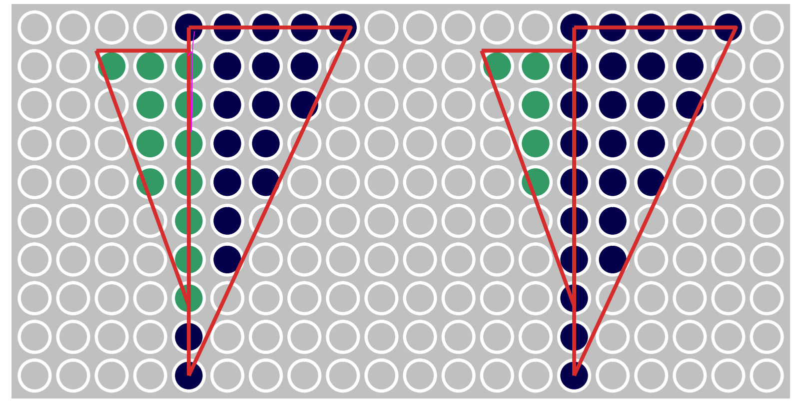
Shared Edge Ordering



Interpolation During Scan Conversion

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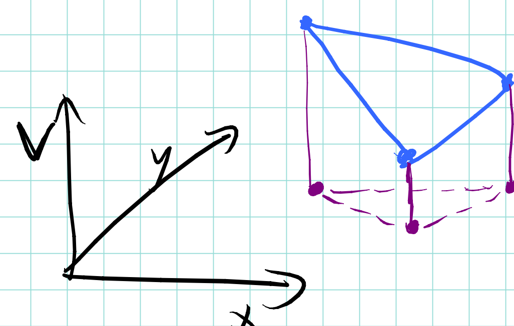
- interpolate between vertices: (demo)
  - *z*
  - *r,g,b* *colour components*
  - *u,v* *texture coordinates*
  - N_x, N_y, N_z *surface normals*
- three equivalent ways of viewing this (for triangles)
  1. *bilinear interpolation*
  2. *plane equation*
  3. *barycentric coordinates*

[remainder of lecture done on board --
get this from a friend if you could not
make this class]



1. Bilinear Interpolation

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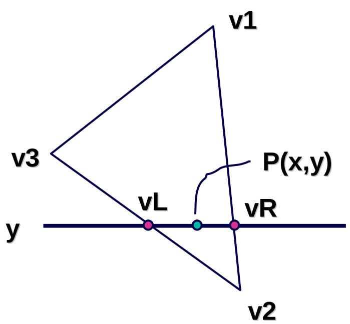
- interpolate quantity along LH and RH edges, as a function of y
  - *then interpolate quantity as a function of x*



2. Plane Equation

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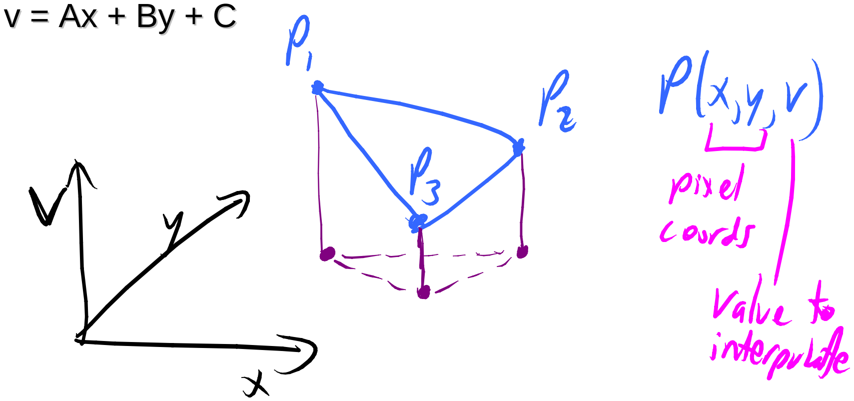
- v = Ax + By + C



$Ax + By + Cv + D = 0$ general implicit plane eqn

then solve for v:

$v = \left(\frac{-A}{C}\right)x + \left(\frac{-B}{C}\right)y + \left(\frac{-D}{C}\right)$

Reminder of how to compute A, B, C, D for a plane equation:

$Ax + By + Cz + D = 0$

$\langle A, B, C\rangle \cdot \langle x, y, z\rangle + D = 0$

$N \cdot P + D = 0$

(1) compute A, B, C by computing a normal,

e.g. $N = (P_3 - P_1) \times (P_2 - P_1)$

(2) compute $D = -N \cdot P$ for any point P on the plane

e.g. $D = -N \cdot P_1$

3. Barycentric Coordinates

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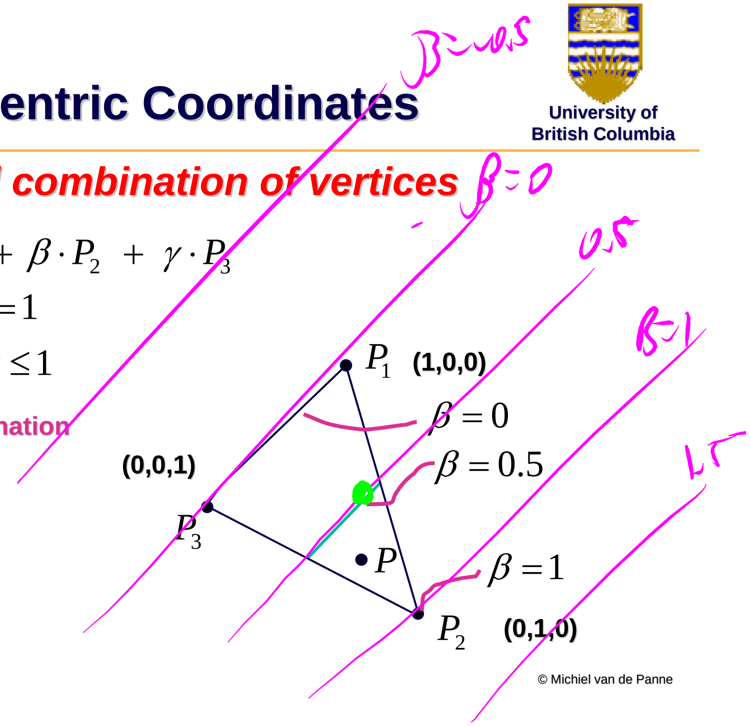
•*weighted combination of vertices*

$$\begin{cases} P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3 \\ \alpha + \beta + \gamma = 1 \\ 0 \le \alpha, \beta, \gamma \le 1 \end{cases}$$

"convex combination of points"



Barycentric Coordinates

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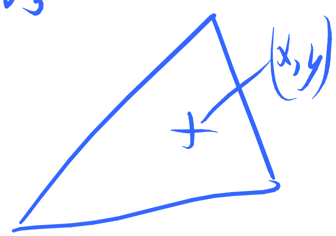
- once computed, use to interpolate any # of parameters from their vertex values

$V = \alpha V_1 + \beta V_2 + \gamma V_3$

$$z = \alpha \cdot z_1 + \beta \cdot z_2 + \gamma \cdot z_3$$
$$r = \alpha \cdot r_1 + \beta \cdot r_2 + \gamma \cdot r_3$$
$$g = \alpha \cdot g_1 + \beta \cdot g_2 + \gamma \cdot g_3$$

etc.



Computing Barycentric Coords

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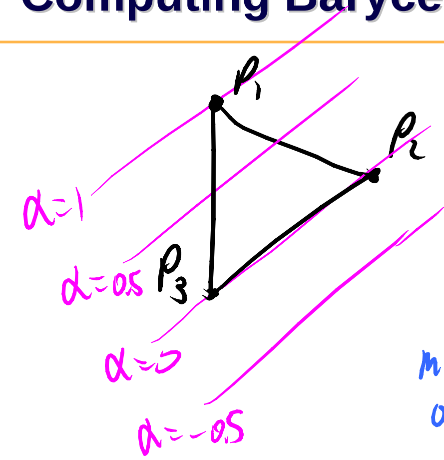

$P = \alpha P_1 + \beta P_2 + \gamma P_3$

Begin with implicit line eq'n for $\overline{P_2P_3}$:

$F(x,y) = Ax + By + C$

much like α, $F(x,y) = 0$ for points on the line. But if we get $F(x_1, y_1) = k$, we should rescale so that

$\alpha(P_1) = F'(x_1, y_1) = 1$

$$F'(x,y) = \frac{F(x,y)}{k}$$

Summary:

$$\alpha = \frac{A}{k}x + \frac{B}{k}y + \frac{C}{k}$$

where $k = Ax_1 + By_1 + C$

$A = y_2 - y_3$
$B = x_2 - x_3$

$C = x_2 y_3 - x_3 y_2$

Similar equations can be computed for β and γ.

Can also use $\gamma = 1 - \alpha - \beta$