CS414 - Solutions for Assignment 2

1(a)

$$Ax + By + Cz + D = N \cdot P + D = 0$$

Where
N = <A, B, C> is normal to the plane, and
P = <x, y, z> is a point on the plane.

Compute N:

$$N = (P_2 - P_3) \times (P_1 - P_3) = \begin{bmatrix} 6 \\ -3 \\ -2 \end{bmatrix} \times \begin{bmatrix} 5 \\ 2 \\ -8 \end{bmatrix}$$

$$\begin{matrix} i & j & k \\ 6 & -3 & -2 \\ 5 & 2 & -8 \end{matrix}$$

$$i = (-3)\cdot(-8) - (-2)\cdot 2 = 24 + 4 = 28$$
$$j = -6\cdot(-8) + (-2)\cdot 5 = 48 - 10 = 38$$
$$k = 6\cdot 2 - (-3)\cdot 5 = 12 + 15 = 27$$

$$N = \begin{bmatrix} 28 \\ 38 \\ 27 \end{bmatrix}$$

Plug in any of the points on the plane to calculate the value of D.

$$N \cdot P_1 + D = 0 \Rightarrow D = -N \cdot P_1 = -\begin{bmatrix} 28 \\ 38 \\ 27 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 6 \\ 2 \end{bmatrix} =$$

$$-(196 + 228 + 54) = -478$$

Therefore, the plane equation is:

$$\boxed{28x + 38y + 27z - 478 = 0}$$

1(b)

x = 7, y = 3, z = **?**

$$28x + 38y + 27z - 478 = 0$$
$$\therefore$$

$$z = \frac{478 - 28x - 38y}{27} = \frac{478 - 28\cdot 7 - 38\cdot 3}{27} = 6.22$$

1(c)

computing α:
implicit line equation $\overline{P_2 P_3}$:

$$A = y_3 - y_2 = 3$$
$$B = x_2 - x_3 = 6$$
$$C = -x_2 \cdot y_3 + x_3 \cdot y_2 = -32 + 2 = -30$$
$$k = \frac{1}{Ax_1 + By_1 + C} = \frac{1}{3\cdot 7 + 6\cdot 6 - 30} = \frac{1}{27}$$
$$\alpha = F(P)' = \frac{1}{27}\cdot(Ax + By + C) =$$
$$\frac{1}{27}\cdot(3\cdot 7 + 6\cdot 3 - 30) = \frac{1}{3}$$

computing β:
implicit line equation $\overline{P_3 P_1}$:

$$A = y_1 - y_3 = 2$$
$$B = x_3 - x_1 = -5$$
$$C = -x_3 \cdot y_1 + x_1 \cdot y_3 = -12 + 28 = 16$$
$$k = \frac{1}{Ax_2 + By_2 + C} = \frac{1}{2\cdot 8 - 5\cdot 1 + 16} = \frac{1}{27}$$
$$\beta = \frac{1}{27}\cdot(Ax + By + C) =$$
$$\frac{1}{27}\cdot(2\cdot 7 - 5\cdot 3 + 16) = \frac{5}{9}$$
$$\gamma = 1 - \beta - \alpha = \frac{1}{9}$$

computing z, r,g, b using the barycentric coordinates: $z = \alpha z_1 + \beta z_2 + \gamma z_3 = \frac{1 \cdot 2}{3} + \frac{5 \cdot 8}{9} + \frac{1 \cdot 10}{9} = 6.22$ $r = \alpha r_1 + \beta r_2 + \gamma r_3 = \frac{0.3}{3} + \frac{5 \cdot 0.2}{9} + \frac{1}{9} = 0.322$	$g = \alpha g_1 + \beta g_2 + \gamma g_3 = 0 + \frac{5 \cdot 0.7}{9} + \frac{0.81}{9} = 0.478$ $b = \alpha b_1 + \beta b_2 + \gamma b_3 = 0 + \frac{5 \cdot 0.1}{9} + \frac{0.21}{9} = 0.078$

1(d)

To Show: $|F'(P)| = d$, where d is the minimal distance of point P from the plane.

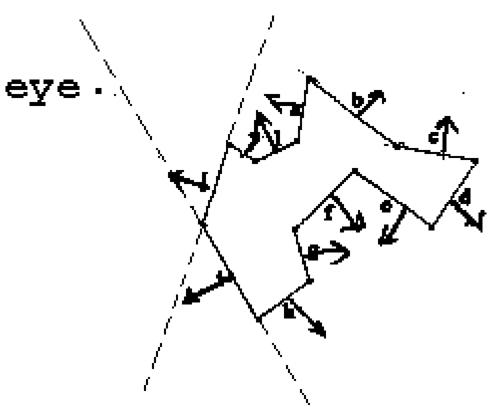
Let $P' = P + d \cdot N'$, where P is a point on the plane.

$$|F'(P')| = |F'(P + dN')| = |A'(x + dA') + B'(y + dB') + C'(z + dC') + D'| =$$

$$|A'x + dA'A' + B'y + dB'B' + C'z + dC'C' + D'| = |A'x + B'y + C'z + D' + dA'A' + dB'B' + dC'C'| =$$

$$|F'(P) + dN' \cdot N'| = |0 + d \cdot 1| = d$$

2

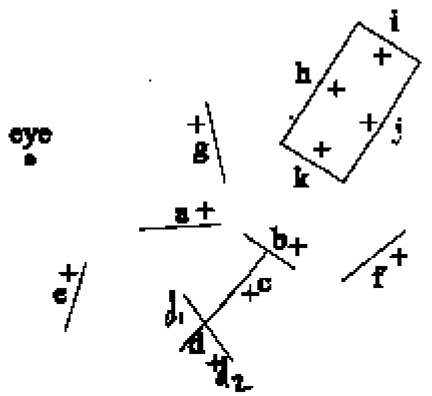


The following faces would be removed:

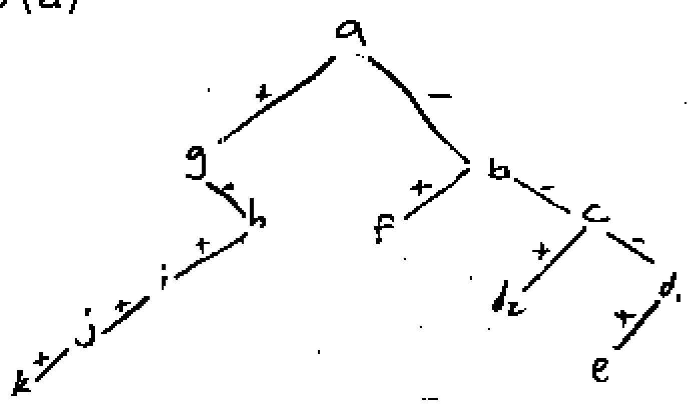
b, d, f, g, h, k

Face **c** would be culled if one made the assumption that the eye must not be directly on the surface plane.

The point is to give each face a normal in some consistent manner. For example, all normals facing the outside of the object. For each face, check if the eye point is on the wrong side of the face's plane. This can be done by checking if $N \cdot P_{eye} < 0$. Draw a line through the face. If the eye is on one side of the line, and the normal is on the other – the face is culled.



3 (a)



3(b)

(a-) a (a+)
(b+) b (b-) a (g-) g (g+)
f b (c+) c (c-) a (h+) h (h-) g
f b (d2-) d2 (d2+) c (d1-) d1 (d1+) a (i-) i (i+) h g
f b d2 c d1 e a i (j-) j (j+) h g
f b d2 c d1 e a i j k h g

4(a)

near plane: $F_n(P) = -z - near = -z - 1 = 0$

far plane: $F_f(P) = z + far = z + 10 = 0$

left plane: $F_l(P) = x + \frac{left \cdot z}{near} = x - \frac{z}{2} = 0$

right plane: $F_r(P) = -x - \frac{right \cdot z}{near} = -x - \frac{z}{2} = 0$

top plane: $F_t(P) = -y - \frac{top \cdot z}{near} = -y - \frac{z}{2} = 0$

bottom plane: $F_t(P) = y + \frac{bottom \cdot z}{near} = y - \frac{z}{2} = 0$

4(b)

near plane:

$F_n(P_1) = -(-4) - 1 = 3 > 0$ (inside volume)

$F_n(P_2) = -(-3) - 1 = 2 > 0$ (inside)

$F_n(P_3) = -(-2) - 1 = 1 > 0$ (inside)

far plane:

$F_f(P_1) = -4 + 10 = 6 > 0$ (inside)

$F_f(P_2) = -3 + 10 = 7 > 0$ (inside)

$F_f(P_3) = -2 + 10 = 8 > 0$ (inside)

left plane:

$F_l(P_1) = 0 - \frac{-4}{2} = 2 > 0$ (inside)

$F_l(P_2) = 1 - \frac{-3}{2} = \frac{5}{2} > 0$ (inside)

$F_l(P_3) = 2 - \frac{-2}{2} = 3 > 0$ (inside)

right plane:

$F_r(P_1) = -0 - \frac{-4}{2} = 2 > 0$ (inside)

$F_r(P_2) = -1 - \frac{-3}{2} = \frac{1}{2} > 0$ (inside)

$F_r(P_1) = -2 - \frac{-2}{2} = -1 < 0$ **(OUT!)**

For line $\overline{P_1P_3}$:

Parametric form of plane:

$$F_r(P_1) + t[F_r(P_3) - F_r(P_1)] = 0$$

$$t = \frac{-F_r(P_1)}{F_r(P_3) - F_r(P_1)} = \frac{-2}{-1-2} = \frac{2}{3}$$

Intersect with parametric form of line:

$$P_{13} = P_1 + t(P_3 - P_1) = \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} 2-0 \\ 2-1 \\ -2-(-4) \end{bmatrix} = \begin{bmatrix} 4/3 \\ 5/3 \\ -8/3 \end{bmatrix}$$

For line $\overline{P_2P_3}$:

$$t = \frac{-F_r(P_2)}{F_r(P_3) - F_r(P_2)} = \frac{-1/2}{-1-1/2} = \frac{1}{3}$$

Intersect with parametric form of line:

$$P_{23} = P_2 + t(P_3 - P_2) = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4/3 \\ 2/3 \\ -8/3 \end{bmatrix}$$

Notice now our clipped coordinates are P_1, P_2, P_{23}, P_{13}.

top plane:

$F_t(P_1) = -1 - \frac{-4}{2} = 1 > 0$ (inside)

$F_t(P_2) = -\frac{-3}{2} = \frac{3}{2} > 0$ (inside)

$F_t(P_{23}) = -\frac{2}{3} - \frac{-8}{6} = \frac{4}{6} > 0$ (inside)

$F_t(P_{13}) = -\frac{5}{3} - \frac{-8}{6} = \frac{-2}{6} < 0$ **(OUT!)**

For line $\overline{P_1P_{13}}$:

$$t = \frac{-F_t(P_1)}{F_t(P_{13}) - F_t(P_1)} = \frac{6}{8}$$

$$P_{113} = P_1 + t(P_{13} - P_1) = \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} + \frac{6}{8}\begin{bmatrix} 4/3 \\ 2/3 \\ 4/3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \\ -3 \end{bmatrix}$$

For line $\overline{P_{13}P_{23}}$:

$$t = \frac{-F_t(P_{13})}{F_t(P_{23}) - F_t(P_{13})} = \frac{2}{6}$$

$$P_{1323} = P_{13} + t(P_{23} - P_{13}) = \begin{bmatrix} 4/3 \\ 5/2 \\ -8/3 \end{bmatrix} + \frac{2}{6}\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4/3 \\ 4/3 \\ -8/3 \end{bmatrix}$$

Notice now our clipped coordinates are $P_1, P_2, P_{23}, P_{1323} P_{113}$.

Bottom plane:

$$F_b(P_1) = 1 - \frac{-4}{2} = 3 > 0 \quad \text{(inside)}$$

$$F_b(P_2) = 0 + \frac{3}{2} > 0 \quad \text{(inside)}$$

$$F_b(P_{113}) = \frac{3}{2} - \frac{-3}{2} = 3 > 0 \quad \text{(inside)}$$

$$F_b(P_{1323}) = \frac{8}{6} - \frac{-8}{6} = \frac{16}{6} > 0 \quad \text{(inside)}$$

$$F_b(P_{23}) = \frac{2}{3} - \frac{-8}{2} = 2 > 0 \quad \text{(inside)}$$

Final coordinates for the clipped triangle are $P_1, P_2, P_{23}, P_{1323} P_{113}$.