Clipping (30 pts)

1. (15 pts) Clip the line segment with endpoints (-17,-3), (5,9) to the box (-1,-1), (1,-1), (1,1), (-1,1). Use the Cohen-Sutherland algorithm and show intermediate work at each step, including outcodes. Use the following clipping order: left, bottom, right, top.

A:

We use the following outcodes:

\[
\begin{pmatrix}
1001 & 0001 & 0011 \\
1000 & 0000 & 0010 \\
1100 & 0100 & 0110
\end{pmatrix}
\]

Pa = (-17,-3), Pb = (5,9); OC(Pa) = 1100, OC(Pb) = 0011

Since \(OC(Pa)\neq 0\), not trivial accept; Since \((OC(Pa)&\&OC(Pb)) = 0\), not trivial reject

check the left edge:

\[x_{intersect} = -1, m = \frac{9 + 3}{5 + 17} = \frac{6}{11}; y_{intersect} = -3 + m(-1 + 17) = \frac{63}{11} = 5.73;\]

We can discard the portion of \((Pa, P_{intersect})\) since it is out of left edge.

Now we have \((P_{intersect}, Pb)\) where \(P_{intersect} = (-1, 5.73)\), \(OC(P_{intersect}) = 0001\)

Since \((OC(P_{intersect})&\&OC(Pb)) = 0\), trivial reject

Then the whole line is rejected. DONE.

2. (15 pts) Clip the triangle with points P1 = (-.3, 1.4), P2 = (.8, -1.8), P3 = (-1.3, -.2) against the box (-1,-1), (1,-1), (1,1), (-1,1). Use the Sutherland-Hodgeman algorithm and show your intermediate work.

A:

We use the same outcodes as Q1

(a) check the left edge

input: P1, P2, P3 OC(P1)=0001, OC(P2)=0100, OC(P3)=1000

P1:

P3 outside, P1 inside

\[x_{intersect} = -1, m = \frac{1.4 + 0.2}{-0.3 + 1.3}; y_{intersect} = -0.2 + (-1 + 1.3) \times m = 2.8\]

output \(P1(1.2, 8), P1\)

P2:

P1,P2 both inside, output P2

P3:

P2 inside, P3 outside

\[x_{intersect} = -1, m = \frac{-0.2 + 1.8}{-1.3 - 0.8}; y_{intersect} = -1.8 + (-1 - 0.8) \times m = -0.43\]

output \(P1(-1, -0.43)\)

so output of left clipping : P1, P1, P2, P1

(b) check the bottom edge

input: P1(-1.2,8), P1(-0.3,1.4), P2(0.8,-1.8), P12(-1,-0.43)

OC(P1)=0000, OC(P1)=0001, OC(P2)=0100, OC(P12)=0000

P1:

P1, P1 both inside, output P1

P1:
Pl1, P1 both inside, output P1
P2:
P1 inside, P2 outside
\[ y_{intersect} = -1, m = \frac{-1.8 - 1.4}{0.8 + 0.3}, x_{intersect} = -0.3 + \frac{-1 - 1.4}{m} = 0.53 \]
output Pb1(0.53, -1)
P2:
P2 outside, P2 inside
\[ y_{intersect} = -1, m = \frac{-0.43 + 1.8}{-1 - 0.8}, x_{intersect} = 0.8 + \frac{-1 + 1.8}{m} = -0.25 \]
output Pb2(-0.25, -1), P1
so output of bottom clipping: Pl1, P1, Pb1, Pb2, P1
(c) check the right edge
input: Pl1(-1.2, 2), P1(-0.3, 1.4), Pb1(0.53, -1), Pb2(-0.25, -1), P1(-1.0.43)
OC(Pl1)= 0000, OC(P1)=0001, OC(Pb1)=0000, OC(Pb2=0000), OC(P1)=0000 all points are inside, so no clipping
(d) check the top edge
input: Pl1(-1.2, 2), P1(-0.3, 1.4), Pb1(0.53, -1), Pb2(-0.25, -1), P1(-1.0.43)
OC(Pl1)= 0000, OC(P1)=0001, OC(Pb1)=0000, OC(Pb2=0000), OC(P1)=0000
Pl1:
P2, Pl1 both inside, output Pl1
P1:
P11 inside, P1 outside
\[ y_{intersect} = 1, m = \frac{1.4 - 0.28}{-0.3 + 1}, x_{intersect} = -1 + \frac{1 - 0.28}{m} = -0.55 \]
output Pt1(-0.55, 1)
Pb1:
P1 outside, Pb1 inside
\[ y_{intersect} = 1, m = \frac{-1 - 1.4}{0.53 + 0.3}, x_{intersect} = -0.3 + \frac{-1 + 1.4}{m} = -0.16 \]
output Pt2(-0.16, 1), Pb1
Pb2:
Pb1, Pb2 both inside, output Pb2
P2:
Pb2, P2 both inside, output P2
so output of top clipping: Pl1, Pt1, Pt2, Pb1, Pb2, P1
After clipping, the accepted polygon is Pl1(-1.0.28), Pt1(-0.55, 1), Pt2(-0.16, 1), Pb1(0.53, -1), Pb2(-0.25, -1), P1(-1.0.43).
DONE.

Visibility (45 pts)

3. (15 pts) Build a BSP tree for the following scene using the polygons (shown as lines) as cutting planes. You should build your BSP tree by inserting the polygons into the BSP tree in numerical order. Represent the labelled side of each cutting plane as the right child in the tree and the unlabelled side as the left child. Show your work.

![BSP Tree Diagram]

4. (15 pts) Traverse your BSP tree to produce a painter’s algorithm ordering from the given eyepoint. Show your work.
The eye is on the unlabeled side of 0, thus first draw the labeled side sub tree, then 0, and then the other side.
Note: R(x)/L(x) is right/left sub-tree of x.
Next, recursively expand each sub tree. Note that the eye is on the labeled side of 1, and unlabeled side of 2.

\[
R(0), 0, L(0)
\]

\[
(3a), 0, L(1), 1, R(1)
\]

\[
3a, 0, 1, R(2), 2, L(2)
\]

\[
3a, 0, 1, 3b, 2, 3c
\]

5. (5 pts) How will a BSP tree deal with 3 cyclically-overlapping polygons?

One polygon is used as the root of the tree. The polygon plane is then used to partition the 3D space. Each of the other two polygons are split if they intersect the first polygon’s plane. Next each of these polygons (more than two, since some were split) is inserted into the tree, again using the planes to partition the sub-spaces (causing additional splits as needed). At the end, no cyclically overlapping polygons remain.

6. (10 pts) For the following scene, the polygons forming a closed solid are represented by edges. Which faces would be removed by backface culling for the given eyepoint?

Faces where the eye is on the back side or below the plane are culled.

BC, CD, EF, FG, HI

**Textures (20 pts)**

7. (10 pts) Given the triangle \( T = (P1, P2, P3) \) with \( P1 = (0,0,0,5), P2 = (2,2,-10,5), \) and \( P3 = (4,0,0,5) \) and with texture \((s, t)\) coordinates at the vertices defined as \((0,0), (5,1), \) and \((1,0)\) respectively, compute the texture coordinates at the midpoint \( P \) of the triangle. Use the standard barycentric coordinate formula.

\[
\alpha = \beta = \gamma = 1/3
\]

\[
S_p = \alpha * 0 + \beta * 0.5 + \gamma * 1 = 0.50;
\]

\[
T_p = \alpha * 0 + \beta * 1 + \gamma * 0 = 0.33;
\]

8. (10 pts) Find the texture coordinates at triangle midpoint \( P \) as above, but this time using perspective-correct barycentric interpolation.

\[
\alpha = \beta = \gamma = 1/3
\]

\[
S_p = \frac{\alpha s_1/w_1 + \beta s_2/w_2 + \gamma s_3/w_3}{\alpha/w_1 + \beta/w_2 + \gamma/w_3}
\]

\[
= \frac{(1/3)0/5 + (1/3)0.5/5 + (1/3)1/5}{(1/3)/5 + (1/3)/5 + (1/3)/5}
\]

\[
= 0.50
\]

\[
T_p = \frac{\alpha t_1/w_1 + \beta t_2/w_2 + \gamma t_3/w_3}{\alpha/w_1 + \beta/w_2 + \gamma/w_3}
\]

\[
= \frac{(1/3)0/5 + (1/3)1/5 + (1/3)0/5}{(1/3)/5 + (1/3)/5 + (1/3)/5}
\]

\[
\approx 0.33
\]
Pipeline (5 pts)

9. (5 pts) Sketch the rendering pipeline and indicate where the following are performed: a) clipping b) Gouraud shading.

   (a) Geometry Database
   (b) ModelView Transformation
   (c) Lighting (Gouraud shading lighting calculation per vertex)
   (d) Perspective Transformation
   (e) Clipping (clipping done here)
   (f) Scan Conversion (Gouraud shading interpolation per pixel)
   (g) Texturing
   (h) Depth Test
   (i) Blending
   (j) Frame Buffer