Modified Gram-Schmidt

For Vectors

input: spanning basis $\{a_k\}$

output: orthonormal basis $\{q_k\}$

algorithm:

for k = 1 to n

$r_{kk} = \sqrt{a_k^T a_k}$

if r_{kk} = 0 then stop

$q_k = a_k / r_{kk}$

for j = k + 1 to n

$r_{kj} = q_k^T a_j$

$a_j = a_j - r_{kj} q_k$

end

end

For Functions

input: spanning basis $\{\psi_k(x)\}$

output: orthonormal basis $\{\phi_k(x)\}$

algorithm:

for k = 1 to n

$r_{kk} = \sqrt{\int_a^b \psi_k^2(x)\, dx}$

if r_{kk} = 0 then stop

$\phi_k(x) = \psi_k(x) / r_{kk}$

for j = k + 1 to n

$r_{kj} = \int_a^b \phi_k(x) \psi_j(x)\, dx$

$\psi_j(x) = \psi_j(x) - r_{kj} \phi_k(x)$

end

end

For more details, see Heath section 3.5.3