



CPSC 314  
21 - GLOBAL ILLUMINATION

Textbook: 20

[UGRAD.CS.UBC.CA/~CS314](http://UGRAD.CS.UBC.CA/~CS314)

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# ILLUMINATION MODELS/ALGORITHMS

Local illumination - Fast

Ignore real physics, approximate the look  
Interaction of each object with light

- Compute on surface (light to viewer)



Global illumination - Slow

Physically based  
Interactions between objects



# ILLUMINATION MODELS/ALGORITHMS

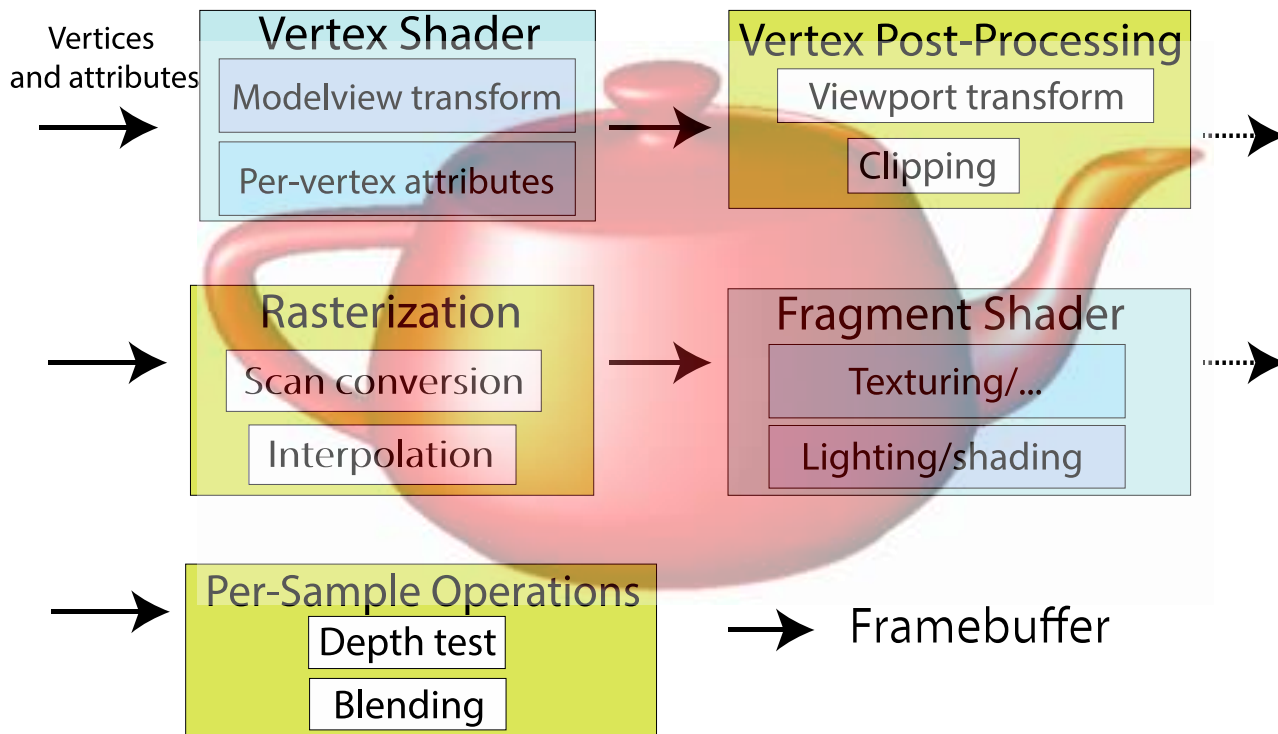
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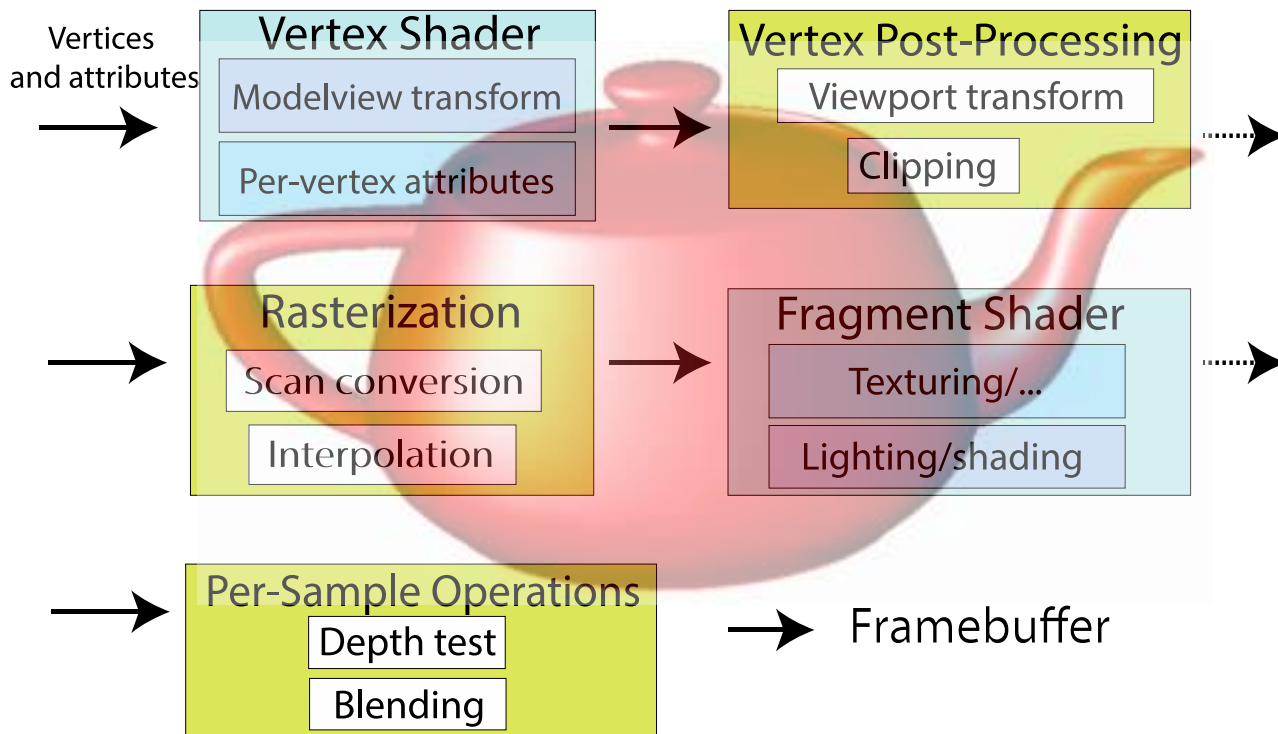
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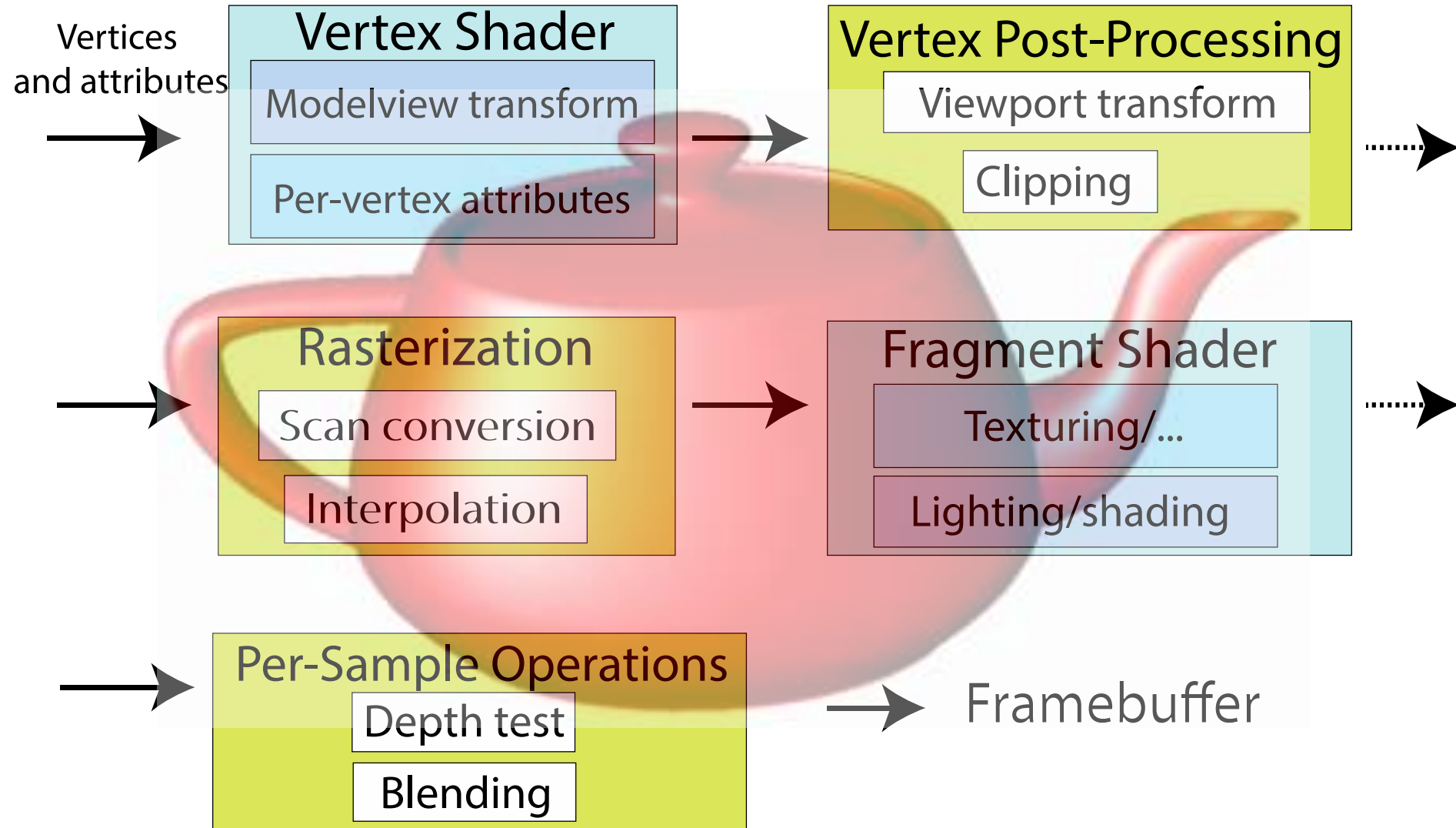
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Physically based  
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# WHAT WAS NON-PHYSICAL IN LOCAL ILLUMINATION?

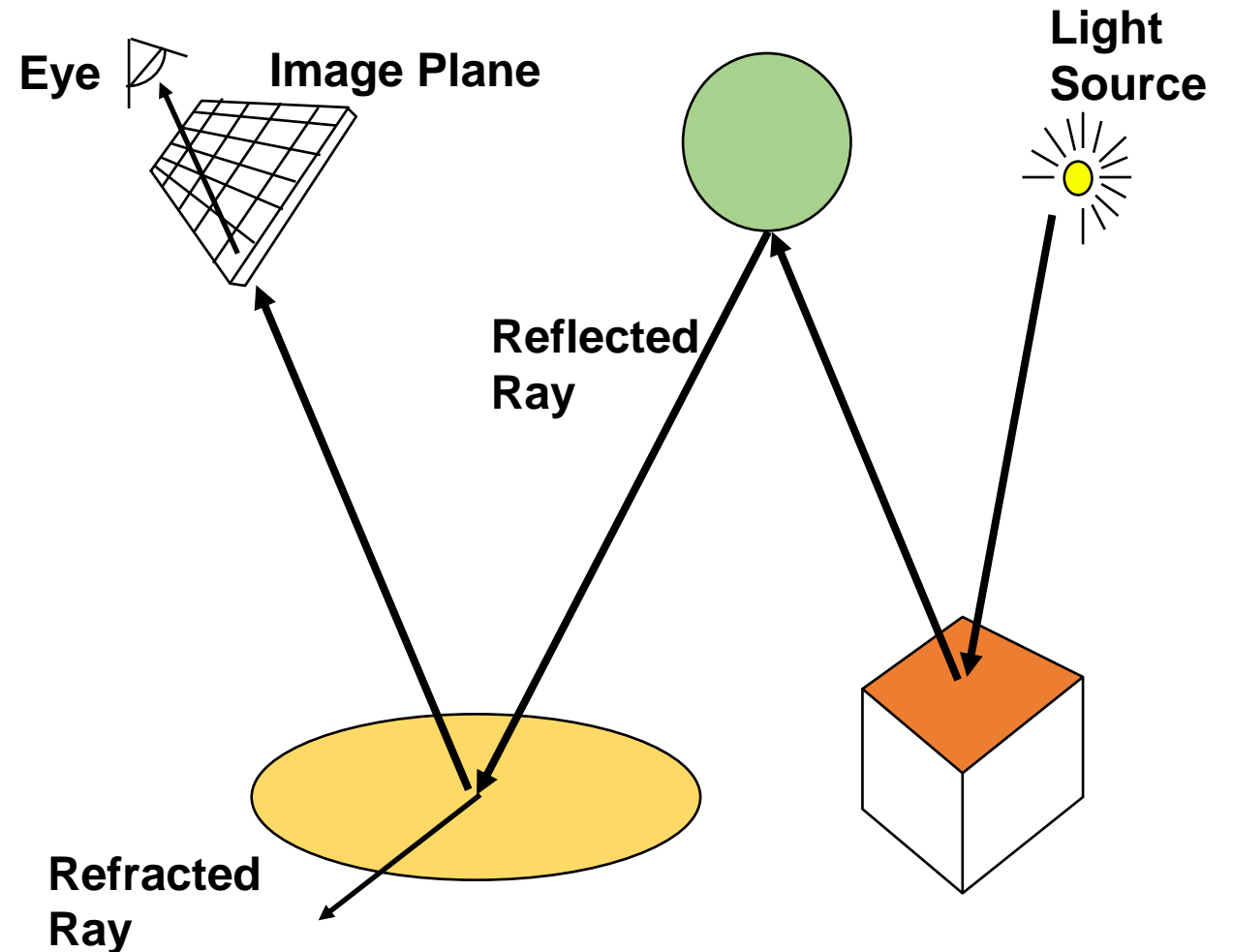


**HOW SHOULD GLOBAL ILLUMINATION WORK?**

# HOW SHOULD GLOBAL ILLUMINATION WORK?

Simulate light

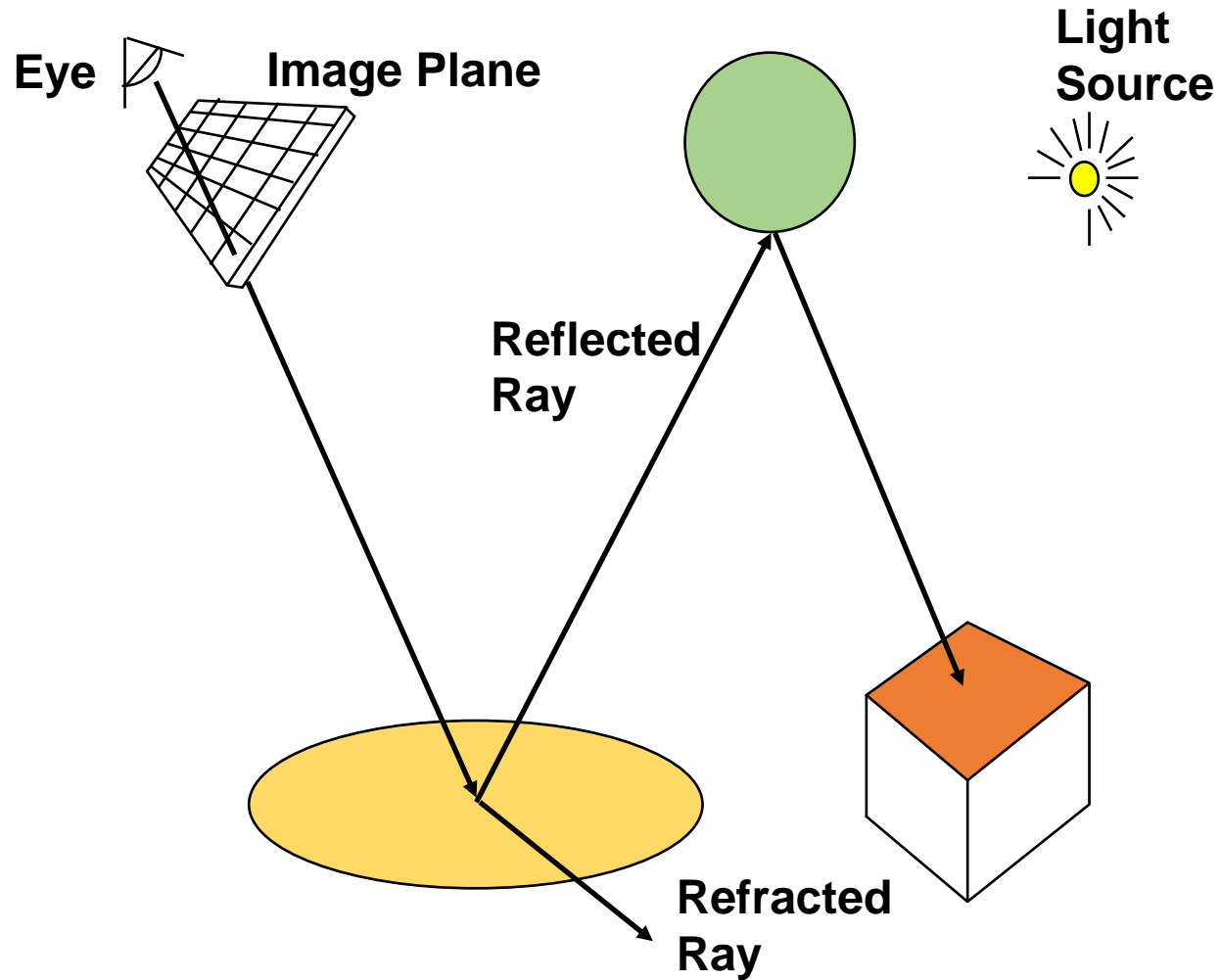
- As it is emitted from light sources
- As it bounces off objects / get absorbed / refracted
- As some of the rays hit the camera



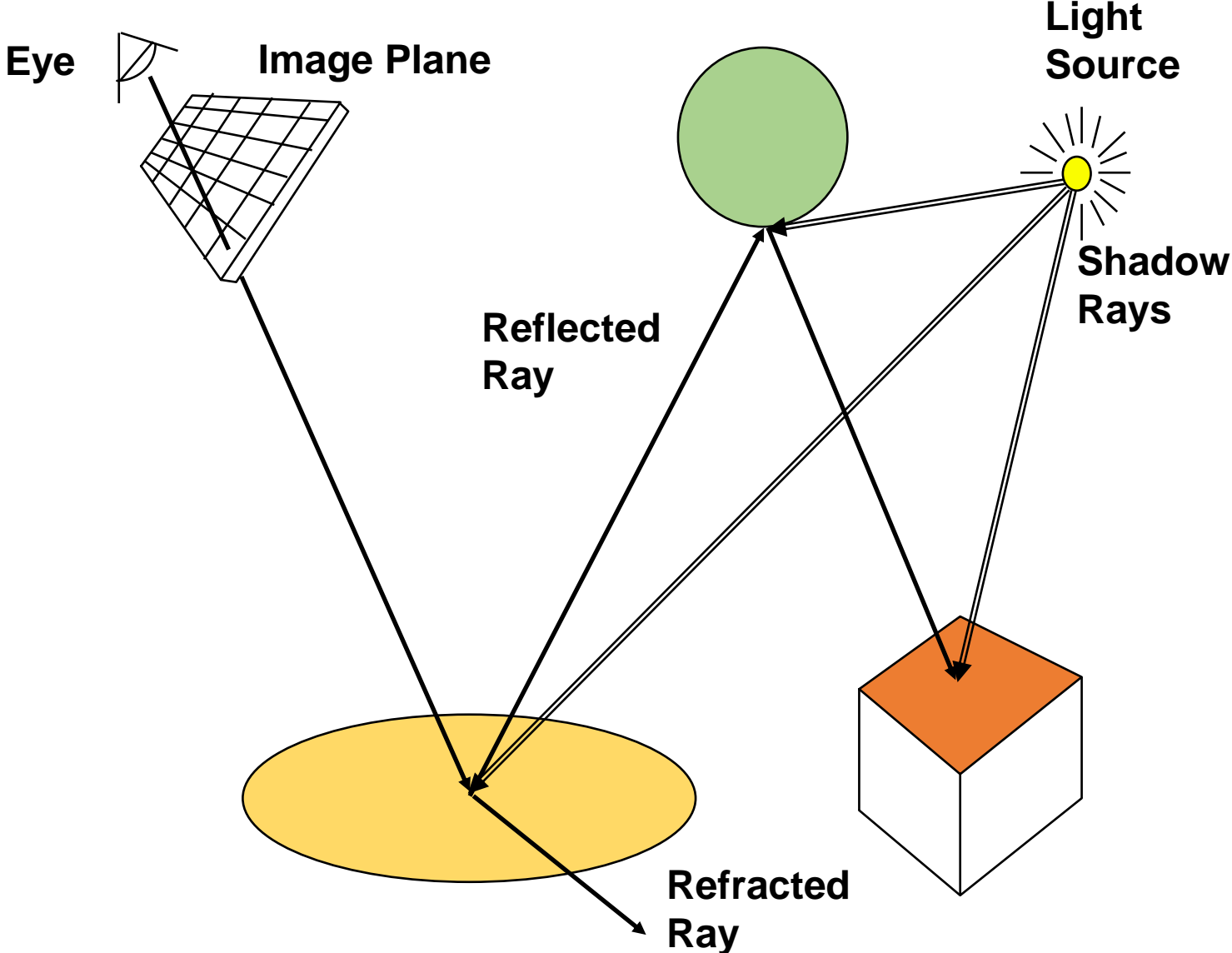
**PROBLEM?**



# RAY TRACING: IDEA



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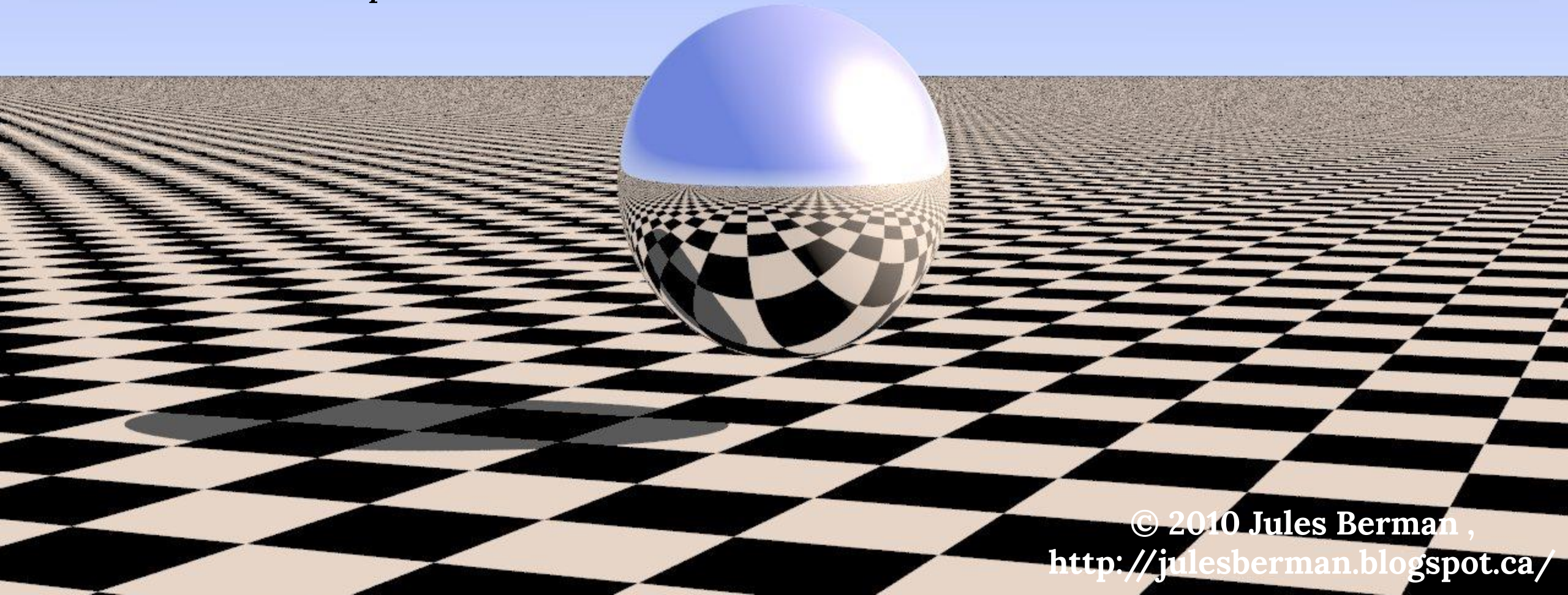
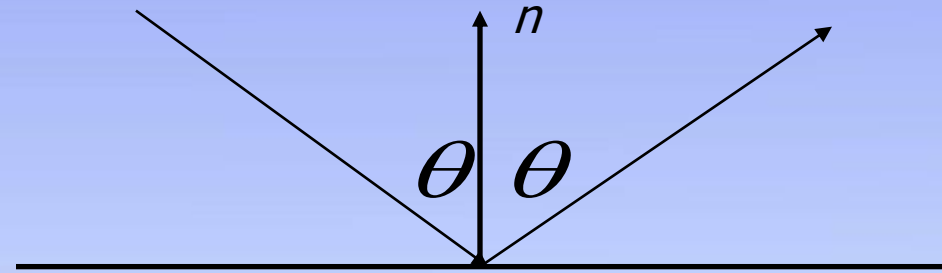


# RAY TRACING

- Invert the direction of rays!
- Shoot rays from CAMERA through each pixel
  - “Trace the rays back”
- Simulate whatever the light rays do:
  - Reflection
  - Refraction
  - ...
- Each interaction of the ray with an object adds to the final color
- Those rays are never gonna hit the light source, so
  - Shoot “shadow rays” to compute direct illumination

# REFLECTION

- Mirror effects
  - Perfect specular reflection



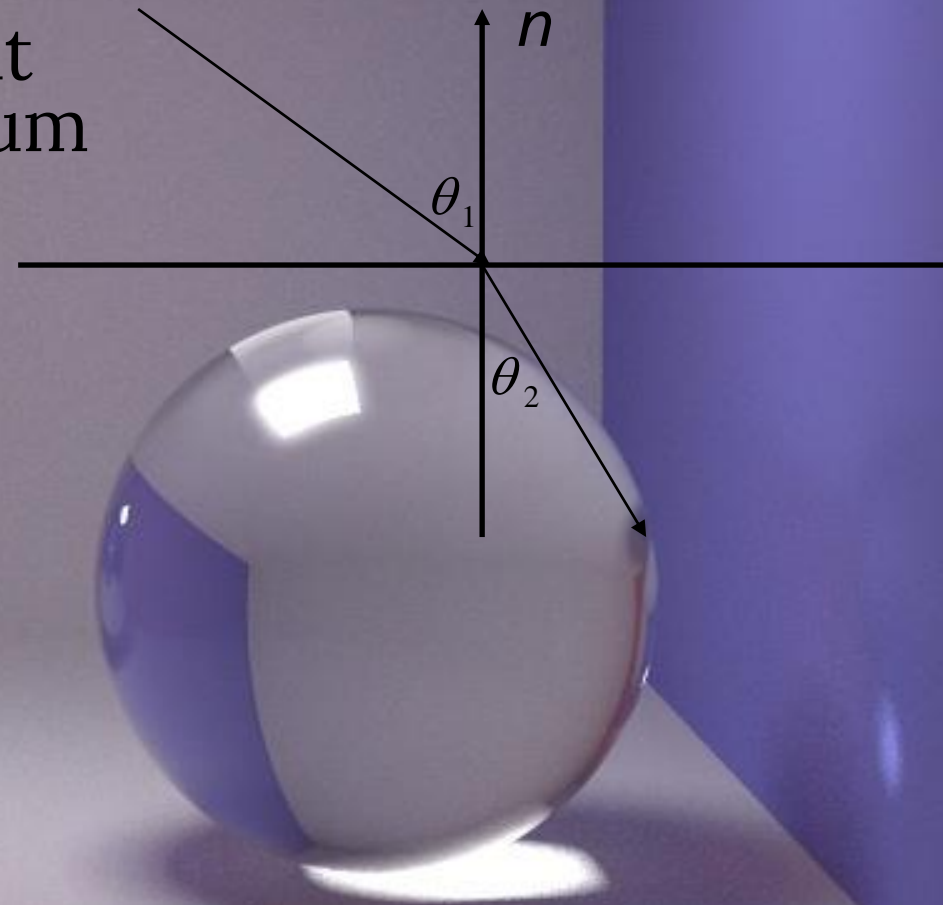
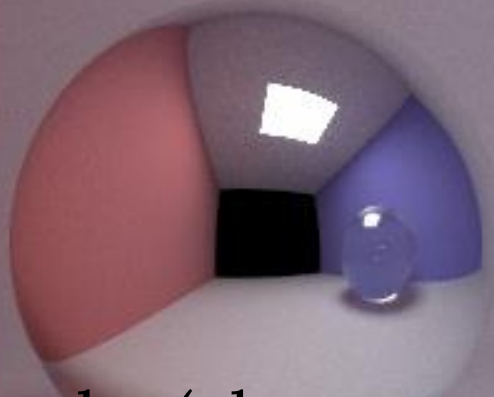
# REFRACTION

- Interface between transparent object and surrounding medium
  - E.g. glass/air boundary

- Light ray breaks (changes direction) based on refractive indices  $c_1$ ,  $c_2$

Snell's Law

$$c_2 \sin \theta_1 = c_1 \sin \theta_2$$



# BASIC RAY-TRACING ALGORITHM

```
RayTrace(r, scene)
obj = FirstIntersection(r, scene)

if (no obj) return BackgroundColor;
else {
    if (Reflect(obj))
        reflect_color = RayTrace(ReflectRay(r, obj));
    else
        reflect_color = Black;

    if (Transparent(obj))
        refract_color = RayTrace(RefractRay(r, obj));
    else
        refract_color = Black;

    return Shade(reflect_color, refract_color, obj);
}
```

# WHEN TO STOP?

- Algorithm above does not terminate
- Termination Criteria
  - No intersection
  - Contribution of secondary ray attenuated below threshold – each reflection/refraction attenuates ray
  - Maximal depth is reached

# SUB-ROUTINES

- `ReflectRay(r,obj)` – computes reflected ray (use obj normal at intersection)
- `RefractRay(r,obj)` – computes refracted ray
  - Note: ray is inside obj
- `Shade(reflect_color,refract_color,obj)` – compute illumination given three components



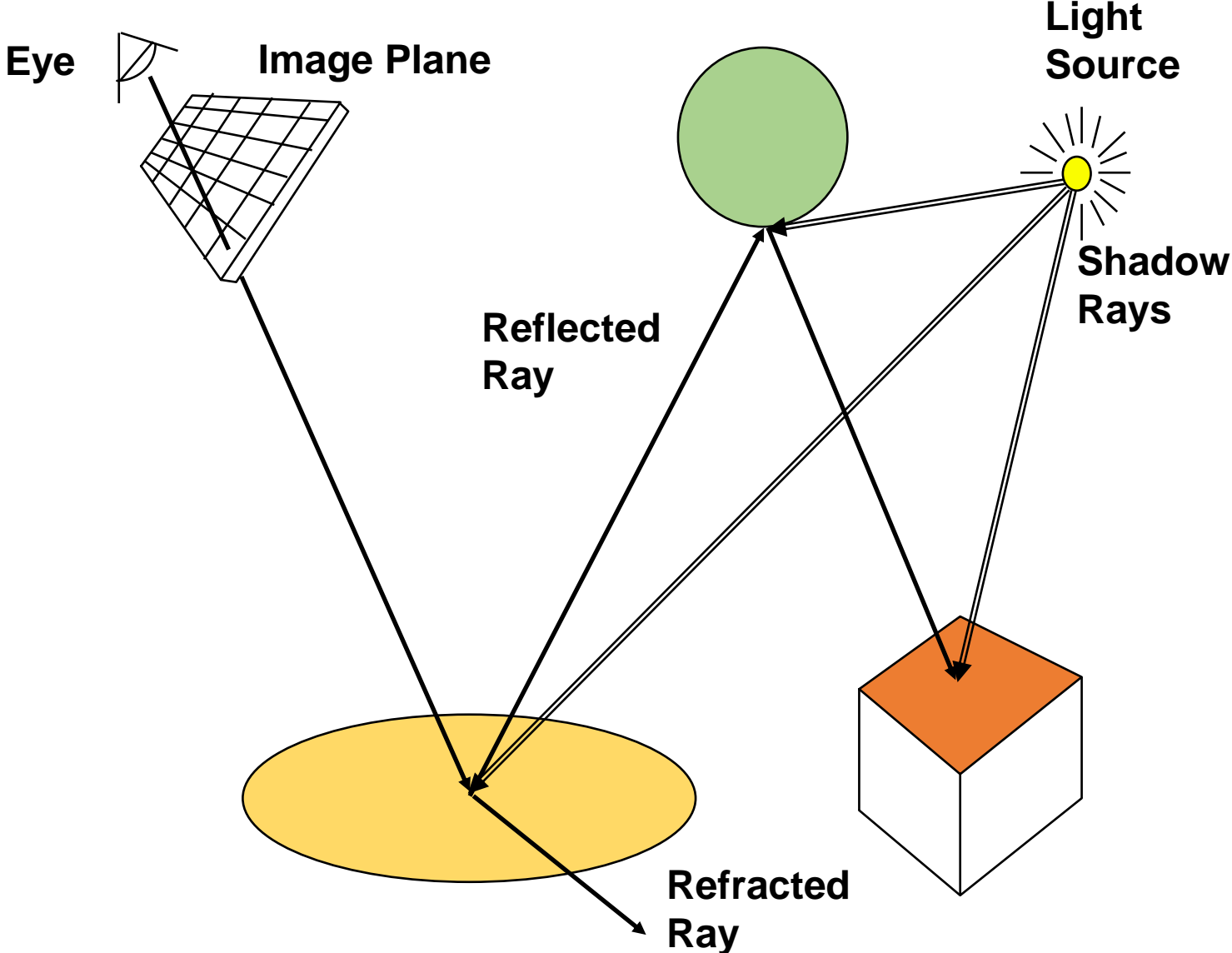
# SIMULATING SHADOWS

- Trace ray from each ray-object intersection point to light sources
  - If the ray intersects an object in between  $\Rightarrow$  point is shadowed from the light source

```
shadow = RayTrace(LightRay(obj,r,light));
```

```
return Shade(shadow,reflect_color,refract_color,obj);
```

# RAY TRACING: IDEA



# RAY-TRACING: PRACTICALITIES

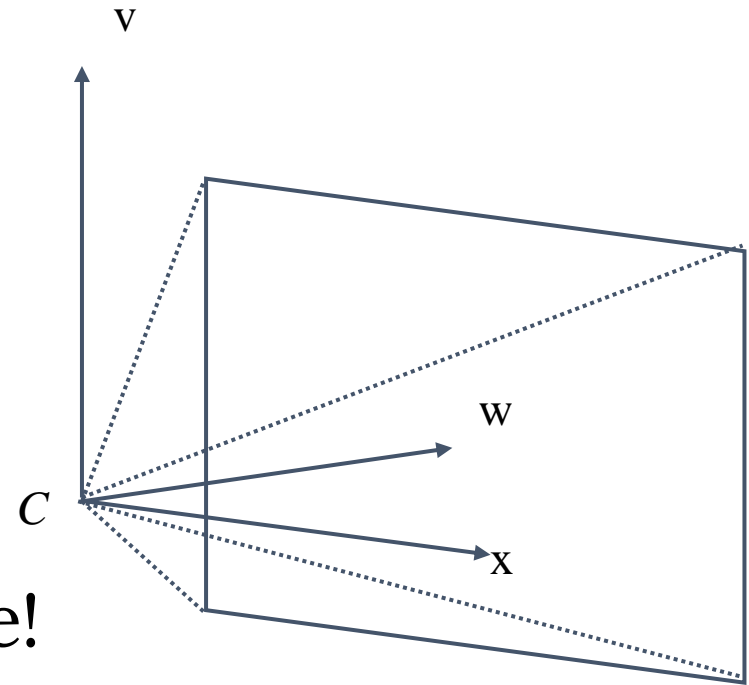
- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Speed: Reducing number of intersection tests
  - E.g. use BSP trees or other types of space partitioning

# RAY-TRACING: GENERATION OF RAYS

- Camera Coordinate System

- Origin:  $C$  (camera position)
- Viewing direction:  $w$
- Up vector:  $v$
- $u$  direction:  $u = w \times v$

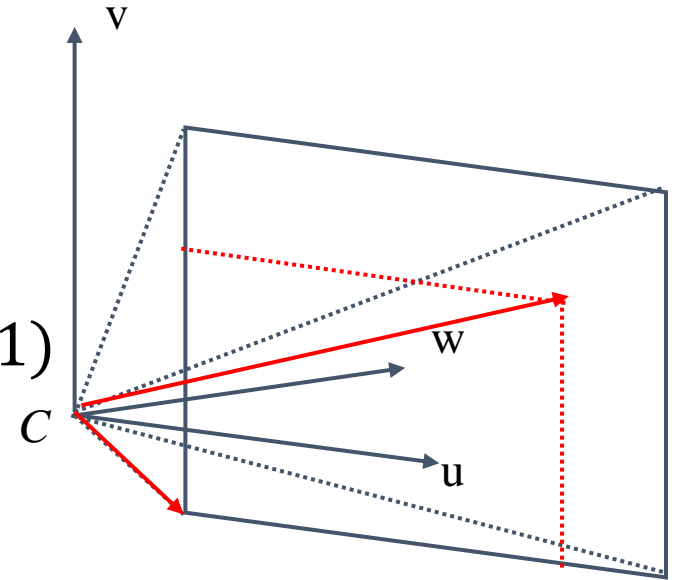
- Corresponds to viewing transformation in rendering pipeline!



# RAY-TRACING: GENERATION OF RAYS

- Distance to image plane:  $d$
- Image resolution (in pixels):  $N_x, N_y$
- Image plane dimensions:  $l, r, t, b$
- Pixel at position  $i, j$  ( $i = 0, \dots, N_x - 1; j = 0, \dots, N_y - 1$ )

$$O = C + d\vec{w} + l\vec{u} + t\vec{v}$$



$$P_{i,j} = O + (i + 0.5) \cdot \frac{r - l}{N_x} \cdot \vec{u} - (j + 0.5) \cdot \frac{t - b}{N_y} \cdot \vec{v}$$
$$= O + (i + 0.5) \cdot \Delta u \cdot \vec{u} - (j + 0.5) \cdot \Delta v \cdot \vec{v}$$

# RAY-TRACING: GENERATION OF RAYS

- Parametric equation of a ray:

$$\mathbf{R}_{i,j}(t) = \mathbf{C} + t \cdot (\mathbf{P}_{i,j} - \mathbf{C}) = \mathbf{C} + t \cdot \mathbf{v}_{i,j}$$

where  $t = 0 \dots \infty$

# RAY-TRACING: PRACTICALITIES

- Generation of rays
- **Intersection of rays with geometric primitives**
- Geometric transformations
- Lighting and shading
- Speed: Reducing number of intersection tests
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# RAY-OBJECT INTERSECTIONS

- In OpenGL pipeline, we were limited to discrete objects:
  - Triangle meshes
- In ray tracing, we can support analytic surfaces!
  - No problem with interpolating  $z$  and normals, # of triangles, etc.
    - Almost



# RAY-OBJECT INTERSECTIONS

- Core of ray-tracing  $\Rightarrow$  must be extremely efficient
- Usually involves solving a set of equations
  - Using implicit formulas for primitives

## Example: Ray-Sphere intersection

ray:  $x(t) = p_x + v_x t$ ,  $y(t) = p_y + v_y t$ ,  $z(t) = p_z + v_z t$

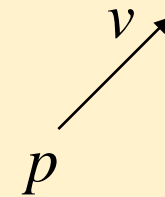
(unit) sphere:  $x^2 + y^2 + z^2 = 1$

quadratic equation in  $t$ :

$$0 = (p_x + v_x t)^2 + (p_y + v_y t)^2 + (p_z + v_z t)^2 - 1$$

$$= t^2 (v_x^2 + v_y^2 + v_z^2) + 2t(p_x v_x + p_y v_y + p_z v_z)$$

$$+(p_x^2 + p_y^2 + p_z^2) - 1$$



# RAY INTERSECTIONS WITH OTHER PRIMITIVES

- Implicit functions:
  - Spheres at arbitrary positions
    - Same thing
  - Conic sections (hyperboloids, ellipsoids, paraboloids, cones, cylinders)
    - Same thing (all are quadratic functions!)
  - Higher order functions (e.g. tori and other quartic functions)
    - In principle the same
    - But root-finding difficult
    - Numerical methods

# RAY INTERSECTIONS WITH OTHER PRIMITIVES

- Polygons:
  - First intersect ray with plane
    - linear implicit function
  - Then test whether point is inside or outside of polygon (2D test)
  - For convex polygons
    - Suffices to test whether point in on the right side of every boundary edge

# RAY-TRACING: PRACTICALITIES

- Generation of rays
- Intersection of rays with geometric primitives
- **Geometric transformations**
- Lighting and shading
- Speed: Reducing number of intersection tests
  - E.g. use BSP trees or other types of space partitioning

# RAY-TRACING: TRANSFORMATIONS

- Note: rays replace perspective transformation
- Geometric Transformations:
  - Similar goal as in rendering pipeline:
    - Modeling scenes convenient using different coordinate systems for individual objects
  - Problem:
    - Not all object representations are easy to transform
      - This problem is fixed in rendering pipeline by restriction to polygons (affine invariance!)

# RAY-TRACING: TRANSFORMATIONS

- Ray Transformation:
  - For intersection test, it is only important that ray is in same coordinate system as object representation
  - Transform all rays into object coordinates
    - Transform camera point and ray direction by inverse of model/view matrix
  - Shading has to be done in world coordinates (where light sources are given)
    - Transform object space intersection point to world coordinates
    - Thus have to keep both world and object-space ray

# RAY-TRACING: PRACTICALITIES

- Generation of rays
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# RAY-TRACING: DIRECT ILLUMINATION

- Light sources:
  - For the moment: point and directional lights
  - More complex lights are possible
    - Area lights
    - Fluorescence



# RAY-TRACING: DIRECT ILLUMINATION

- Local surface information (normal...)
  - For implicit surfaces  $F(x,y,z)=0$ :  
normal  $\mathbf{n}(x,y,z)$  is gradient of F:

$$\mathbf{n}(x, y, z) = \nabla F(x, y, z) = \begin{pmatrix} \partial F(x, y, z) / \partial x \\ \partial F(x, y, z) / \partial y \\ \partial F(x, y, z) / \partial z \end{pmatrix}$$

- Example:

$$F(x, y, z) = x^2 + y^2 + z^2 - r^2$$

$$\mathbf{n}(x, y, z) = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

Needs to be normalized!

# RAY-TRACING: DIRECT ILLUMINATION

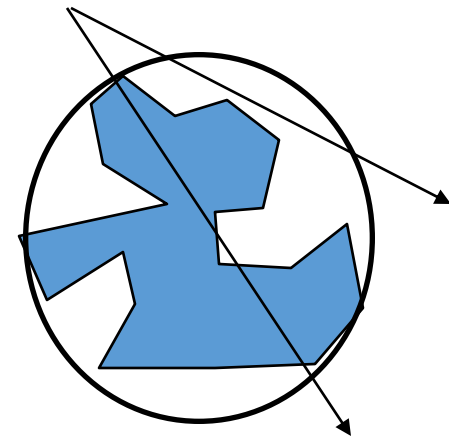
- For triangle meshes
  - Interpolate per-vertex information as in rendering pipeline
    - Phong shading!
    - Same as discussed for rendering pipeline
  - Difference to rendering pipeline:
    - Have to compute Barycentric coordinates for every intersection point (e.g. plane equation for triangles)

# RAY-TRACING: PRACTICALITIES

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- **Speed:** Reducing number of intersection tests

# OPTIMIZED RAY-TRACING

- Basic algorithm is simple but VERY expensive
- Optimize...
  - Reduce number of rays traced
  - Reduce number of ray-object intersection calculations
- Parallelize
  - Cluster
  - GPU
- Methods
  - Bounding Boxes
  - Spatial Subdivision
    - Visibility, Intersection/Collision
  - Tree Pruning

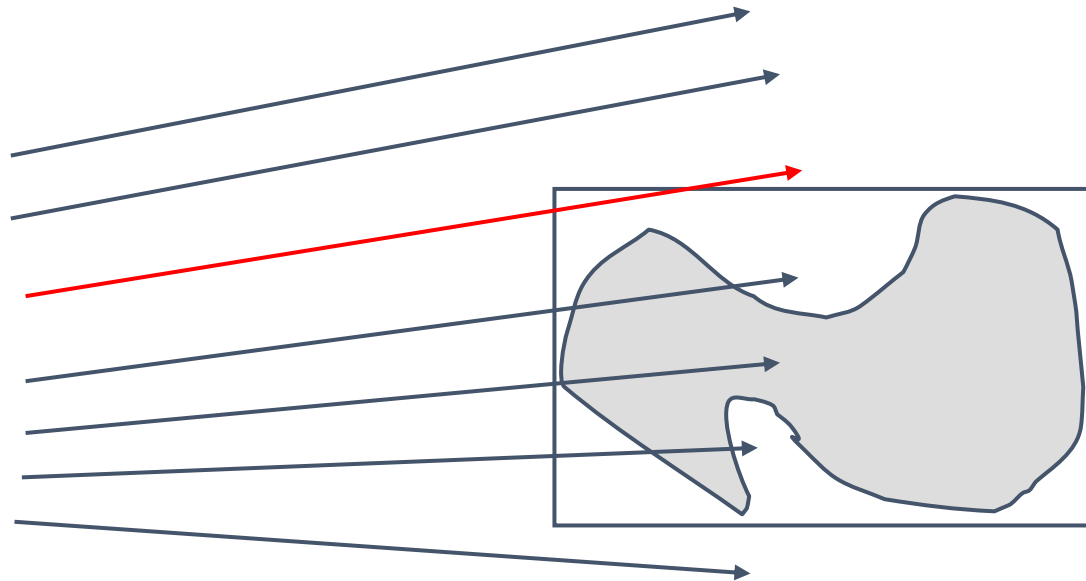


# SPATIAL SUBDIVISION DATA STRUCTURES

- Goal: reduce number of intersection tests per ray
- Lots of different approaches:
  - (Hierarchical) bounding volumes
  - Hierarchical space subdivision
    - Octree, k-D tree, BSP tree

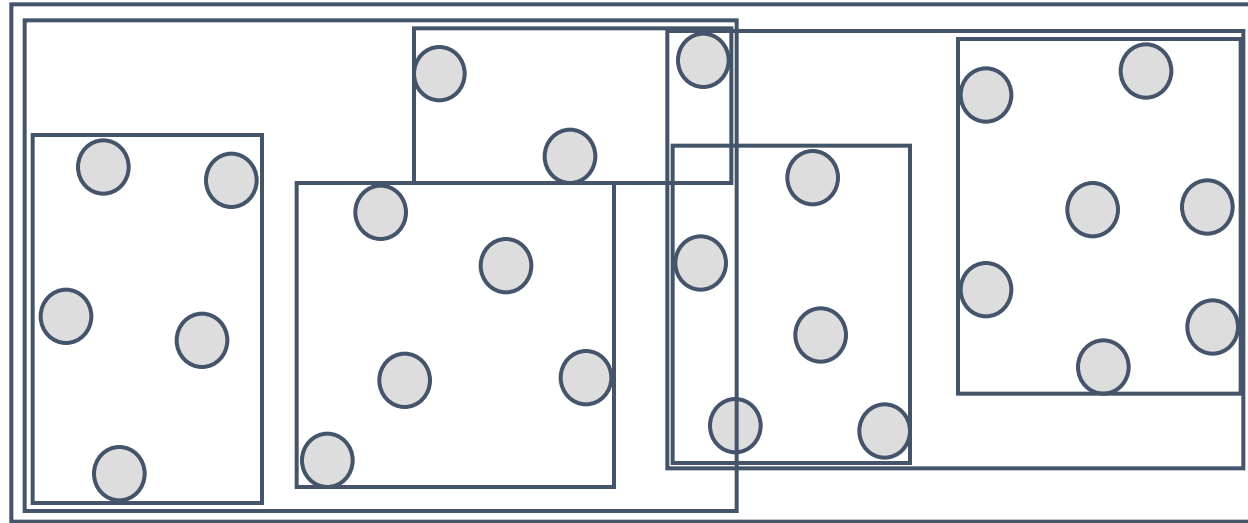
# BOUNDING VOLUMES: IDEA

- Don't test each ray against complex objects (e.g. triangle mesh)
- Do a quick conservative test first which eliminates most rays
  - Surround complex object by simple, easy to test geometry (e.g. sphere or axis-aligned box)
    - Reduce false positives: make bounding volume as tight as possible!



# HIERARCHICAL BOUNDING VOLUMES

- Extension of previous idea:
  - Use bounding volumes for groups of objects



# SPATIAL SUBDIVISION DATA STRUCTURES

- Bounding Volumes:
  - Find simple object completely enclosing complicated objects
    - Boxes, spheres
  - Hierarchically combine into larger bounding volumes
- Spatial subdivision data structure:
  - Partition the whole space into cells
    - Grids, octrees, (BSP trees)
  - Simplifies and accelerates traversal
  - Performance less dependent on order in which objects are inserted

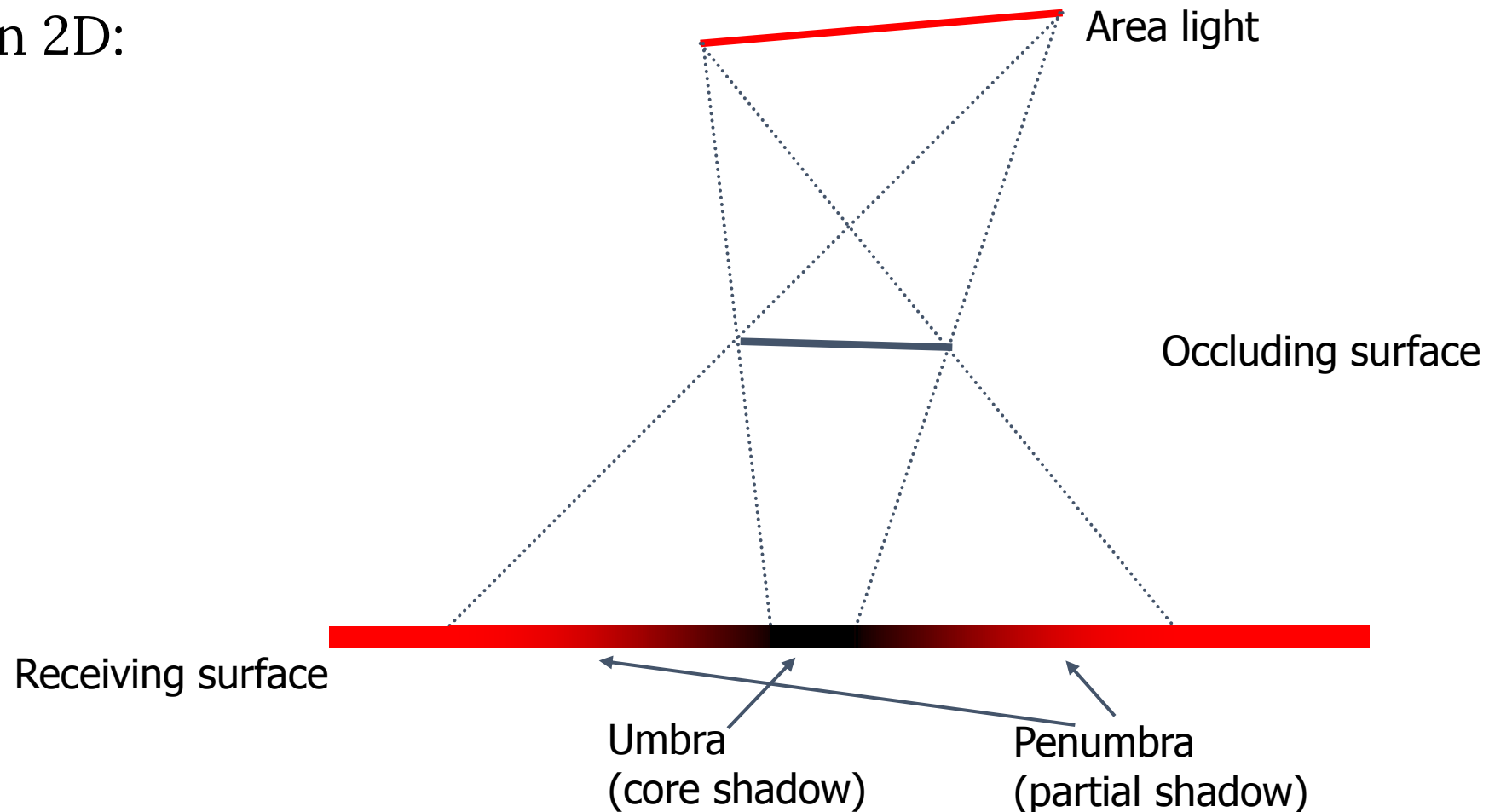


# SOFT SHADOWS: AREA LIGHT SOURCES

- So far:
  - All lights were either point-shaped or directional
    - Both for ray-tracing and the rendering pipeline
  - Thus, at every point, we only need to compute lighting formula and shadowing for **ONE** direction per light
- In reality:
  - All lights have a finite area
  - Instead of just dealing with one direction, we now have to **integrate** over all directions that go to the light source

# AREA LIGHT SOURCES

- Area lights produce soft shadows:
  - In 2D:

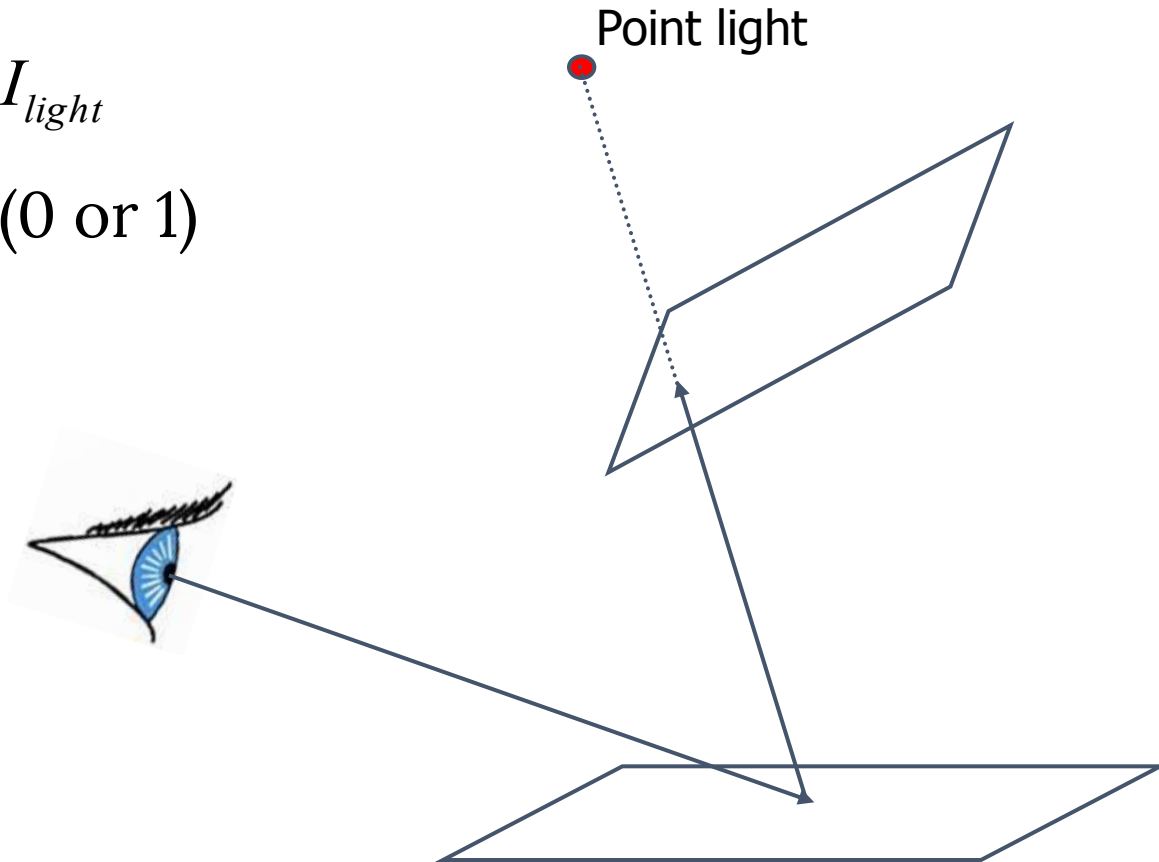


# AREA LIGHT SOURCES

- Point lights:
  - Only one light direction:

$$I_{reflected} = \rho \cdot V \cdot I_{light}$$

- $V$  is visibility of light (0 or 1)
- $\rho$  is lighting model (e.g. diffuse or Phong)

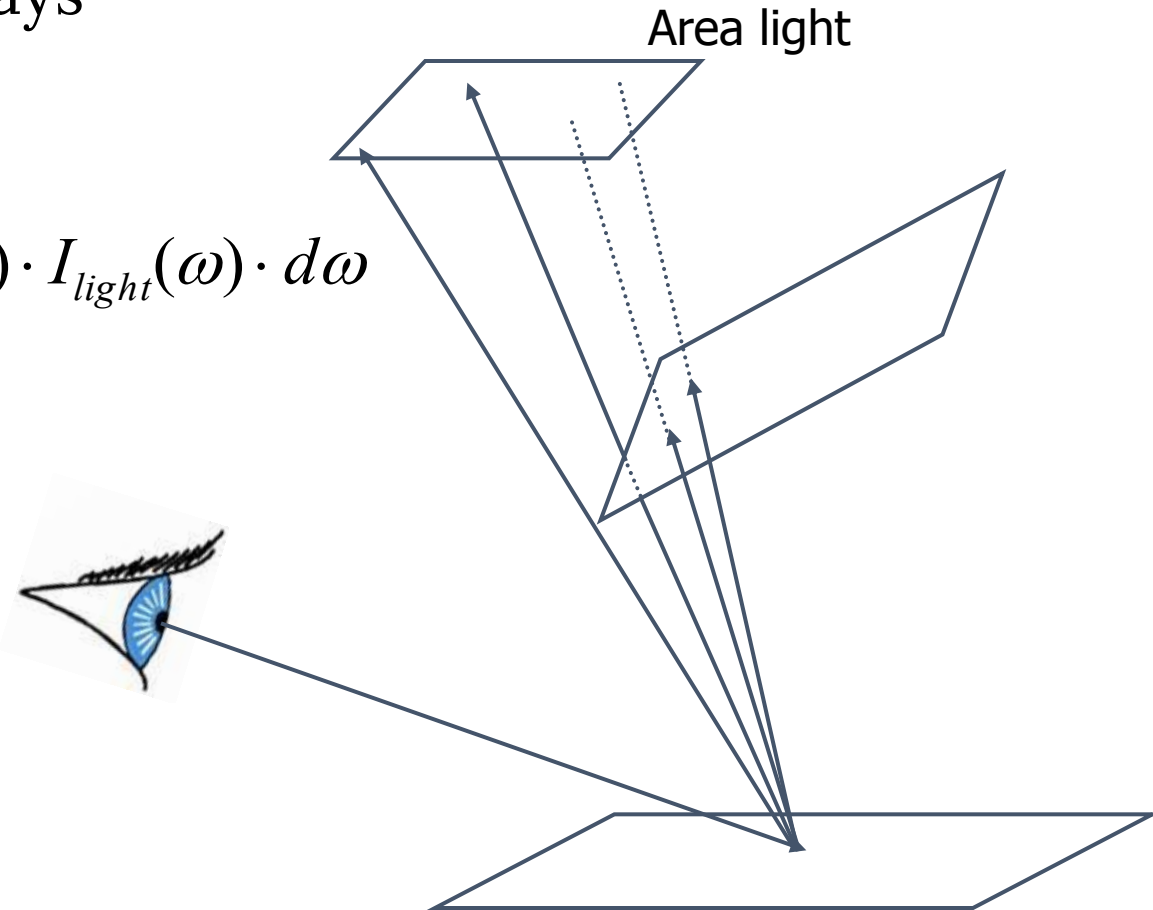


# AREA LIGHT SOURCES

- Area Lights:
  - Infinitely many light rays
  - Need to integrate over all of them:

$$I_{reflected} = \int_{\text{light directions}} \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

- Lighting model visibility and light intensity can now be different for every ray!



# INTEGRATING OVER LIGHT SOURCE

- Rewrite the integration
  - Instead of integrating over directions

$$I_{reflected} = \int_{\substack{\text{light} \\ \text{directions}}} \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

integrate over points on the light source

$$I_{reflected}(q) = \int_{s,t} \rho(p - q) \cdot V(p - q) I_{light}(p) \cdot ds \cdot dt$$

- q point on reflecting surface
- p= F(s,t) point on the area light
- We are integrating over p

# INTEGRATION

## ■ Problem:

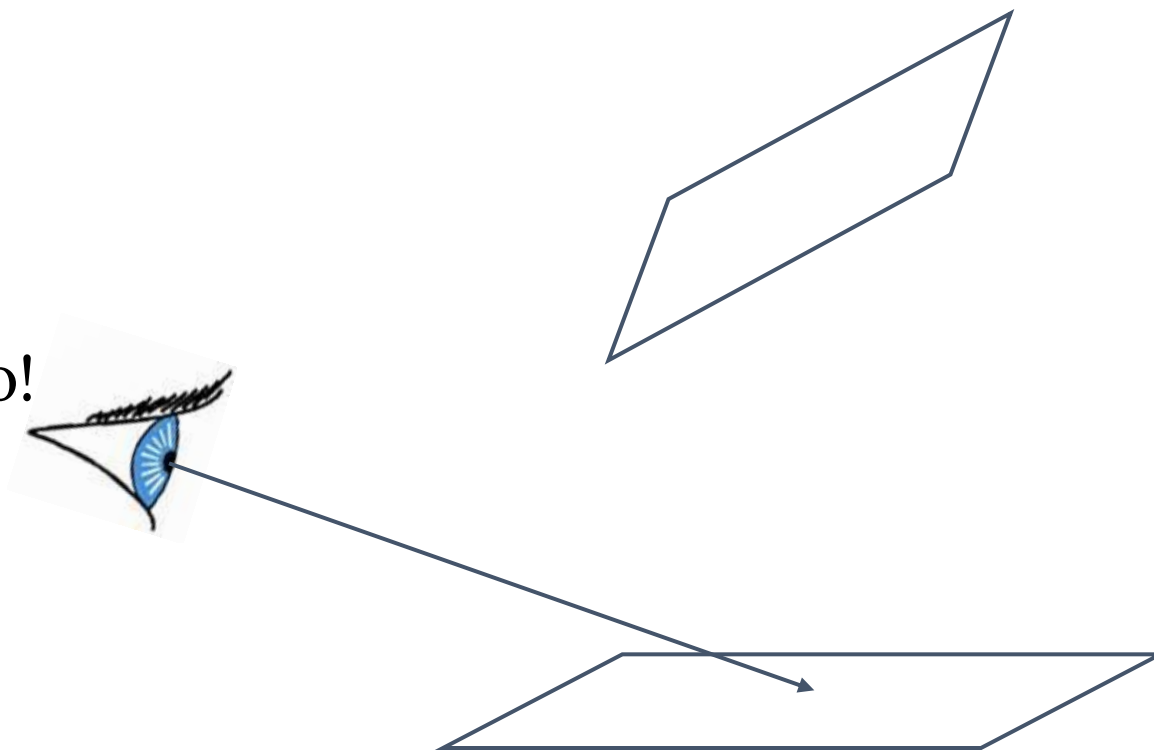
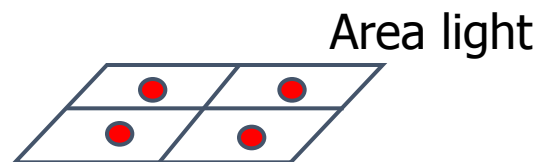
- Except for basic case **not solvable analytically!**
  - Largely due to the visibility term

## ■ So:

- Use numerical integration = approximate light with lots of point lights

# NUMERICAL INTEGRATION

- Regular grid of point lights
  - Problem: Too regular  
see 4 hard shadows
  - Need LOTS of points  
to avoid this problem
  - Solution: Monte-Carlo!



# GLOBAL ILLUMINATION ALGORITHMS

- Ray Tracing
- Path Tracing
- Photon Mapping
- Radiosity
- Metropolis light transport
- ...