CPSC 221: Written Assignment 3

Last Updated: November 9, 2016 Due: 21.00 Wednesday November 23, 2016.

Submission Instructions

Handin your solutions using `handin`. You can write your solutions by hand and scan the pages or take pictures of them with your phone; or use a word processing package to typeset your solutions and produce a .pdf file.

To handin: Copy the files that contain your solutions to the directory `~/cs221/assign3` in your home directory on an undergraduate machine. (You may have to create this directory using `mkdir ~/cs221/assign3`.) Then run `handin cs221 assign3` from your home directory.

We encourage you to work in pairs. **Be sure to include the names and ugrad login IDs of both partners on all solution pages, but only one partner should handin the assignment.**

Late submissions are accepted subject to the following penalty: You lose $100 \times (2^{\lfloor m/5 \rfloor})/64$ percent of your mark, where m is the number of minutes late your assignment is. For example, if you hand in 10 minutes late, you'll lose $100 \times (2^2)/64 = 6.25\%$ of the mark, but if you hand in 25 minutes late, you'll lose 50% of the mark. You cannot hand in more than 30 minutes late.

Acknowledge all collaborators or sources of assistance (besides the course staff, handouts, and required textbooks) on the first page of your assignment by name. If you quote from or derive work from any source, you *must* acknowledge that source where it is used as is usual for citations. We don't need a formal citations list, although that's not too hard to produce with LaTeX.

Questions

1. Draw the Dictionary data structure obtained after inserting:

$$120, 130, 70, 30, 50, 20, 40, 140, 150, 60, 10, 97, 110, 96, 93$$

one after the other into the following initially empty structures.

   (a) (5 points) A Hash Table of size 11 that uses chaining (with unsorted linked-list chains, recently inserted items inserted at the front of the chain). Use hash function $\text{hash}(k) = k \bmod 11$.

   (b) (5 points) A Hash Table of size 23 that uses open addressing with double hashing. The first hash function is: $\text{hash}_1(k) = k$. The second is: $\text{hash}_2(k) = 13 - (k \bmod 13)$. Use mod 23 to restrict the probe sequence to the Hash Table. For each key, indicate the table slots probed.

   (c) (5 points) An AVL Tree. Draw the tree before and after every double or single rotation.

   (d) (5 points) A B^+-Tree with $M = 5$ and $L = 3$. When nodes split, put half of the items on the left and half on the right, and put the extra item on the left if there is one. Draw the tree after every insertion that causes a split as well as the final tree.

2. (5 points) A room contains 6 computers. Each computer is directly connected to 0 or more of the other computers in the room. Show that there are at least two computers in the room that are directly connected to the same number of other computers.

   Hint: You might first try the problem assuming each computer is directly connected to 1 or more of the other computers.

3. (5 points) Let $N(h)$ be the smallest number of nodes in an AVL tree of height h. For example, $N(0) = 1$ and $N(1) = 2$. Prove that $N(h) = F(h + 3) - 1$ where $F(i)$ is the ith Fibonacci number ($F(0) = 0$, $F(1) = 1$, and $F(i) = F(i - 1) + F(i - 2)$). To do this, you need to come up with a recurrence relation that defines $N(h)$ and argue why it is correct. Then you need to prove, by induction, that $N(h) = F(h + 3) - 1$.