Curves and Surfaces

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- how to model curves?
- how to model surfaces ?

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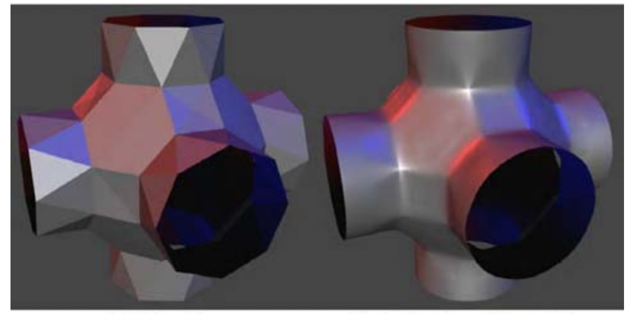
1. Subdivision curves and surfaces



Initial mesh Butterfly scheme interpolation

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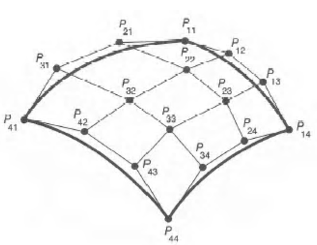
2. Parametric curves and surfaces



http://www.sjbaker.org/teapot/NewellTeaset.jpg

Parametric Curve examples

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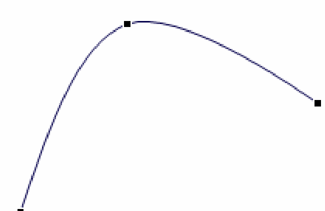
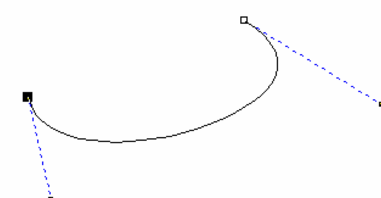
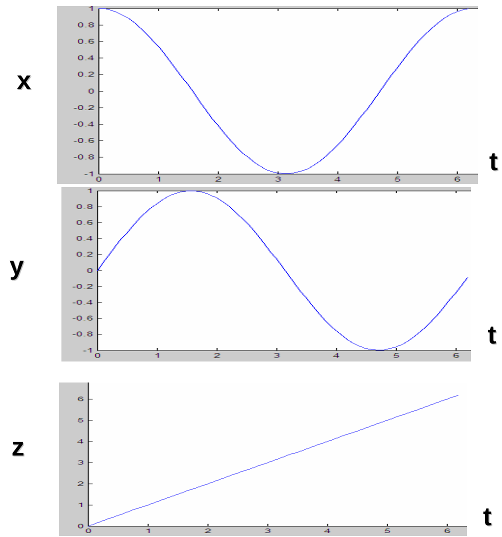


Powerpoint



CorelDraw

Test yourself...



cubic parametric curves

$$x(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

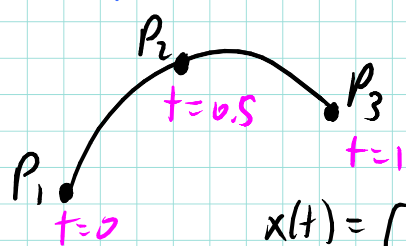
$$y(t) = b_3 t^3 + b_2 t^2 + b_1 t + b_0$$

$$z(t) = \dots$$

$$t \in [0,1]$$

$$x(t) = \underbrace{[t^3 \ \ t^2 \ \ t \ \ 1]}_{T} \underbrace{\begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}}_{A} = T \cdot A$$

Example with a Quadratic Parametric Curve



$$x(t) = [t^2 \ \ t \ \ 1] \cdot A$$

$$x_1 = [0 \ \ 0 \ \ 1] \cdot A$$

$$x_2 = [0.5^2 \ \ 0.5 \ \ 1] \cdot A$$

$$x_3 = [1 \ \ 1 \ \ 1] \cdot A$$

rewrite using a matrix

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0.5^2 & 0.5 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

$$G_x = C \cdot A$$

$$G = C \cdot A$$

coefficients of polynomial

constraint matrix

geometry vector, i.e., control points

We can now solve for A:

$$A = C^{-1} G_x$$

Now we can write out $x(t)$:

$$x(t) = T \cdot A = T C^{-1} G_x$$

$M \equiv C^{-1}$
$\equiv$ "basis matrix"

$$= T M G_x$$

$$x(t) = [t^2 \ \ t \ \ 1] \begin{bmatrix} 0 & 0 & 1 \\ 0.25 & 0.5 & 1 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

To render the curve:

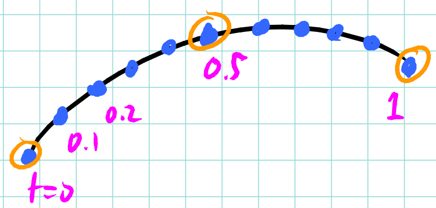
Compute a_2, a_1, a_0 : $A = M G_x$
compute b_2, b_1, b_0 : $B = M G_y$

for(t=0; t<=1; t+=Δt) {

$$x = a_2 t^2 + a_1 t + a_0$$
$$y = b_2 t^2 + b_1 t + b_0$$

draw line from previous point to (x,y)

}

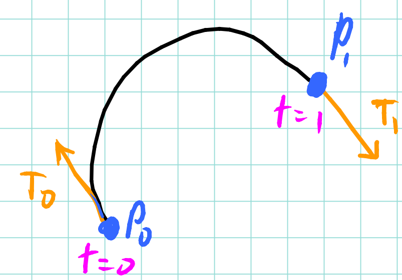


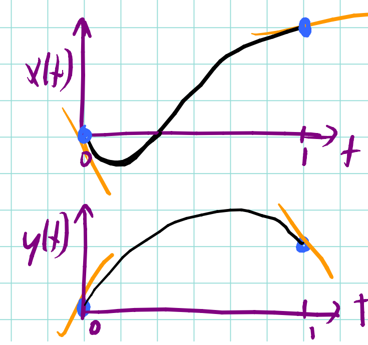
o ≡ control points
• ≡ sample points used for drawing curve

Hermite Curves

- parametric cubic curve
- defined by:
  - start, end points
  - start, end tangents

think of this as a velocity





$$x(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$
$$= \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

$$\frac{dx}{dt} = x'(t) = \begin{bmatrix} 3t^2 & 2t & 1 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

$$\left.\frac{dx}{dt}\right|_{t=0}$$

$$x(0) = x_0 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} A$$
$$x(1) = x_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} A$$
$$x'(0) = dx/dt|_{t=0} = t_{0x} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} A$$
$$x'(1) = dx/dt|_{t=1} = t_{1x} = \begin{bmatrix} 3 & 2 & 1 & 0 \end{bmatrix} A$$

$$\begin{bmatrix} x_0 \\ x_1 \\ t_{0x} \\ t_{1x} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

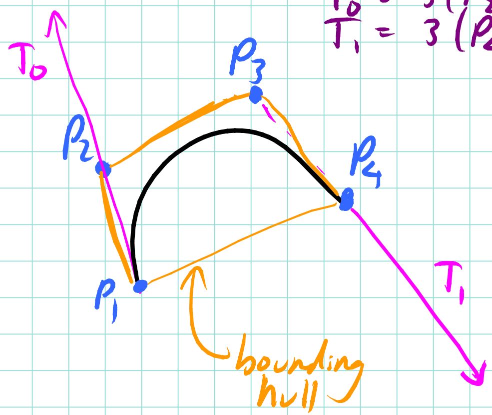
$$G_x = CA$$
$$\Rightarrow \quad A = C^{-1} G_x$$
$$A = \underbrace{\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{\text{Hermite basis matrix}} \begin{bmatrix} x_0 \\ x_1 \\ t_{x0} \\ t_{x1} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} \; M_H \; \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ t_{x0} \\ t_{x1} \end{bmatrix}$$

Bézier Curves

- variation on Hermite curves
- four points :
- Hermite equivalent:

$T_0 = 3(P_2 - P_1)$
$T_1 = 3(P_4 - P_3)$



Piecewise Hermite and Bezier Curves