

ILLUMINATION MODELS/ALGORITHMS

Local illumination - Fast
Ignore real physics, approximate the look
Interaction of each object with light
• Compute on surface (light to viewer)

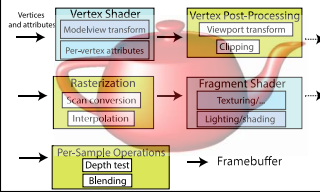


Global illumination - Slow
Physically based
Interactions between objects



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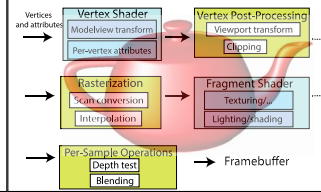


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ILLUMINATION MODELS/ALGORITHMS

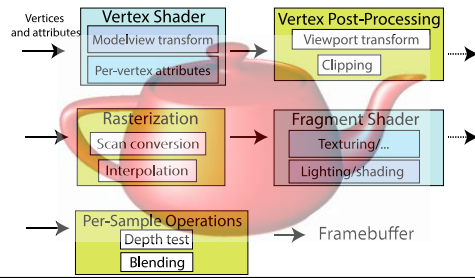
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Global illumination - Slow
Physically based
Interactions between objects



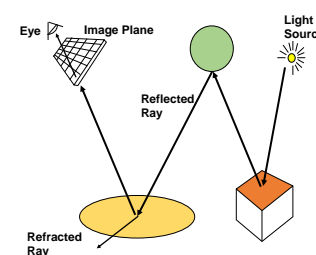
WHAT WAS NON-PHYSICAL IN LOCAL ILLUMINATION?



HOW SHOULD GLOBAL ILLUMINATION WORK?

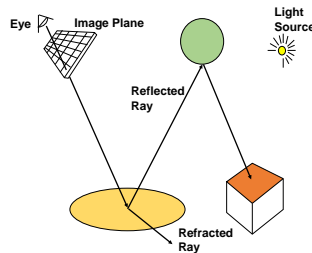
HOW SHOULD GLOBAL ILLUMINATION WORK?

- Simulate light
 - As it is emitted from light sources
 - As it bounces off objects / get absorbed / refracted
 - As some of the rays hit the camera

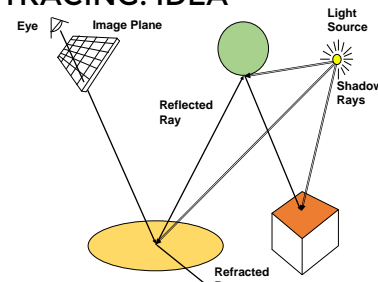


PROBLEM?

RAY TRACING: IDEA



RAY TRACING: IDEA

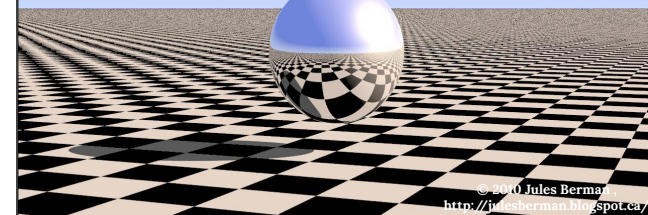


RAY TRACING

- Invert the direction of rays!
- Shoot rays from CAMERA through each pixel
 - "Trace the rays back"
- Simulate whatever the light rays do:
 - Reflection
 - Refraction
 - ...
- Each interaction of the ray with an object adds to the final color
- Those rays are never gonna hit the light source, so
 - Shoot "shadow rays" to compute direct illumination

REFLECTION

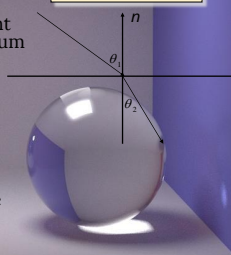
- Mirror effects
 - Perfect specular reflection



REFRACTION

Snell's Law
 $c_2 \sin \theta_1 = c_1 \sin \theta_2$

- Interface between transparent object and surrounding medium
 - E.g. glass/air boundary



- Light ray breaks (changes direction) based on refractive indices c_1, c_2

BASIC RAY-TRACING ALGORITHM

```
RayTrace(r,scene)
obj = FirstIntersection(r,scene)
if (no obj) return BackgroundColor;
else {
  if (Reflect(obj))
    reflect_color = RayTrace(ReflectRay(r,obj));
  else
    reflect_color = Black;
  if (Transparent(obj))
    refract_color = RayTrace(RefractRay(r,obj));
  else
    refract_color = Black;
  return Shade(reflect_color, refract_color, obj);
}
```

WHEN TO STOP?

- Algorithm above does not terminate
- Termination Criteria
 - No intersection
 - Contribution of secondary ray attenuated below threshold - each reflection/refraction attenuates ray
 - Maximal depth is reached

SUB-ROUTINES

- ReflectRay(r,obj) - computes reflected ray (use obj normal at intersection)
- RefractRay(r,obj) - computes refracted ray
 - Note: ray is inside obj
- Shade(reflect_color,refract_color,obj) - compute illumination given three components

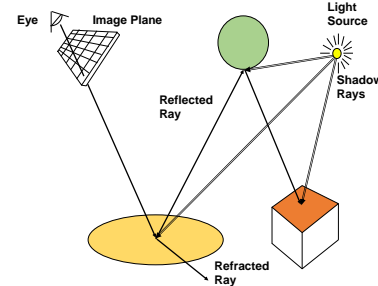
SIMULATING SHADOWS

- Trace ray from each ray-object intersection point to light sources
 - If the ray intersects an object in between \Rightarrow point is shadowed from the light source

```
shadow = RayTrace(LightRay(obj,r,light));

return Shade(shadow,reflect_color,refract_color,obj);
```

RAY TRACING: IDEA

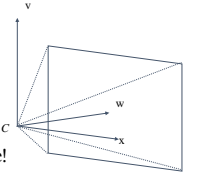


RAY-TRACING: PRACTICALITIES

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Speed: Reducing number of intersection tests
 - E.g. use BSP trees or other types of space partitioning

RAY-TRACING: GENERATION OF RAYS

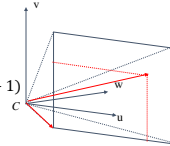
- Camera Coordinate System
 - Origin: C (camera position)
 - Viewing direction: w
 - Up vector: v
 - u direction: $u = w \times v$



- Corresponds to viewing transformation in rendering pipeline!

RAY-TRACING: GENERATION OF RAYS

- Distance to image plane: d
- Image resolution (in pixels): N_x, N_y
- Image plane dimensions: l, r, t, b
- Pixel at position i, j ($i = 0, \dots, N_x - 1; j = 0, \dots, N_y - 1$)



$$O = C + d\vec{w} + l\vec{u} + t\vec{v}$$

$$P_{i,j} = O + (i + 0.5) \cdot \frac{r-l}{N_x} \cdot \vec{u} - (j + 0.5) \cdot \frac{t-b}{N_y} \cdot \vec{v}$$

$$= O + (i + 0.5) \cdot \Delta u \cdot \vec{u} - (j + 0.5) \cdot \Delta v \cdot \vec{v}$$

RAY-TRACING: GENERATION OF RAYS

- Parametric equation of a ray:

$$R_{i,j}(t) = C + t \cdot (P_{i,j} - C) = C + t \cdot \mathbf{v}_{i,j}$$

where $t = 0 \dots \infty$

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RAY-OBJECT INTERSECTIONS

- In OpenGL pipeline, we were limited to discrete objects:
 - Triangle meshes
- In ray tracing, we can support analytic surfaces!
 - No problem with interpolating z and normals, # of triangles, etc.
 - Almost

RAY-OBJECT INTERSECTIONS

- Core of ray-tracing \Rightarrow must be extremely efficient
- Usually involves solving a set of equations
 - Using implicit formulas for primitives

Example: Ray-Sphere intersection

ray: $x(t) = p_x + v_x t, y(t) = p_y + v_y t, z(t) = p_z + v_z t$

(unit) sphere: $x^2 + y^2 + z^2 = 1$

quadratic equation in t :

$$0 = (p_x + v_x t)^2 + (p_y + v_y t)^2 + (p_z + v_z t)^2 - 1$$

$$= t^2 (v_x^2 + v_y^2 + v_z^2) + 2t(p_x v_x + p_y v_y + p_z v_z) + (p_x^2 + p_y^2 + p_z^2) - 1$$



RAY INTERSECTIONS WITH OTHER PRIMITIVES

- Implicit functions:
 - Spheres at arbitrary positions
 - Same thing
 - Conic sections (hyperboloids, ellipsoids, paraboloids, cones, cylinders)
 - Same thing (all are quadratic functions!)
 - Higher order functions (e.g. tori and other quartic functions)
 - In principle the same
 - But root-finding difficult
 - Numerical methods

RAY INTERSECTIONS WITH OTHER PRIMITIVES

- Polygons:
 - First intersect ray with plane
 - linear implicit function
 - Then test whether point is inside or outside of polygon (2D test)
- For convex polygons
 - Suffices to test whether point is on the right side of every boundary edge

RAY-TRACING: PRACTICALITIES

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RAY-TRACING: TRANSFORMATIONS

- Note: rays replace perspective transformation
- Geometric Transformations:
 - Similar goal as in rendering pipeline:
 - Modeling scenes convenient using different coordinate systems for individual objects
 - Problem:
 - Not all object representations are easy to transform
 - This problem is fixed in rendering pipeline by restriction to polygons (affine invariance!)

RAY-TRACING: TRANSFORMATIONS

- Ray Transformation:
 - For intersection test, it is only important that ray is in same coordinate system as object representation
 - Transform all rays into object coordinates
 - Transform camera point and ray direction by inverse of model/view matrix
 - Shading has to be done in world coordinates (where light sources are given)
 - Transform object space intersection point to world coordinates
 - Thus have to keep both world and object-space ray

RAY-TRACING: PRACTICALITIES

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RAY-TRACING: DIRECT ILLUMINATION

- Light sources:
 - For the moment: point and directional lights
 - More complex lights are possible
 - Area lights
 - Fluorescence

RAY-TRACING: DIRECT ILLUMINATION

- Local surface information (normal...)

- For implicit surfaces $F(x,y,z)=0$:
normal $\mathbf{n}(x,y,z)$ is gradient of F:

$$\mathbf{n}(x,y,z) = \nabla F(x,y,z) = \begin{pmatrix} \partial F(x,y,z)/\partial x \\ \partial F(x,y,z)/\partial y \\ \partial F(x,y,z)/\partial z \end{pmatrix}$$

- Example:

$$F(x,y,z) = x^2 + y^2 + z^2 - r^2$$

$$\mathbf{n}(x,y,z) = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix} \quad \text{Needs to be normalized!}$$

RAY-TRACING: DIRECT ILLUMINATION

- For triangle meshes

- Interpolate per-vertex information as in rendering pipeline
 - Phong shading!
 - Same as discussed for rendering pipeline

- Difference to rendering pipeline:

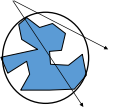
- Have to compute Barycentric coordinates for every intersection point (e.g. plane equation for triangles)

RAY-TRACING: PRACTICALITIES

- Generation of rays
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- Lighting and shading
- Speed:** Reducing number of intersection tests

OPTIMIZED RAY-TRACING

- Basic algorithm is simple but VERY expensive
- Optimize...
 - Reduce number of rays traced
 - Reduce number of ray-object intersection calculations
- Parallelize
 - Cluster
 - GPU
- Methods
 - Bounding Boxes
 - Spatial Subdivision
 - Visibility, Intersection/Collision
 - Tree Pruning

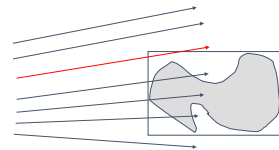


SPATIAL SUBDIVISION DATA STRUCTURES

- Goal: reduce number of intersection tests per ray
- Lots of different approaches:
 - (Hierarchical) bounding volumes
 - Hierarchical space subdivision
 - Ocree, k-D tree, BSP tree

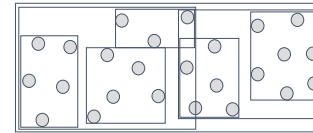
BOUNDING VOLUMES: IDEA

- Don't test each ray against complex objects (e.g. triangle mesh)
- Do a quick conservative test first which eliminates most rays
 - Surround complex object by simple, easy to test geometry (e.g. sphere or axis-aligned box)
 - Reduce false positives: make bounding volume as tight as possible!



HIERARCHICAL BOUNDING VOLUMES

- Extension of previous idea:
 - Use bounding volumes for groups of objects



SPATIAL SUBDIVISION DATA STRUCTURES

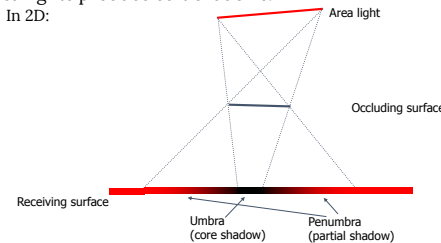
- Bounding Volumes:
 - Find simple object completely enclosing complicated objects
 - Boxes, spheres
 - Hierarchically combine into larger bounding volumes
- Spatial subdivision data structure:
 - Partition the whole space into cells
 - Grids, octrees, (BSP trees)
 - Simplifies and accelerates traversal
 - Performance less dependent on order in which objects are inserted

SOFT SHADOWS: AREA LIGHT SOURCES

- So far:
 - All lights were either point-shaped or directional
 - Both for ray-tracing and the rendering pipeline
 - Thus, at every point, we only need to compute lighting formula and shadowing for **ONE** direction per light
- In reality:
 - All lights have a finite area
 - Instead of just dealing with one direction, we now have to **integrate** over all directions that go to the light source

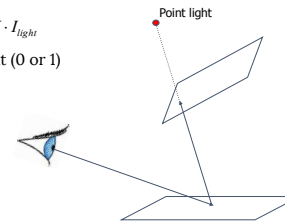
AREA LIGHT SOURCES

- Area lights produce soft shadows:
 - In 2D:



AREA LIGHT SOURCES

- Point lights:
 - Only one light direction:
- $$I_{reflected} = \rho \cdot V \cdot I_{light}$$
- V is visibility of light (0 or 1)
 - ρ is lighting model (e.g. diffuse or Phong)

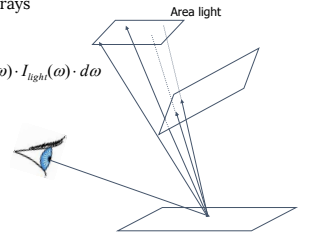


AREA LIGHT SOURCES

- Area Lights:
 - Infinitely many light rays
 - Need to integrate over all of them:

$$I_{reflected} = \int \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

- Lighting model visibility and light intensity can now be different for every ray!



INTEGRATING OVER LIGHT SOURCE

- Rewrite the integration
 - Instead of integrating over directions

$$I_{reflected} = \int \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

integrate over points on the light source

$$I_{reflected}(q) = \int \rho(p-q) \cdot V(p-q) \cdot I_{light}(p) \cdot ds \cdot dt$$

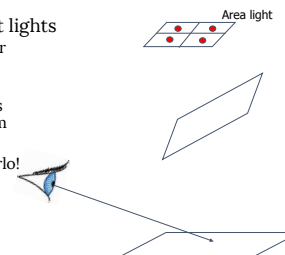
- q point on reflecting surface
- p = F(s,t) point on the area light
- We are integrating over p

INTEGRATION

- Problem:
 - Except for basic case **not solvable analytically!**
 - Largely due to the visibility term
- So:
 - Use numerical integration = approximate light with lots of point lights

NUMERICAL INTEGRATION

- Regular grid of point lights
 - Problem: Too regular see 4 hard shadows
- Need LOTS of points to avoid this problem
- Solution: Monte-Carlo!



GLOBAL ILLUMINATION ALGORITHMS

- Ray Tracing
- Path Tracing
- Photon Mapping
- Radiosity
- Metropolis light transport
- ...