Unit #0: Introduction

CPSC 221: Algorithms and Data Structures

Will Evans and Jan Manuch[1]

2016W1

hello

hello

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[1] Thanks to Steve Wolfman for the content of most of these slides with additional material from Alan Hu, Ed Knorr, and Kim Voll.

Unit Outline

- Course logistics
- Course overview
- Fibonacci Fun
- Arrays
- Queues
- Stacks
- Deques

Course Information

Instructors

Will Evans	`will@cs.ubc.ca`	ICCS X841
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TAs

Alexander Lim   Chris Hunter   David Zheng   Harman Gakhal
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Zheng Dong

Office hours

See `www.ugrad.cs.ubc.ca/~cs221`

thurs after class

Texts

Epp Discrete Math, Koffman Data Structs C++

Course Work

No late work; may be flexible with advance notice

10%	Labs
15%	Programming projects (~ 3)
15%	Written homework (~ 3)
20%	Midterm exam
40%	Final exam

Must pass the final and combo of labs/assignments to pass the course.

Collaboration

You may work in groups of two people on:

- Labs
- Programming projects
- Written homework

You may also collaborate with others as long as you follow the rules (see the website) and **acknowledge** their help on your assignment.

Don't violate the collaboration policy.

Course Mechanics

encourage non-anonymous posting

- Web page: `www.ugrad.cs.ubc.ca/~cs221`
- Piazza:
  `https://piazza.com/ubc.ca/winterterm12016/cpsc221/home`
- UBC Connect site: `www.connect.ubc.ca`
- Labs are in ICCS X350
  - Use the `Xshell` program on the lab machines to `ssh` into a undergrad Unix machine (e.g. `lulu.ugrad.cs.ubc.ca`)
- Programming projects will be graded on `UNIX/g++`

What is a Data Structure?

List
Array
Map
Stack
Heap

Tree
Queue
Graph
Hashtable

A method of storing data that provides, through a set of operations, a way to manipulate and access the data.

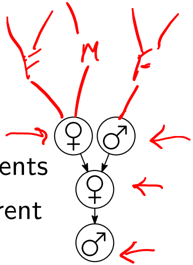
Observation

- All programs manipulate data
  - programs process, store, display, gather data
  - data can be information, numbers, images, sound
- You ~~The programmer~~ must decide how to store and manipulate data
- This choice influences the program in many ways
  - execution speed
  - memory requirements
  - maintenance (debugging, extending, etc.)

Goals of the Course

- Become familiar with some of the fundamental data structures and algorithms in computer science
  - Learn when to use them
- Improve your ability to solve problems abstractly
  - Data structures and algorithms are the building blocks
- Improve your ability to analyze algorithms
  - Prove correctness
  - Gauge, compare, and improve time and space complexity
- Become modestly skilled with C++ and UNIX, but this is largely on your own!

Analysis Example: Fibonacci numbers



Bee ancestory:

1. Fertilized egg becomes a female bee with two parents
2. Unfertilized egg becomes a male bee with one parent

How many great-grandparents does a male bee have?
great-great-grandparents? ...

Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

First two numbers are 1; each succeeding number is the sum of the previous two numbers.

Recursive Fibonacci

Problem: Calculate the *n*th Fibonacci number.

Recursive definition:

$$fib_n = \begin{cases} 1 & \text{if } n = 1 \\ 1 & \text{if } n = 2 \\ fib_{n-1} + fib_{n-2} & \text{if } n \geq 3 \end{cases}$$

C++ code:

```
int fib(int n) {
   if (n <= 2) return 1;
   else        return fib(n-1) + fib(n-2);
}
```

Too slow!

Iterative Fibonacci

Idea: Use an array

```
int fib(int n) {
  int F[n+1];

  F[0]=0; F[1]=1; F[2]=1;
  for( int i=3; i<=n; ++i ) {
    F[i] = F[i-1] + F[i-2];
  }
  return F[n];
}
```

(We don't really need the array.)

Can we do better?

Fibonacci by formula

Idea: Use a formula (a *closed form solution* to the recursive definition.)

$$fib_n = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}}$$

where $\varphi = (1 + \sqrt{5})/2 \approx 1.61803$.

```
#include <cmath>
int fib(int n) {
   double phi = (1 + sqrt(5))/2;
   return (pow(phi, n) - pow(-phi,-n))/sqrt(5);
}
```

Sadly, it's impossible to represent $\sqrt{5}$ exactly on a digital computer.

Can we do better?

Fibonacci with Matrix Multiplication

$$\overset{A}{\boxed{\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}1+1\\ 1\end{bmatrix} = \begin{bmatrix}fib_3\\ fib_2\end{bmatrix}$$

$$\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = \underline{\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}2\\ 1\end{bmatrix}} = \begin{bmatrix}fib_4\\ fib_3\end{bmatrix} \qquad \begin{bmatrix}3\\ 2\end{bmatrix}$$

$$\boxed{\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}^{n-2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}fib_n\\ fib_{n-1}\end{bmatrix}$$

How do we calculate $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}^{n-2}$?

Repeated Squaring

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{aligned} A \times A &= A^2 \\ A^2 \times A^2 &= A^4 \\ A^4 \times A^4 &= A^8 \\ A^8 \times A^8 &= A^{16} \\ A^{16} \times A^{16} &= A^{32} \\ &\vdots \end{aligned}$$

$2^5\,2^4\,2^3\,2^2\,2^1\,2^0$

$1\;1\;1\;1\;1\;1$

$= 63$

$2^6 - 1$ ''

n	R	i	m
43	39sec	—	—
⋮	X		
2^{20}		7sec	—
2^{22}		2:39sec	—
⋮		X	
2^{25}			16sec (3sec)
⋮			
2^{27}			(18.5sec)
⋮			

Example: $A^{\overset{n}{100}} = A^{64} \times A^{32} \times A^4$. 8 instead of 99 multiplications.
Generally, about $\log_2 n$ multiplications.

Is this better than iterative Fibonacci?

Abstract Data Type

Abstract Data Type

Mathematical description of an object and the set of operations on the object

Example: **Dictionary ADT**

- Stores pairs of strings: (word, definition)
- Operations:
  - Insert(word,definition)
  - Delete(word)
  - Find(word) ← returns def. for word

Another Example: Array ADT

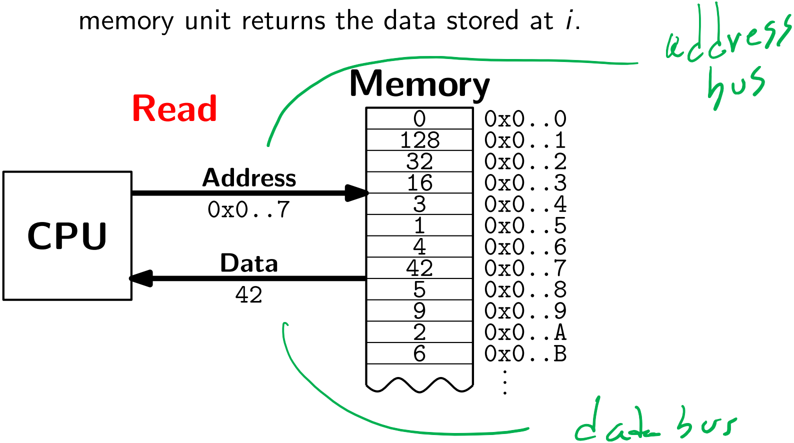
- Store things like integers, (pointers to) strings, etc.
- Operations:
  - Initialize an empty array that can hold *n* things.
    `thing A[n];`
  - Access (read or write) the *i*th thing in the array ($0 \leq i \leq n-1$).
    `thing1 = A[i];` Read
    `A[i] = thing2;` Write

Why Arrays?

- Computer memory is an array.

  Read: CPU provides address *i*,

  memory unit returns the data stored at *i*.



Read

CPU

Address

0x0..7

Data

42

Memory

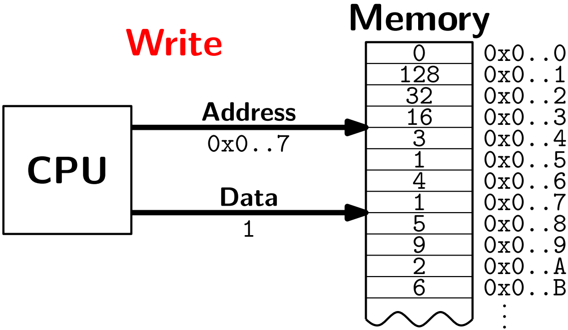
0	0x0..0
128	0x0..1
32	0x0..2
16	0x0..3
3	0x0..4
1	0x0..5
4	0x0..6
42	0x0..7
5	0x0..8
9	0x0..9
2	0x0..A
6	0x0..B
	⋮

address bus

data bus

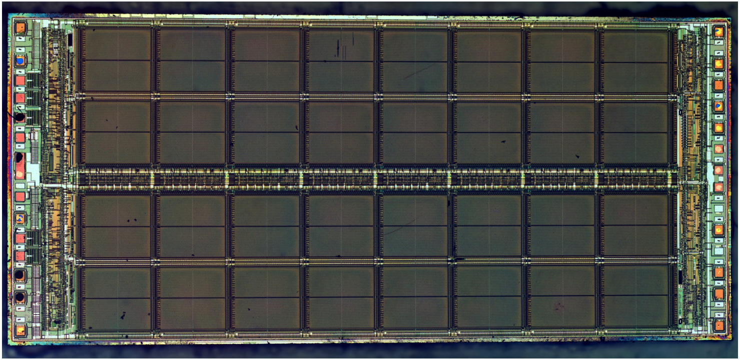
Why Arrays?

- Computer memory is an array.
  Write: CPU provides address *i* and data *d*,
  memory unit stores data *d* at *i*.



Why Arrays?

- Computer memory is an array. Every bit has a physical location.



http://zeptobars.ru/en/read/how-to-open-microchip-asic-what-inside licensed under Creative Commons Attribution 3.0 Unported.

Why Arrays?

- Computer memory is an array.
- Simple and fast.
- Used in almost every program.
- Used to implement other data structures.

Array limitations

- Need to know size when array is created.

  Fix: Resizeable arrays.
  If the array fills up, allocate a new, bigger array and copy the old contents to the new array.

- Indices are integers 0,1,2,...

  Fix: Hashing.
  (more later)

AWK has
associative arrays
(indexed by anything)

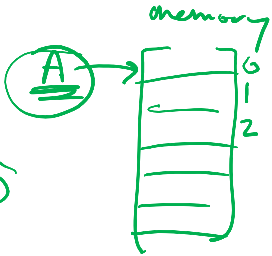
<u>man</u> awk

How would you implement the Array ADT?

memory management (tricky)

address arithmetic

convert i to mem address



A's value is an address

pointer

0x0007

pointer arithmetic

A[3] is located at address 0x0007

\+ 3

0x000A

How would you implement the Array ADT?

Arrays in C++

32-bit

Create `int A[100];`

Access
```
for( int i=0; i<100; i++ )
    A[i] = (i+1) * A[i-1];
```

$*(A+i)$

How would you implement the Array ADT?

Arrays in C++

Create `int A[100];`

Access

```
for( int i=0; i<100; i++ )
    A[i] = (i+1) * A[i-1];
```

Warning No bounds checking!

Data Structures as Algorithms

Algorithm

a high level, language independent description of a step-by-step process for solving a problem

Data Structure

a way of storing and organizing data so that it can be manipulated as described by an ADT

A data structure is defined by the algorithms that implement the ADT operations.

Why so many data structures?

Ideal data structure

fast, elegant, memory efficient

Trade-offs

- time vs. space
- performance vs. elegance
- generality vs. simplicity
- one operation's performance vs. another's

Data structures for Dictionary ADT

- List
- Skip list
- Binary search tree
- AVL tree
- Splay tree
- B-tree
- Red-Black tree
- Hash table

...

Code Implementation

~~Theory~~ Ideal

- abstract base class (interface) describes ADT
- descendents implement data structures for the ADT
- data structures can change without affecting client code

array implementat of dictionary
is_full()

Practice

- different implementations sometimes suggest different interfaces (generality vs. simplicity)
- performance of a data structure may influence the form of the client code (time vs. space, one operation vs. another)

if delete is slow, you may not call delete
client

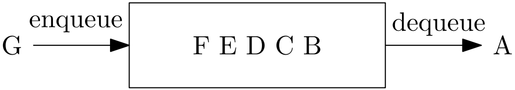
ADT Presentation Algorithm

1. Present an ADT
2. Motivate with some applications
3. Repeat
   3.1 develop a data structure for the ADT
   3.2 analyze its properties
      - efficiency
      - correctness
      - limitations
      - ease of programming
4. Contrast data structure's strengths and weaknesses
   - understand when to use each one

Queue ADT

Queue operations

- create
- destroy
- enqueue
- dequeue
- is_empty



Queue property

If x is enqueued before y is enqueued, then x will be dequeued before y is dequeued. → x is removed from Queue

FIFO: First In First Out

Applications of the Q

- Hold jobs for a printer
- Store packets on network routers
- Hold memory “freelists”
- Make waitlists fair
- Breadth first search

Abstract Q Example

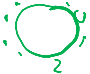
enqueue R
enqueue O
dequeue
enqueue T
enqueue A
enqueue T
dequeue
dequeue
enqueue E
dequeue

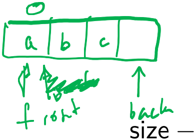
In order, what letters are dequeued?

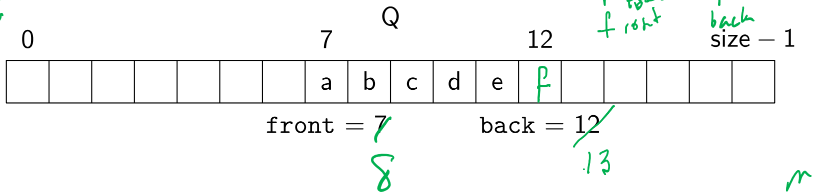
a. OATE
[✓ b. ROTA] ?
c. OTAE
d. None of these, but it **can** be determined from just the ADT.
e. None of these, and it **cannot** be determined from just the ADT.

initial entries?
Size ?
RRRR ?

Circular Array Q Data Structure







mod or %

```
void enqueue(Object x) {
  Q[back] = x;
  back = (back + 1) % size;
}

bool is_empty() {
  return (front == back);
}
```

f

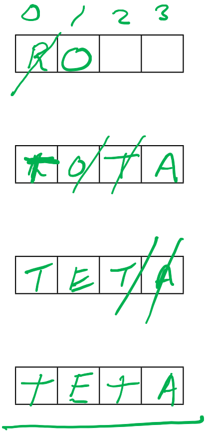
```
Object dequeue() {
  x = Q[front];
  front = (front + 1) % size;
  return x;
}

bool is_full() {
  return (front ==
    (back + 1) % size);
}
```

a ←

if is_full() explode();

if is_empty() implode();

Circular Array Q Example



	front	back
	0	0
enqueue R	0	1
enqueue O	0	2
dequeue	1	2
enqueue T	1	3
enqueue A	1	0
✱ enqueue T	1	1
dequeue		
dequeue		
enqueue E		
dequeue		

What are the final contents of the array?

a. RTE

b. RTET

c. TETA ✓

d. TE ✓

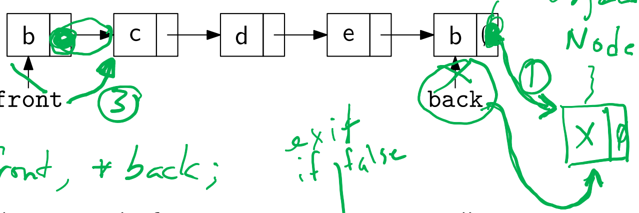
e. None

✱ explode

Linked List Q Data Structure

struct Node{
Object data;
Node* next;
}

front->next
= NULL;



Node *front, *back;

```
void enqueue(Object x) {
  if (is_empty())
    front = back = new Node(x)
  else {
①   back->next = new Node(x);
②   back = back->next;
  }
}

bool is_empty() {
  return (front == NULL);
}
```

exit
if false

```
Object dequeue()
  assert(!is_empty());
  Object ret = front->data;
  Node *temp = front;
  front = front->next; ③
  delete temp;
  return ret;
```

why?

Why not Ⓐ front = front->next ?

memory leak

DIY memory management

Ⓑ delete front;
front = front->next ?

Circular Array vs. Linked List

Ease of implementation same

Generality size limit — dynamic resize ✩✩

— Speed — same except ✩ and ✩✩

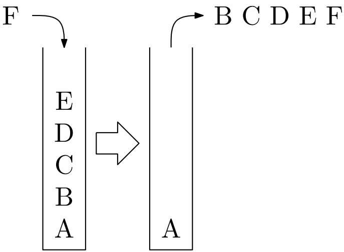
Memory use next pointers increase memory

✩ Cache performance better for Array

Stack ADT

Stack operations

- create
- destroy
- push
- pop
- top
- is_empty



Stack property

if x is pushed before y is pushed, then x will be popped after y is popped.

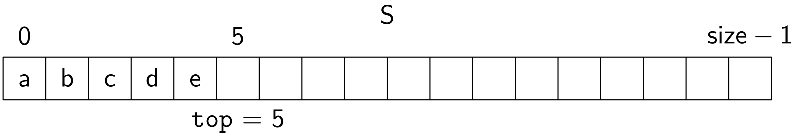
LIFO: Last In First Out

Stacks in Practice

- Function call stack
- Removing recursion
- Balancing symbols (parentheses)
- Evaluating Reverse Polish Notation
- Depth first search

postscript
java VM
HP 1980s

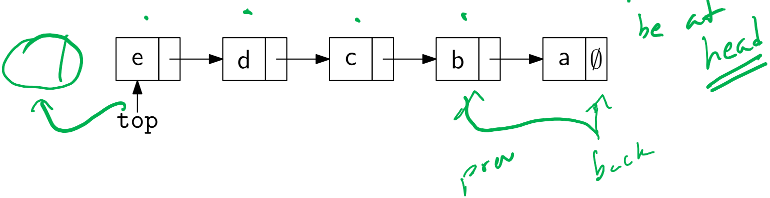
Array Stack Data Structure



```
void push(Object x) {
  assert(!is_full());
  S[top] = x;
  top++;
}

Object top() {
  assert(!is_empty());
  return S[top-1];
}
```

```
Object pop() {
  assert(!is_empty());
  top--;
  return S[top];
}
bool is_empty() {
  return( top == 0 );
}
bool is_full() {
  return( top == size);
}
```

Linked List Stack Data Structure



top should be at head

```
void push(Object x) {
  Node *temp = top;
  top = new Node(x);
  top->next = temp;
}

Object top() {
  assert(!is_empty());
  return top->data;
}
```

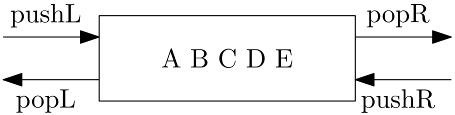
```
Object pop() {
  assert(!is_empty());
  Object ret = top->data;
  Node *temp = top;
  top = top->next;
  delete temp;
  return ret;
}

bool is_empty() {
  return( top == NULL );
}
```

Deque ADT

icon

Deque (Double-ended queue) operations

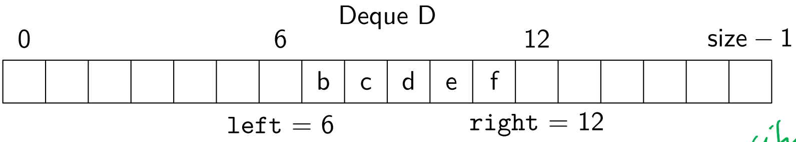
- create/destroy
- pushL/pushR
- popL/popR
- is_empty



Deque property

Deque maintains a list of items.
push/pop adds to/removes from front(L)/back(R) of list.

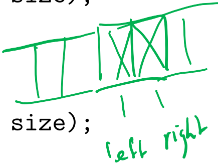
Circular Array Deque Data Structure



Capacity of deque is $\boxed{size-2}$

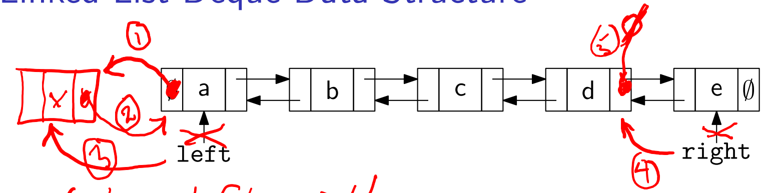
```
void pushL(Object x) {
  assert(!is_full());
  D[left] = x;
  left = (left - 1) % size;
}
Object popR() {
  assert(!is_empty());
  right = (right - 1) % size;
  return D[right];
}
...
```

```
bool is_empty() {
  return( left ==
      (right - 1) % size);
}
bool is_full() {
  return( left ==
      (right + 1) % size);
}
```



is empty Deque

Linked List Deque Data Structure



Node *left, *right;

```
void pushL(Object x) {
  if( is_empty() )
    left = right = new Node(x);
  else {
①  left->prev = new Node(x);
②  left->prev->next = left;
③  left = left->prev;
  }
}
```

```
Object popR() {
  assert(!is_empty());
  Object ret = right->data;
  Node *temp = right;
  right = right->prev; ④
⑤ if( right ) right->next = NULL;
  else left = NULL;
  delete temp;
  return ret;
}
```

right ≠ NULL

```
bool is_empty() {return left==NULL;}
```

Data structures you should already know (a bit)

- Arrays
- Linked lists
- Trees
- Queues
- Stacks