CPSC 314, Written Homework 2

Out: Fri Feb 25
Due: Fri Mar 4, 4pm
Value: 5% of final grade
Total Points: 100

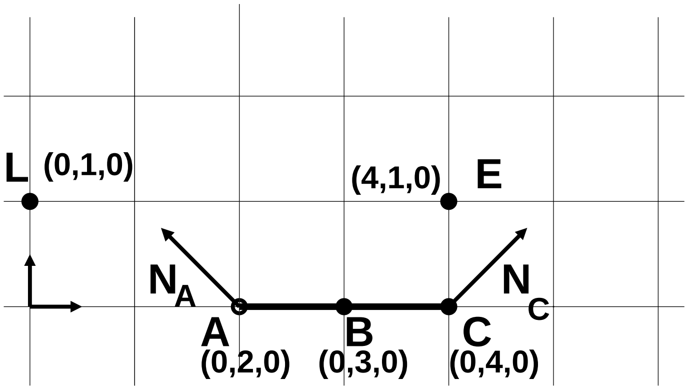
Lighting (36 pts)

1. (12 pts) Give the ambient, diffuse, specular, and combined total illumination at each of points A, B, and C under the flat shading model (assume the point used for the flat shading calculation is A). Show your work. In all cases use the Phong illumination model given by

$$I = I_a k_a + k_d I_L (N \cdot L) + k_s I_L (R \cdot V)^n$$

with parameters

$$I_a = (.3, .3, .3), I_L = (1.0, 1.0, 1.0), k_a = (.2, .2, .2), k_d = (.5, .5, .9), k_s = (.9, .9, .1), n = 10.$$



Answer:

Under the flat shading model, points A, B, C will have the same color (the color specified at point 'A'. as assumed by the question).

For point A:

$$ambient_A = I_a k_a = \begin{bmatrix} 0.3 \\ 0.3 \\ 0.3 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 0.06 \\ 0.06 \\ 0.06 \end{bmatrix}$$

Notice the component-wise multiplication

$$diffuse_A = I_L k_d (\hat{N}_A \cdot \hat{L}_A)$$

$$I_L k_d = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ 0.9 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.9 \end{bmatrix}$$

$$\vec{N}_A = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \hat{N}_A = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

Where $\hat{N}_A$ is the normalized version of $\vec{N}_A$:

$$\vec{L}_A = L - A = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \hat{L}_A = \begin{bmatrix} \frac{-2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ 0 \end{bmatrix}$$

Where $\hat{L}_A$ is the normalized version of $\vec{L}_A$:

$$\hat{N}_A \cdot \hat{L}_A = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{-2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ 0 \end{bmatrix} = \frac{-1}{\sqrt{2}} * \frac{-2}{\sqrt{5}} + \frac{1}{\sqrt{2}} * \frac{1}{\sqrt{5}} + 0 * 0 = \frac{3}{\sqrt{10}}$$

$$diffuse_A = I_L k_d (\hat{N}_A \cdot \hat{L}_A) = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.9 \end{bmatrix} \frac{3}{\sqrt{10}} = \begin{bmatrix} 0.474 \\ 0.474 \\ 0.854 \end{bmatrix}$$

$$specular_A = I_L k_s (\hat{R}_A \cdot \hat{V}_A)^n$$

$$I_L k_s = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0.9 \\ 0.9 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.9 \\ 0.1 \end{bmatrix}$$

$$\hat{R}_A = 2 * (\hat{N}_A \cdot \hat{L}_A) * \hat{N}_A - \hat{L}_A = 2 * \frac{3}{\sqrt{10}} * \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{-2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \\ 0 \end{bmatrix}$$

$$\vec{V}_A = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \hat{V}_A = \begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ 0 \end{bmatrix}$$

$$(\hat{R}_A \cdot \hat{V}_A)^n = \left(\begin{bmatrix} \frac{-1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ 0 \end{bmatrix} \right)^{10} = (\frac{-2}{5} + \frac{2}{5} + 0)^{10} = 0$$

$$specular_A = I_L k_s (\hat{R}_A \cdot \hat{V}_A)^n = \begin{bmatrix} 0.9 \\ 0.9 \\ 0.1 \end{bmatrix} * 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Sum the ambient, diffuse, and specular components to get the combined total illumination:

$$total_A = ambient_A + diffuse_A + specular_A = \begin{bmatrix} 0.06 \\ 0.06 \\ 0.06 \end{bmatrix} + \begin{bmatrix} 0.474 \\ 0.474 \\ 0.854 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.534 \\ 0.534 \\ 0.914 \end{bmatrix}$$

2. (12 pts) Same as above, for the Gouraud shading model.

   Answer:

   For the Gouraud shading model, we calculate the illumination intensity at each vertex (in this case A and C) and interpolate for inbetween points (in this case B).

   Illumination intensity at point $A(I_A) = \begin{bmatrix} 0.534 \\ 0.534 \\ 0.914 \end{bmatrix}$ (From question 1).

Illumination intensity at point $C(I_C) = I_a k_a + k_d I_L(\hat{N}_C \cdot \hat{L}_C) + k_s I_L(\hat{R}_C \cdot \hat{V}_C)^n$

$$\vec{N}_C = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \hat{N}_C = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$\vec{L}_C = L - C = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix}, \hat{L}_C = \begin{bmatrix} \frac{-4}{\sqrt{17}} \\ \frac{1}{\sqrt{17}} \\ 0 \end{bmatrix}$$

$$\hat{N}_C \cdot \hat{L}_C = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{-4}{\sqrt{17}} \\ \frac{1}{\sqrt{17}} \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} * \frac{-4}{\sqrt{17}} + \frac{1}{\sqrt{2}} * \frac{1}{\sqrt{17}} + 0 * 0 = -\frac{3}{\sqrt{34}} = 0(clamped)$$

$$\hat{R}_C = 2 * (\hat{N}_C \cdot \hat{L}_C) * \hat{N}_C - \hat{L}_C = 2 * 0 * \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{-4}{\sqrt{17}} \\ \frac{1}{\sqrt{17}} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{4}{\sqrt{17}} \\ -\frac{1}{\sqrt{17}} \\ 0 \end{bmatrix}$$

$$\vec{V}_C = \hat{V}_C = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$(\hat{R}_C \cdot \hat{V}_C)^n = \left(\begin{bmatrix} \frac{4}{\sqrt{17}} \\ -\frac{1}{\sqrt{17}} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right)^{10} = (\frac{-1}{17})^{10} \approx 0$$

We've already calculated $I_a k_a$, $I_L k_d$, and $I_L k_s$ in question 1.

$$\begin{aligned} I_C &= I_a k_a + k_d I_L(\hat{N}_C \cdot \hat{L}_C) + k_s I_L(\hat{R}_C \cdot \hat{V}_C)^n \\ &= \begin{bmatrix} 0.06 \\ 0.06 \\ 0.06 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \\ 0.9 \end{bmatrix} * 0 + \begin{bmatrix} 0.9 \\ 0.9 \\ 0.1 \end{bmatrix} * 0 = \begin{bmatrix} 0.06 \\ 0.06 \\ 0.06 \end{bmatrix} \end{aligned}$$

At point B, $I_B = \frac{I_A + I_C}{2} = \frac{1}{2} \begin{bmatrix} 0.594 \\ 0.594 \\ 0.974 \end{bmatrix} = \begin{bmatrix} 0.297 \\ 0.297 \\ 0.487 \end{bmatrix}$ since B is half way between A and C.

3. (12 pts) Same as above, for the Phong shading model.

   Answer:

   For the Phong shading model, rather than interpolating the colors at each point, we interpolate the normal instead. Using the interpolated normal, we compute the illumination intensity at that point.

   Illumination intensity at point $A(I_A) = \begin{bmatrix} 0.534 \\ 0.534 \\ 0.914 \end{bmatrix}$ (From question 1).

   Illumination intensity at point $C(I_C) = \begin{bmatrix} 0.06 \\ 0.06 \\ 0.06 \end{bmatrix}$ (From question 2).

The surface normal at point $B = \vec{N}_B = \hat{N}_B = \frac{\vec{N}_A+\vec{N}_C}{2} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\vec{L}_B = L - B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \hat{L}_B = \begin{bmatrix} \frac{-3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \\ 0 \end{bmatrix}$$

$$\hat{N}_B \cdot \hat{L}_B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{-3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \\ 0 \end{bmatrix} = 0 + 1 * \frac{1}{\sqrt{10}} + 0 = \frac{1}{\sqrt{10}}$$

$$\hat{R}_B = 2 * (\hat{N}_B \cdot \hat{L}_B) * \hat{N}_B - \hat{L}_B = 2 * \frac{1}{\sqrt{10}} * \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{-3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \\ 0 \end{bmatrix}$$

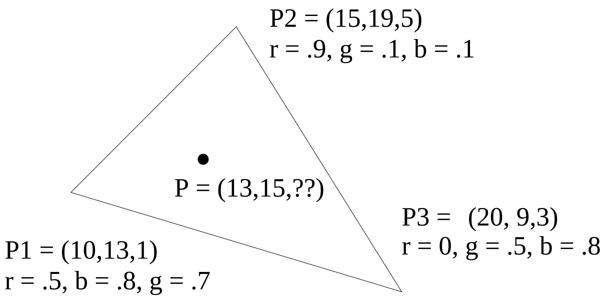
$$\vec{V}_B = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \hat{V}_B = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$(\hat{R}_B \cdot \hat{V}_B)^n = \left(\begin{bmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \right)^{10} = (\frac{4}{\sqrt{20}})^{10} = 0.328$$

Again, we've already calculated $I_a k_a$, $I_L k_d$, and $I_L k_s$ in question 1.

$$I_B = I_a k_a + k_d I_L(\hat{N}_B \cdot \hat{L}_B) + k_s I_L(\hat{R}_B \cdot \hat{V}_B)^n$$
$$= \begin{bmatrix} 0.06 \\ 0.06 \\ 0.06 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \\ 0.9 \end{bmatrix} * \frac{1}{\sqrt{10}} + \begin{bmatrix} 0.9 \\ 0.9 \\ 0.1 \end{bmatrix} * 0.328 = \begin{bmatrix} 0.06 \\ 0.06 \\ 0.06 \end{bmatrix} + \begin{bmatrix} 0.158 \\ 0.158 \\ 0.285 \end{bmatrix} + \begin{bmatrix} 0.295 \\ 0.295 \\ 0.0328 \end{bmatrix} = \begin{bmatrix} 0.513 \\ 0.513 \\ 0.378 \end{bmatrix}$$

Interpolation (30 pts)



4. (10 pts) Interpolate the value of z and the red color component r for the point P by writing a plane equation in x, y, and z, and solving this for the given point P. Show your work.

   Answer:

   Interpolate z:

   Plane Equation:

$$Ax + By + Cz + D = 0 \Longrightarrow [A, B, C]^T \cdot \vec{P} + D = 0,$$

where $\vec{N} = [A, B, C]^T$ is the vector normal to the plane, and $\vec{P} = (x, y, z)$ is a point on the plane.

First, compute the normal to the plane using the cross product:

$$\begin{aligned} \vec{N} &= (P_3 - P_1) \times (P_2 - P_1) \\ &= \begin{bmatrix} 10 \\ -4 \\ 2 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} N_x &= (-4) * 4 - 6 * 2 = -28 \\ N_y &= 5 * 2 - 10 * 4 = -30 \\ N_z &= 10 * 6 - 5 * (-4) = 80 \end{aligned}$$

Compute D by substituting any point on the plane. Using P_1:

$$\begin{aligned} D = -\vec{N} \cdot \vec{P}, &= - \begin{bmatrix} -28 \\ -30 \\ 80 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 13 \\ 1 \end{bmatrix} \\ &= 28 * 10 + 40 * 13 - 80 * 1 = 280 + 390 - 80 = 590 \end{aligned}$$

Thus, The plane equation is:

$$\begin{bmatrix} -28 \\ -30 \\ 80 \end{bmatrix} \cdot \vec{P} + 590 = 0,$$

for any point $\vec{P}$ on the plane.

Now we can plug in the known x, y values at point P, for which we would like to calculate z:

$$\begin{bmatrix} -28 \\ -30 \\ 80 \end{bmatrix} \cdot \begin{bmatrix} 13 \\ 15 \\ z \end{bmatrix} + 590 = 0,$$

Isolate variable z:

$$z = \frac{28 * 13 + 30 * 15 - 590}{80} = \frac{224}{80} = 2.8$$

Interpolate r:

Plane Equation:

$$Ax + By + Cz + D = 0 \Longrightarrow [A, B, C]^T \cdot \vec{P} + D = 0,$$

where $\vec{N} = [A, B, C]^T$ is the vector normal to the plane, and $\vec{P} = (x, y, r)$ is a point on the plane. Notice that we now construct a plane in the (x, y, r) coordinate system, instead of (x, y, z) system.

First, compute the normal to the plane using the cross product:

$$\begin{aligned} \vec{N} &= (P_3 - P_1) \times (P_2 - P_1) \\ &= \begin{bmatrix} 10 \\ -4 \\ -0.5 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 0.4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} N_x &= (-4) * 0.4 - 6 * (-0.5) = 1.4 \\ N_y &= 5 * (-0.5) - 10 * 0.4 = -6.5 \\ N_z &= 10 * 6 - 5 * (-4) = 80 \end{aligned}$$

Compute D by substituting any point on the plane. Using $P_1 = (10, 13, 0.5)$:

$$D = -\vec{N} \cdot \vec{P}, = -\begin{bmatrix} 1.4 \\ -6.5 \\ 80 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 13 \\ 0.5 \end{bmatrix}$$
$$= -1.4 * 10 + 6.5 * 13 - 80 * 0.5 = -14 + 84.5 - 40 = 30.5$$

Thus, The plane equation is:

$$\begin{bmatrix} 1.4 \\ -6.5 \\ 80 \end{bmatrix} \cdot \vec{P} + 30.5 = 0,$$

for any point $\vec{P}$ on the plane.

Now we can plug in the known x, y values at point P, for which we would like to calculate z:

$$\begin{bmatrix} 1.4 \\ -6.5 \\ 80 \end{bmatrix} \cdot \begin{bmatrix} 13 \\ 15 \\ r \end{bmatrix} + 30.5 = 0,$$

Isolate variable r:

$$r = \frac{(-1.4) * 13 + 6.5 * 15 - 30.5}{80} = \frac{76.2}{80} = 0.61$$

5. (5 pts) Interpolate the value of z and r for the point P above using bilinear interpolation. Show your work.

   Answer:

$$\begin{aligned}
&L = P_1 + t_1(P_2 - P_1) \\
&\begin{bmatrix} x_L \\ 15 \end{bmatrix} = \begin{bmatrix} 10 \\ 13 \end{bmatrix} + t_1(\begin{bmatrix} 15 \\ 19 \end{bmatrix} - \begin{bmatrix} 10 \\ 13 \end{bmatrix}) \\
&x_L = 10 + 5t_1 \\
&15 = 13 + 6t_1 \Longrightarrow t_1 = \frac{1}{3} \Longrightarrow x_L = \frac{35}{3}
\end{aligned} \tag{1}$$

$$\begin{aligned}
&R = P_3 + t_2(P_2 - P_3) \\
&\begin{bmatrix} x_R \\ 15 \end{bmatrix} = \begin{bmatrix} 20 \\ 9 \end{bmatrix} + t_2(\begin{bmatrix} 15 \\ 19 \end{bmatrix} - \begin{bmatrix} 20 \\ 9 \end{bmatrix}) \\
&x_R = 20 - 5t_2 \\
&15 = 9 + 10t_2 \Longrightarrow t_2 = \frac{3}{5} \Longrightarrow x_R = 17
\end{aligned} \tag{2}$$

$$\begin{aligned}
&P = L + t_3(R - L) \\
&\begin{bmatrix} 13 \\ 15 \end{bmatrix} = \begin{bmatrix} \frac{35}{3} \\ 15 \end{bmatrix} + t_3(\begin{bmatrix} 17 \\ 15 \end{bmatrix} - \begin{bmatrix} \frac{35}{3} \\ 15 \end{bmatrix}) \\
&13 = \frac{35}{3} + T_3(17 - \frac{35}{3}) \\
&t_3 = \frac{1}{4}
\end{aligned}$$

$$\begin{aligned}
z_L &= z_1 + t_1(z_2 - z_1) = 1 + \frac{1}{3}(5-1) = \frac{7}{3} \\
z_R &= z_3 + t_2(z_2 - z_3) = 3 + \frac{3}{5}(5-3) = \frac{21}{5} \\
z_P &= z_L + t_3(z_R - z_L) = \frac{7}{3} + \frac{1}{4}(\frac{21}{5} - \frac{7}{3}) = \frac{168}{60} = 2.8
\end{aligned} \tag{3}$$

$$\begin{aligned}
r_L &= r_1 + t_1(r_2 - r_1) = 0.5 + \frac{1}{3}(0.9 - 0.5) = \frac{19}{30} = 0.633 \\
r_R &= r_3 + t_2(r_2 - r_3) = 0 + \frac{3}{5}(0.9 - 0) = \frac{27}{50} = 0.54 \\
r_P &= r_L + t_3(r_R - r_L) = 0.633 + \frac{1}{4}(0.54 - 0.633) = 0.61
\end{aligned} \tag{4}$$

6. (15 pts) Determine the barycentric coordinates α, β, and γ of point P above. Interpolate z and r using these coordinates. Show your work.

   Answer:

   t_1, t_2, t_3 taken from question 5.

$$\begin{aligned}
L &= P_1 + t_1(P_2 - P_1) = (1 - t_1)P_1 + t_1 P_2 \\
R &= P_3 + t_2(P_2 - P_3) = (1 - t_2)P_3 + t_2 P_2
\end{aligned}$$

$$\begin{aligned}
P &= L + t_3(R - L) \\
&= (1 - t_3)L + t_3 R \\
&= (1 - t_3)[(1 - t_1)P_1 + t_1 P_2] + t_3[(1 - t_2)P_3 + t_2 P_2] \\
&= (1 - t_3)(1 - t_1)P_1 + (1 - t_3)t_1 P_2 + t_3[(1 - t_2)P_3 + t_3 t_2 P_2 \\
&= (1 - t_3)(1 - t_1)P_1 + ((1 - t_3)t_1 + t_3 t_2)P_2 + t_3[(1 - t_2)P_3 \\
&= (1 - \frac{1}{4})(1 - \frac{1}{3})P_1 + ((1 - \frac{1}{4})\frac{1}{3} + \frac{1}{4}\frac{3}{5})P_2 + \frac{1}{4}[(1 - \frac{3}{5})P_3 \\
&= \frac{1}{2}P_1 + \frac{2}{5}P_2 + \frac{1}{10}P_3
\end{aligned}$$

   Therefor,

$$\alpha = \frac{1}{2}, \beta = \frac{2}{5}, \gamma = \frac{1}{10}$$

$$z = \alpha z_1 + \beta z_2 + \gamma z_3 = 0.5 * 1 + 0.4 * 5 + 0.1 * 3 = 2.8$$

$$r = \alpha r_1 + \beta r_2 + \gamma r_3 = 0.5 * 0.5 + 0.4 * 0.9 + 0.1 * 0 = 0.61$$

   Alternative method for computing barycentric coordinates:

   To compute α:

   Setup an implicit equation for the line $\overline{P_2 P_3}$ such that F(P) = 0 for any point on the line $\overline{P_2 P_3}$.

   General form of a line with endpoints P_0 and P_1:

$$F(x, y) = Ax + By + C = 0,$$

where

$$\begin{aligned} A &= y_1 - y_0 \\ B &= x_0 - x_1 \\ C &= -x_0 y_1 + x_1 y_0 \end{aligned}$$

For $\overline{P_2 P_3}$: Let $P_2 \longrightarrow P_0$ and $P_3 \longrightarrow P_1$

$$\begin{aligned} A &= y_3 - y_2 = -10 \\ B &= x_2 - x_3 = -5 \\ C &= -x_2 y_3 + x_3 y_2 = 245 \end{aligned}$$

Then scale $F(P)$ such that $F'(P) = kF(P)$ and $F'(P_1) = 1$, where $P_1 = (10, 13, 1)$

$$F'(P_1) = 1 = k * (Ax_1 + By_1 + C)$$

We can isolate k:

$$k = \frac{1}{Ax_1 + By_1 + C)} = \frac{1}{(-10)(10) + (-5)(13) + 245} = \frac{1}{80}$$

$$\alpha = F'(P) = \frac{1}{80}(-10x - 5y + 245)$$

Notice how α is constrained to be between 0 and 1, where it is 1 at P_1 and is 0 along the edge furthest away from P_1, that is - along $\overline{P_2 P_3}$.

Similarly for β

$$F(x, y) = Ax + By + C = 0,$$

$$\begin{aligned} A &= y_1 - y_3 = 4 \\ B &= x_3 - x_1 = 10 \\ C &= -x_3 y_1 + x_1 y_3 = -170 \end{aligned}$$

Then scale $F(P)$ such that $F'(P) = kF(P)$ and $F'(P_2) = 1$

$$F'(P_2) = 1 = k * (Ax_1 + By_1 + C)$$

We can isolate k:

$$k = \frac{1}{Ax_2 + By_2 + C)} = \frac{1}{(4)(15) + (10)(19) - 170} = \frac{1}{80}$$

$$\beta = F'(P) = \frac{1}{80}(4x + 10y - 170)$$

Again, notice that β is constrained to be between 0 and 1.

Since by definition all barycentric coordinates sum to 1:

$$\gamma = 1 - \alpha - \beta$$

For Point $P = (13, 15, ??)$

$$\alpha = F'(P) = \frac{1}{80}(-10x - 5y + 245) = \frac{1}{80}(-10 * 13 - 5 * 15 + 245) = 0.5$$

$$\beta = F'(P) = \frac{1}{80}(4x + 10y - 10) = \frac{1}{80}(4 * 13 + 10 * 15 - 170) = 0.4$$

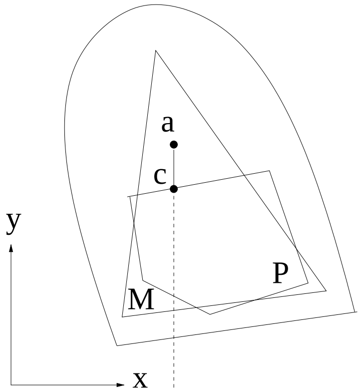
$$\gamma = 1 - \alpha - \beta = 1 - 0.5 - 0.4 = 0.1$$

Color (12 pts)

7. (4 pts) If a light with RGB color triplet (1,.5,0) shines on a surface with diffuse color (0,1,1), what is the resulting color triplet?

   Using componentwise multiplication (1*0,.5*1,0*1)=(0,.5,0)

8. (8 pts) Point a, shown in the 2D CIE chromaticity diagram, lies within the monitor gamut M but not the printer gamut P. One way to do gamut mapping is shown, using the intersection between the y coordinate of the point and the boundary of the gamut. Sketch a better approach, and concisely explain why it is better.

   The chromaticity diagram is of hue and saturation (not intensity). Note that the suggested mapping does not define a mapping for all screen colors (bottom corners). Also, the above mapping changes the saturation of points without adjusting the hue, thus greens will become bluer (or redder depending on the hue).

   Different possibilities are to intersect M with a line from a to the center of M, find the closest point in M or scale P to fit in M. All these solve the first problem and give a better solution for the second.

Scan Conversion (22 pts)

9. (6 pts) Briefly describe how to use parity when scan converting a general polygon.

   Scanning the bbox of each polygon horizontally from left to right. Assume first point is outside of polygon. Use an edge counter initialized to zero (assume staring outside of polygon). Each time the scan line crosses an edge the counter is increased. Pixels are drawn only when the counter is odd, i.e. the pixels are inside the polygon.

   Note: A consistent rule for drawing edges is needed, otherwise the results depend on the order in which polygons are drawn.

10. (16 pts) Give an algorithm for scan-converting a line with the Bresenham approach that works in the third octant (lines with slope between infinity and negative 1), rather than the first octant as described in class (lines with slope between 0 and 1).

```
function Bresenham3 { x0 y0 x1 y1 } {
  dx = x1 - x0;
  dy = y1 - y0;
  x = x0;
  e = 0;

  for (y=y0; y<=y1; ++y) {
    drawPixel(x,y);
    if (2(e+dx)>-dy) {
      e += dx;
    } else {
      e += dx + dy;
      --x;
    }
  }
}
```