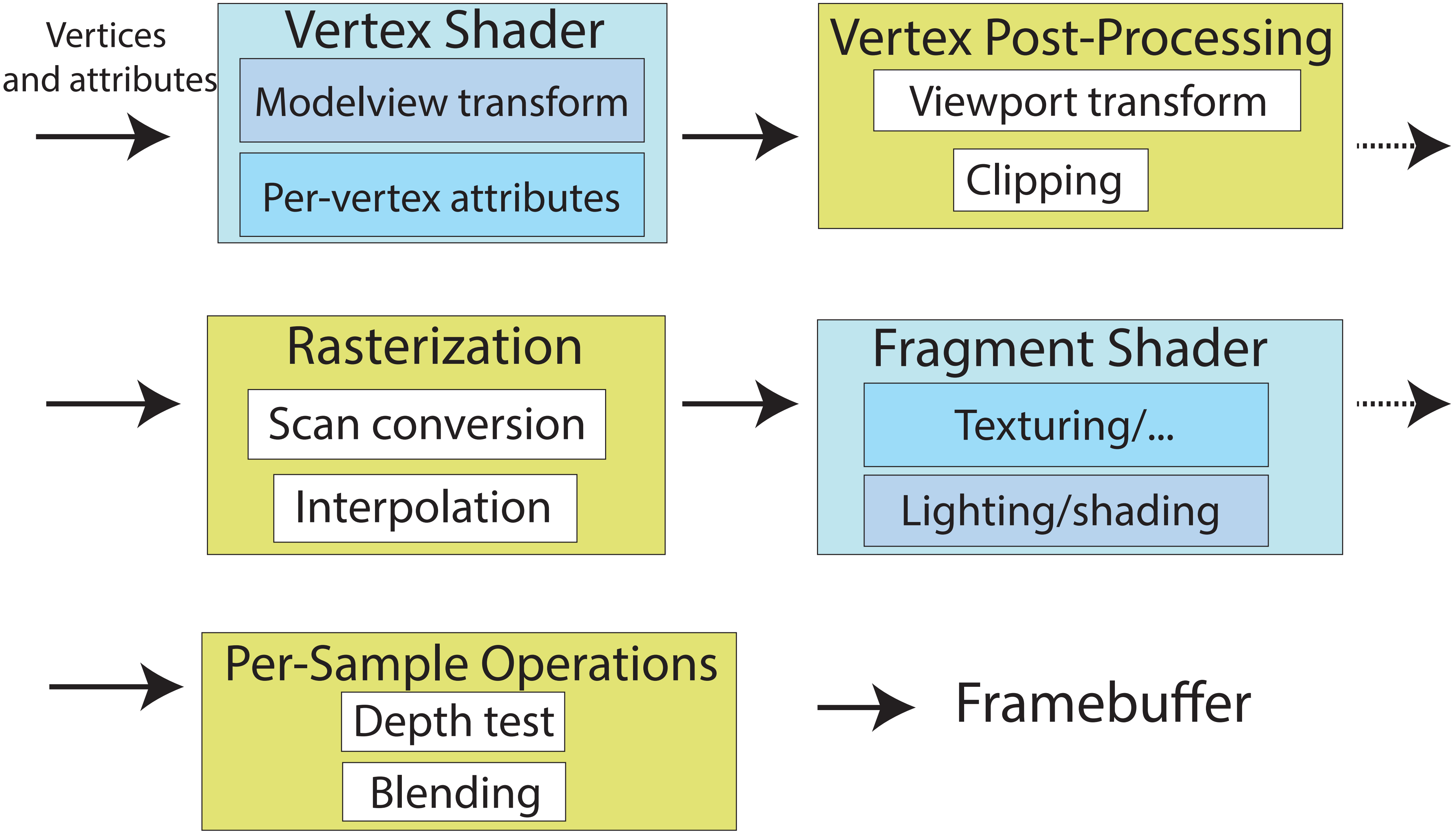
CPSC 314
RASTERIZATION

UGRAD.CS.UBC.CA/~CS314

slides: Mikhail Bessmeltsev

THE RENDERING PIPELINE



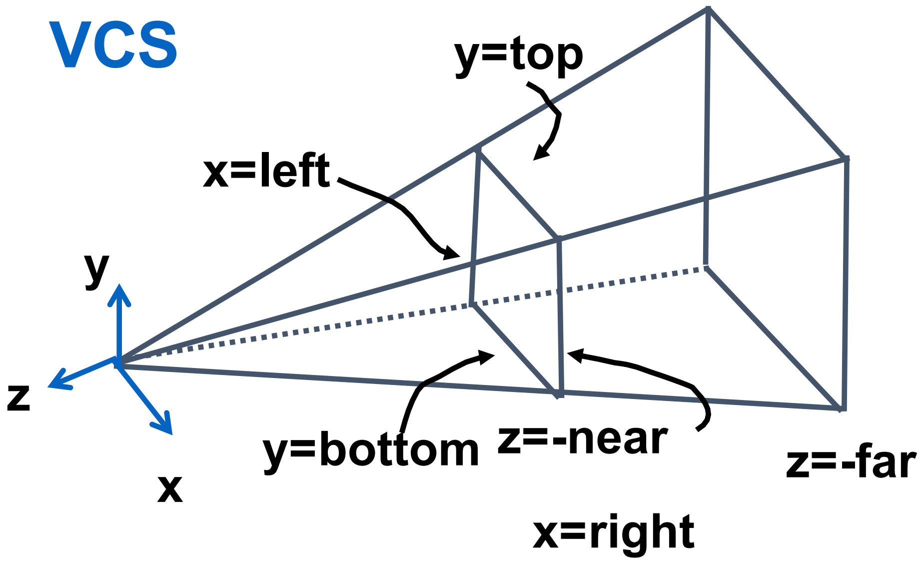
VIEWPORT MATRIX

- We need a transform that maps the lower left corner to $[-0.5, -0.5]^t$ and upper right corner to $[W-0.5, H-0.5]^t$
- The appropriate scale and shift can be done using the viewport matrix:

$$\begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} = \begin{bmatrix} W/2 & 0 & 0 & (W-1)/2 \\ 0 & H/2 & 0 & (H-1)/2 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix}$$

CLIPPING

- We have to clip what's outside our view volume
- Outside to the left/right, top/bottom
- More importantly, front/near:



CLIPPING

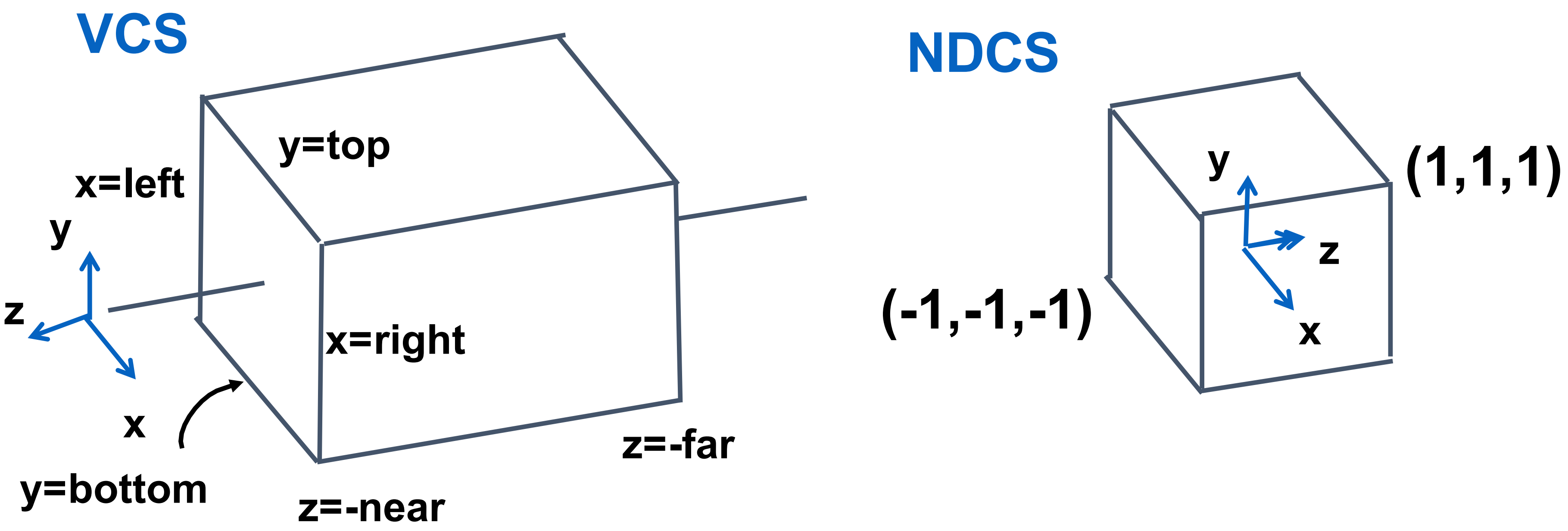
- Where to do it in pipeline?

CLIPPING

- Option 1: Before projection
- Option 2: After NDCS
- Option 3: In between?

UNDERSTANDING Z

- z axis flip changes coord system handedness
  - RHS before projection (eye/view coords)
  - LHS after projection (clip, norm device coords)



CLIPPING

- ~~Option 1: Before projection~~
  - Then it would have to know all the camera info
- Option 2: After NDCS
- Option 3: In between?

CLIPPING

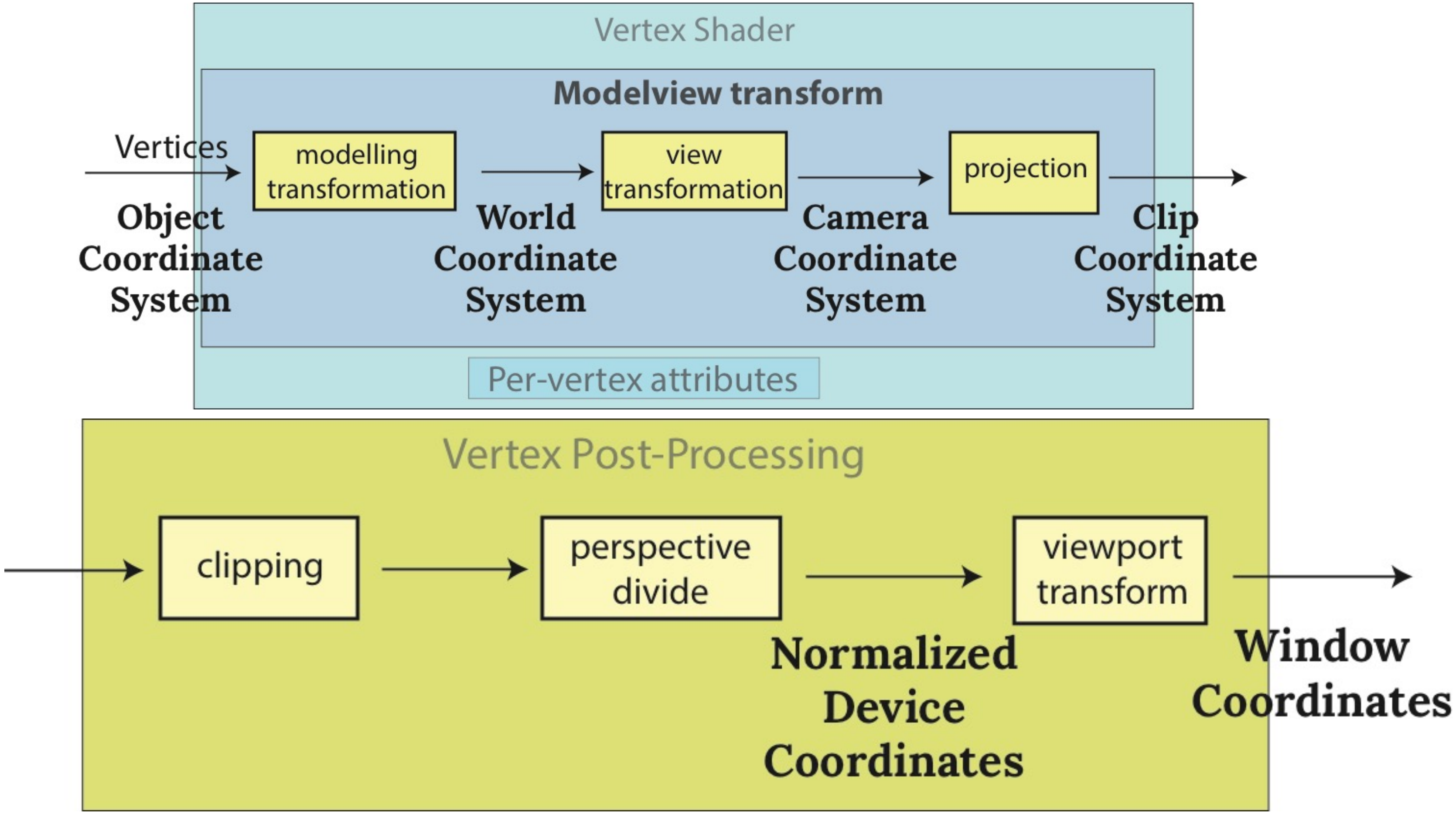
- ~~Option 1: Before projection~~
  - Then it would have to know all the camera info
- ~~Option 2: After NDCS~~
  - Flip already occurred
  - Too many calculations
- Option 3: In between?

CLIPPING

- ~~Option 1: Before projection~~
  - Then it would have to know all the camera info
- ~~Option 2: After NDCS~~
  - Flip already occurred
  - Too many calculations
- Option 3: In between?

CLIPPING

- Perform clipping in clip-coordinates!
  - After projection and before dividing by w



CLIPPING

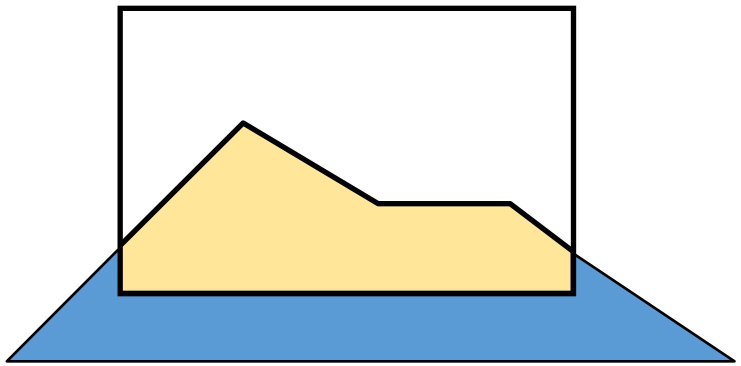
- Perform clipping in clip-coordinates!
  - After projection and before dividing by w

$$-w_c < x_c < w_c$$
$$-w_c < y_c < w_c$$
$$-w_c < z_c < w_c$$

We have not performed any divisions =>
no flip; efficiency

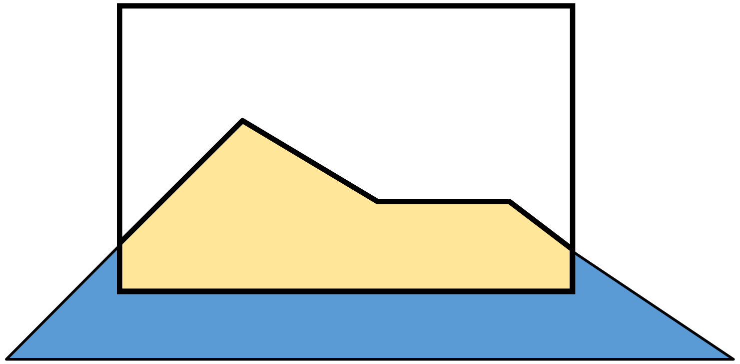
CLIPPING: UNDER THE HOOD

- Creates new vertices
- Done automatically, we won't study the actual algorithm



CLIPPING: UNDER THE HOOD

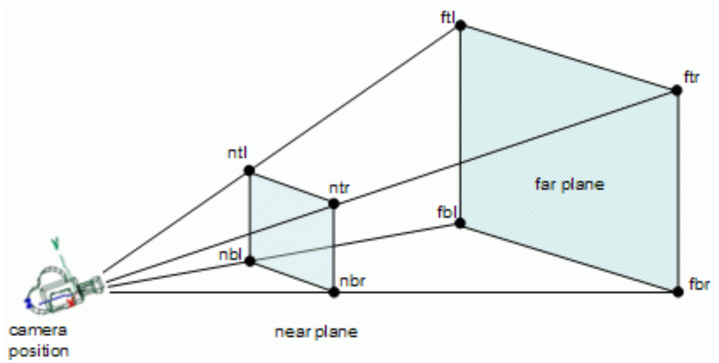
- Creates new vertices
- Done automatically, we won't study the actual algorithm
- Clip:
  - Points -> discard
  - Triangles -> clip



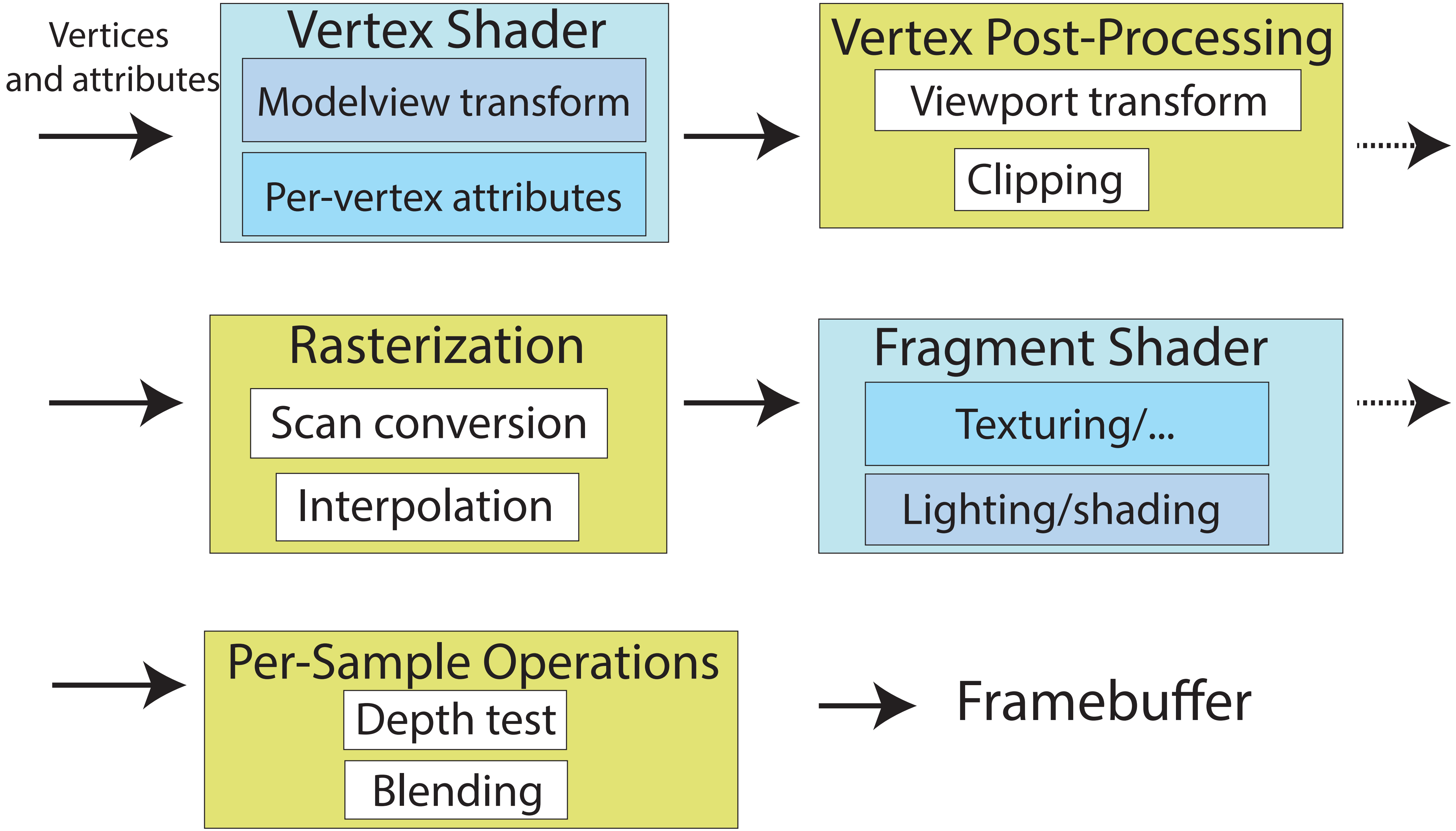
CLIPPING COORDINATES

- Eye coordinates (projected) → clip coordinates → normalized device coordinates (NDCs)
- Dividing clip coordinates (x_c, y_c, z_c, w_c) by the $w_c (w_c = w_n)$ component (the fourth component in the homogeneous coordinates) yields normalized device coordinates (NDCs).

$$\begin{bmatrix} x_n w_n \\ y_n w_n \\ z_n w_n \\ w_n \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} s_x & 0 & -c_x & 0 \\ 0 & s_y & -c_y & 0 \\ 0 & 0 & \dfrac{f+n}{f-n} & -\dfrac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix}$$



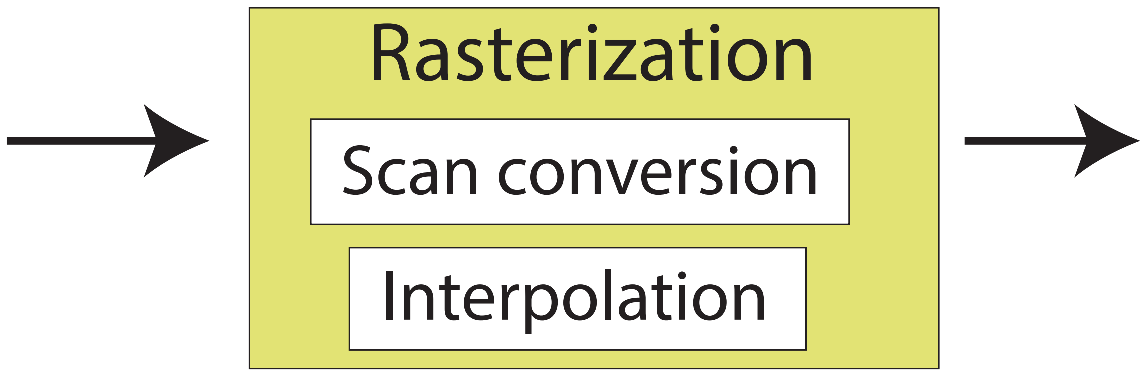
THE RENDERING PIPELINE



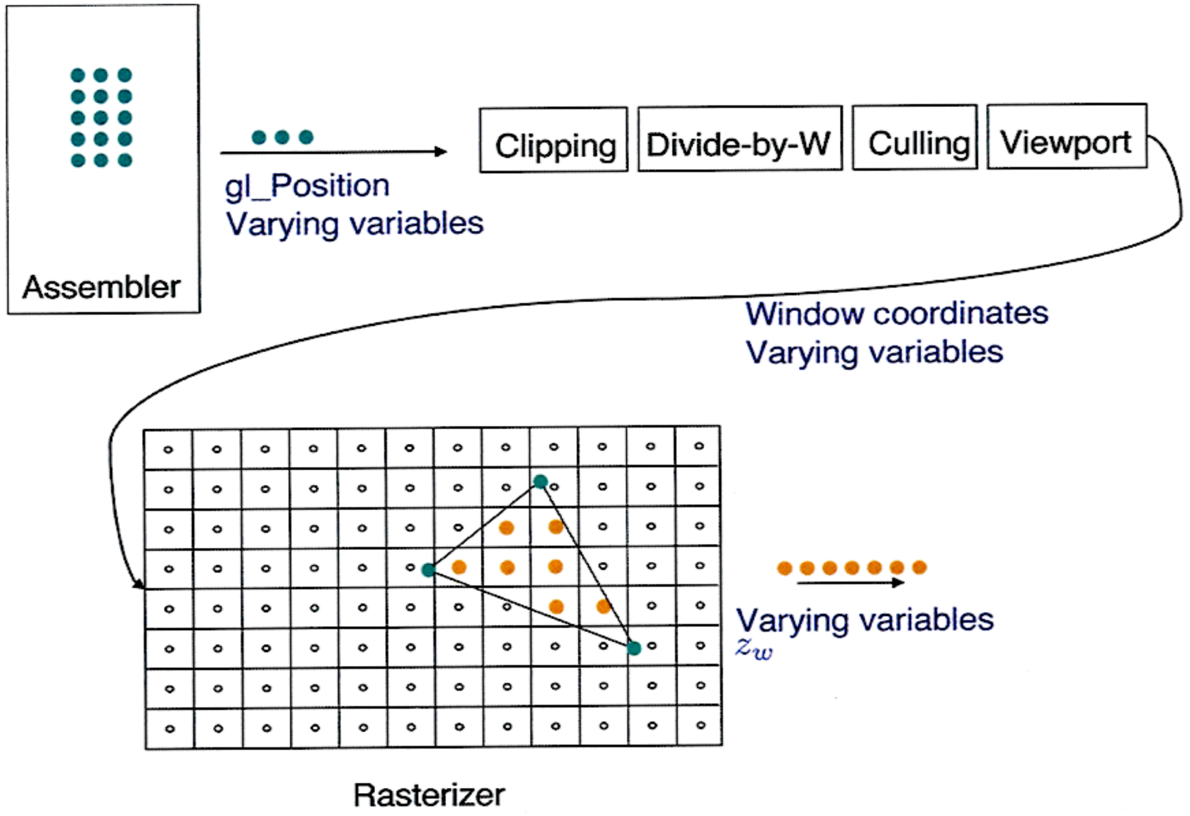
RASTERIZATION

- This is part of the fixed function pipeline

- Input: all polygons are clipped
- Output: fragments (with **varying variables** interpolated)



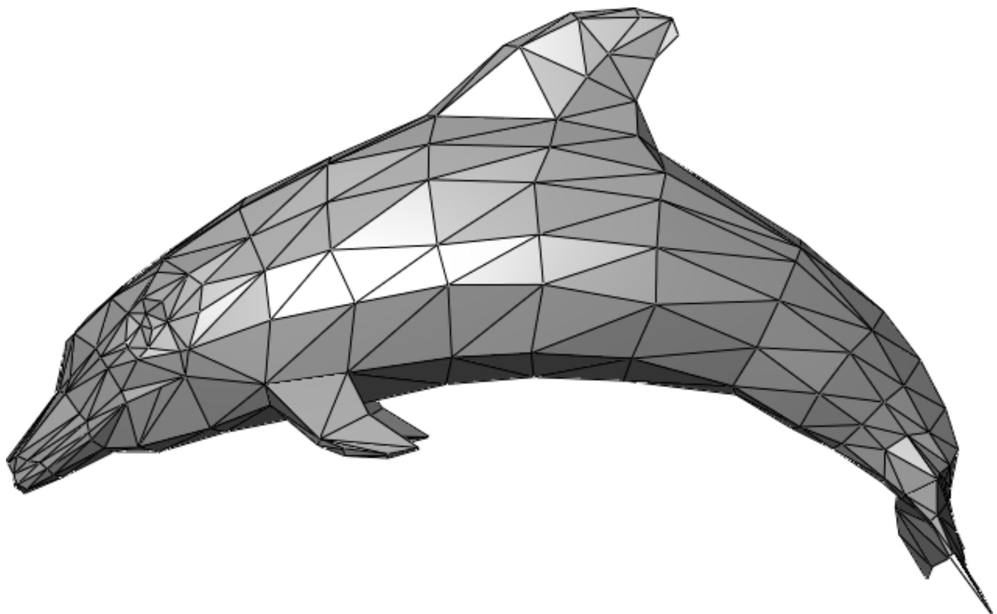
PATH FROM VERTEX TO PIXEL



POLYGONS

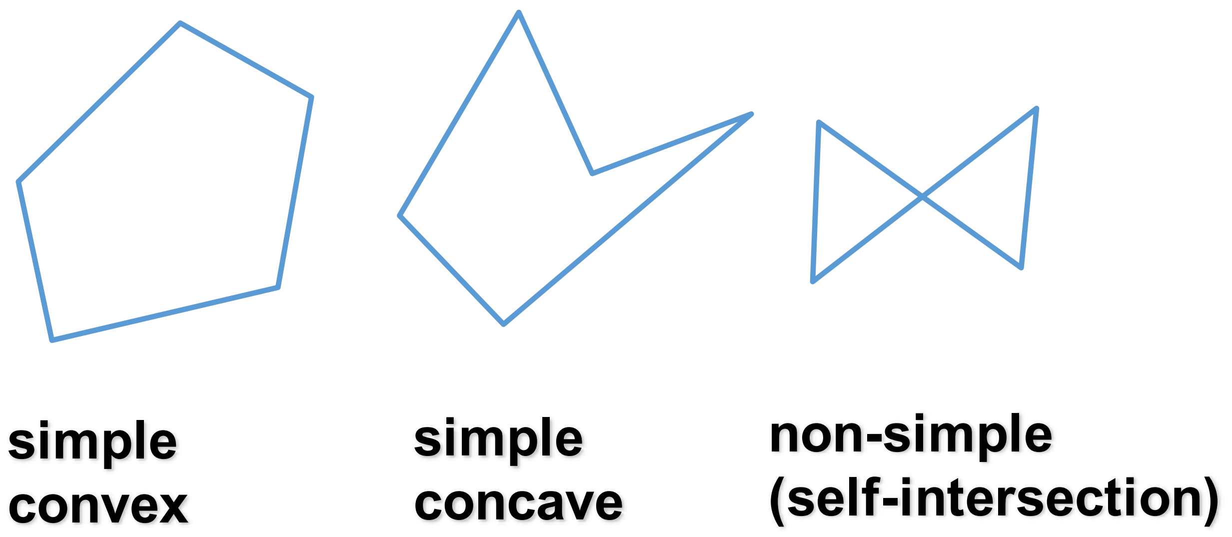
Interactive graphics uses Polygons

- Can represent any surface *with arbitrary accuracy*
  - *Splines, mathematical functions, ...*
- simple, regular rendering algorithms
  - *embed well in hardware*



POLYGONS

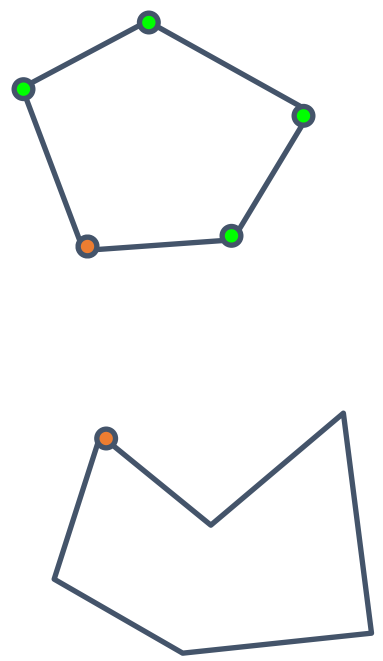
- Basic Types



FROM POLYGONS TO TRIANGLES

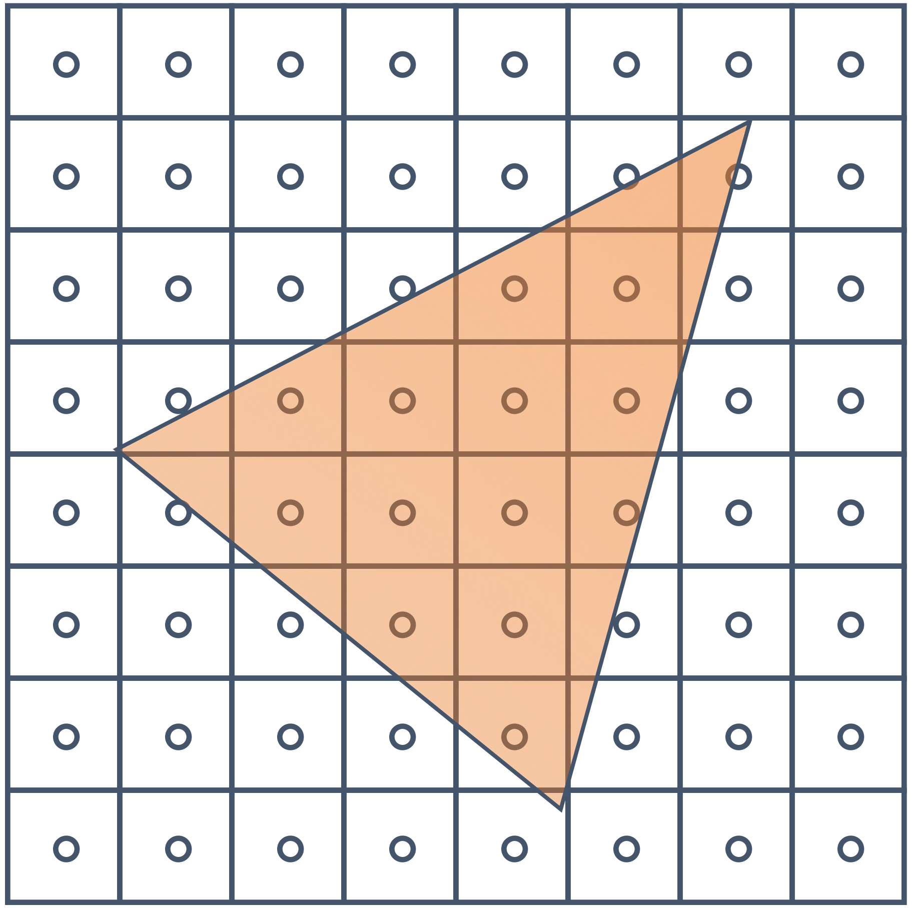
- why?   triangles are always planar, always convex

- simple convex polygons
  - trivial to break into triangles
- concave or non-simple polygons
  - more effort to break into triangles

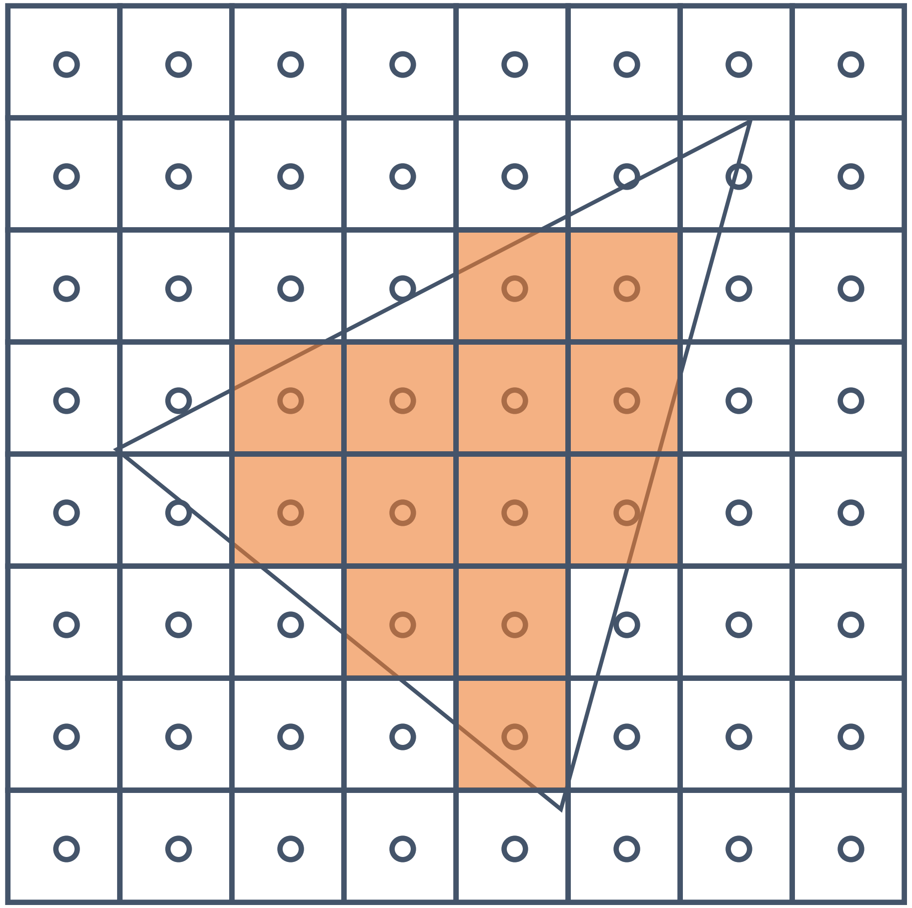


WHAT IS SCAN CONVERSION? (A.K.A. RASTERIZATION)

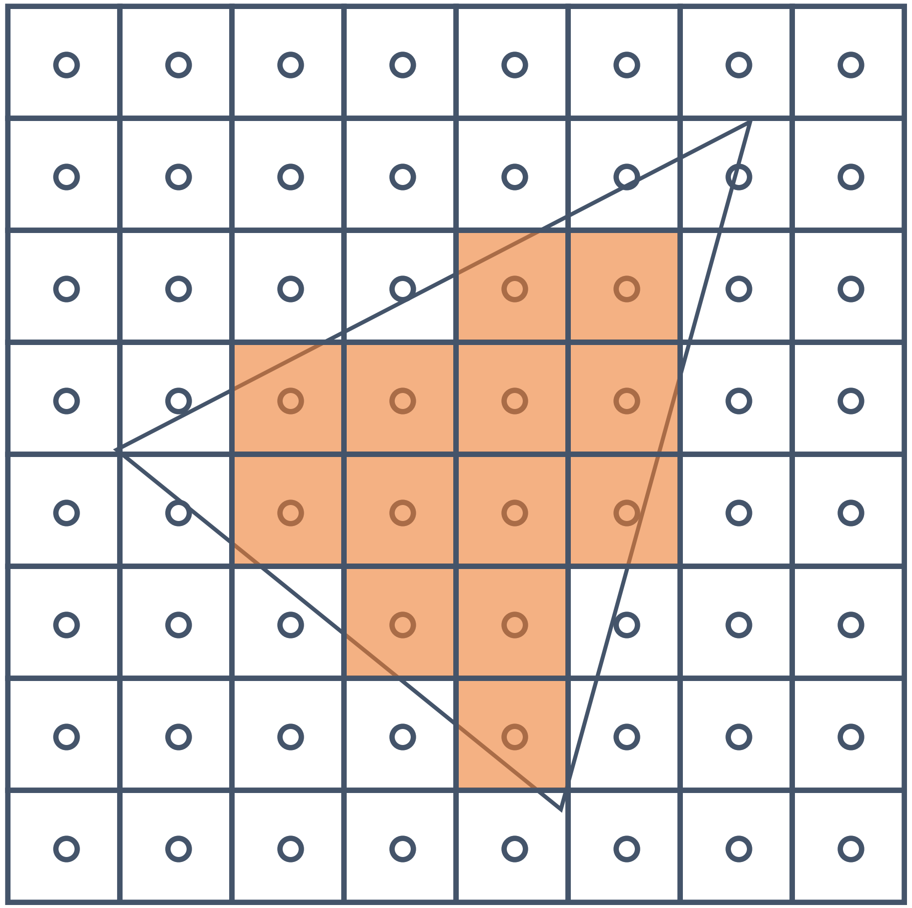
- screen is discrete



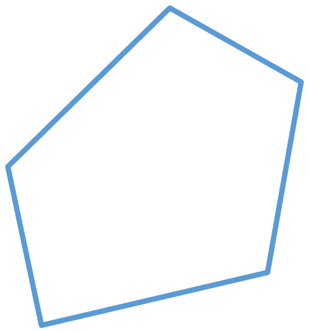
- one possible scan conversion



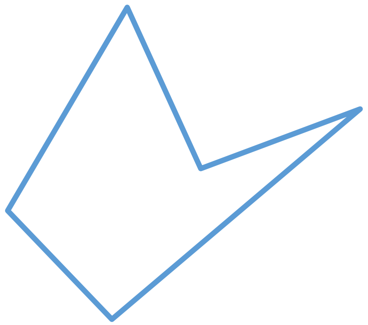
HOW TO CHECK IF A PIXEL IS INSIDE?



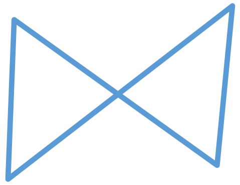
HOW TO TEST IF A POINT IS IN A POLYGON?



simple
convex



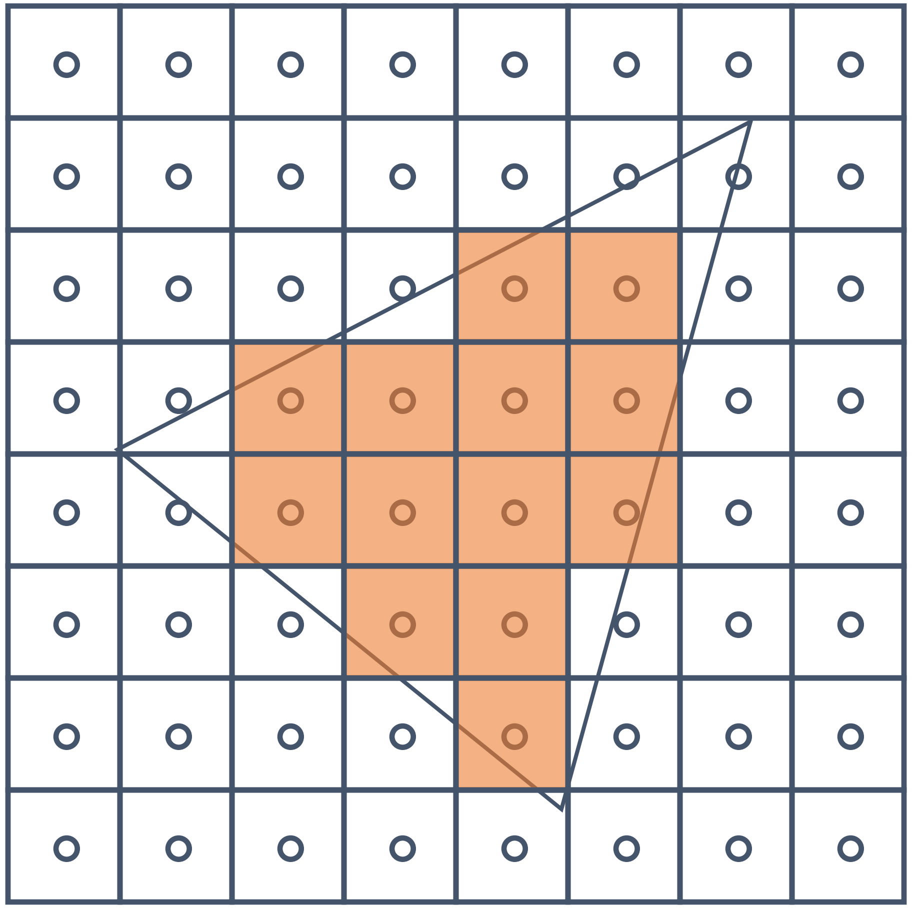
simple
concave



non-simple
(self-intersection)

HOW TO CHECK IF A PIXEL IS INSIDE?

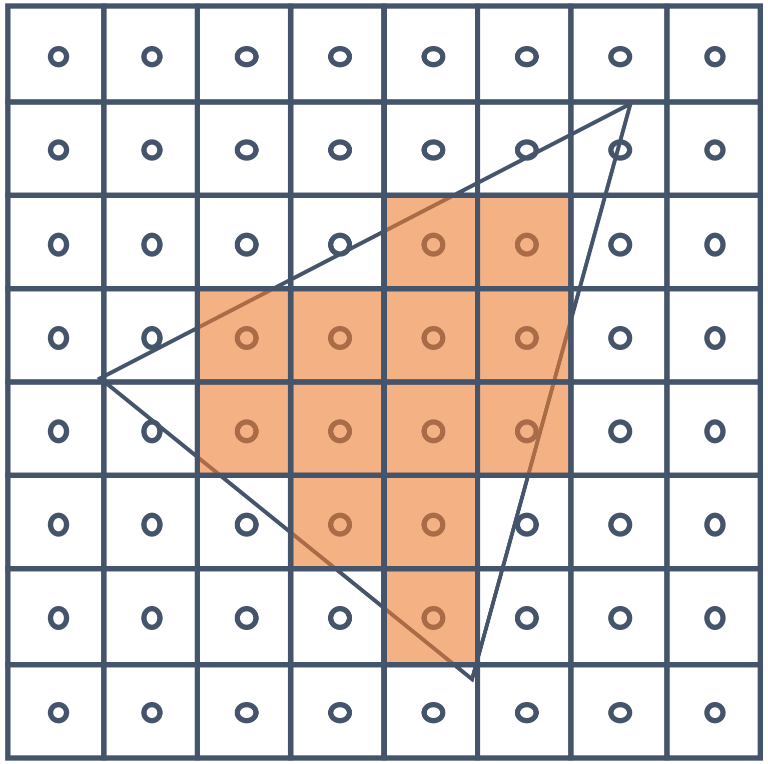
- Use implicit line equation:
  - $Ax + By + C = 0$
  - What is geometric meaning of A,B,C?
    - **(A,B) is a normal (not unit!) to the line**
    - **C is translation of that line**
- How to find A,B,C?
  - Option 1. Solve a system of 2 equations
  - Option 2. Find any normal
- Orientation?
  - Normal points in positive side



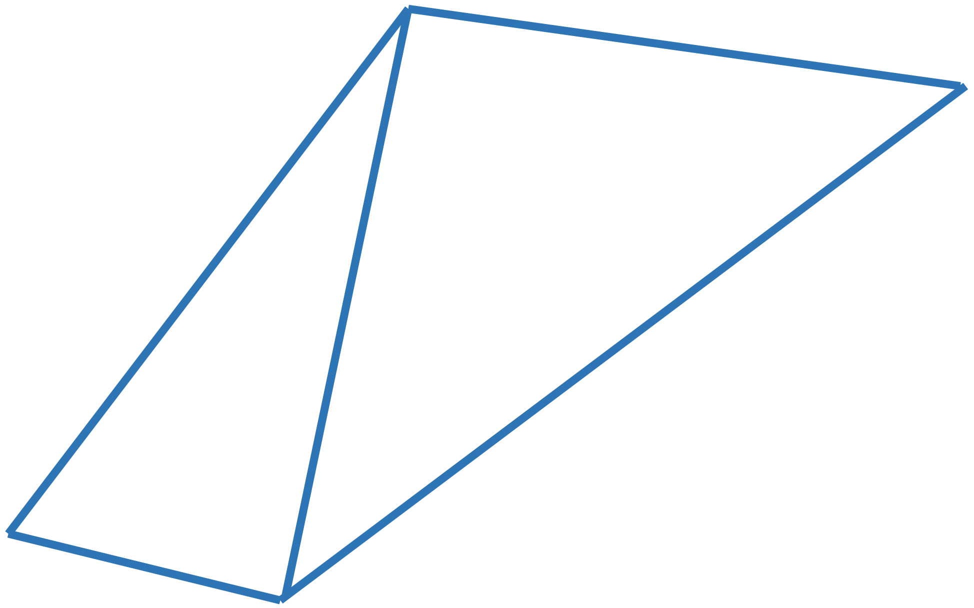
HOW TO CHECK IF A PIXEL IS INSIDE?

A point is inside ⇔

$$A_i x + B_i y + C > 0, i = 1, \ldots, 3$$



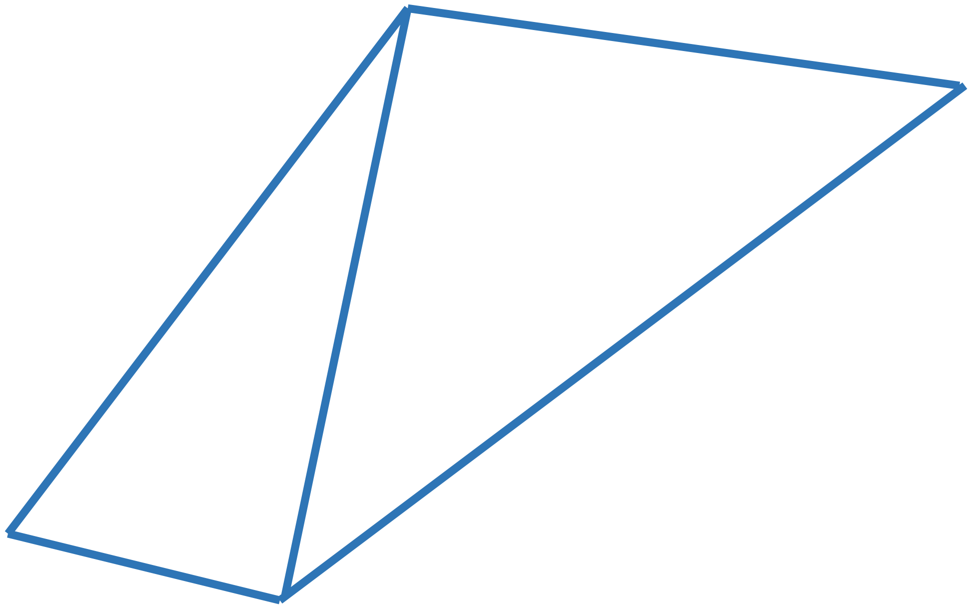
HOW TO TREAT BOUNDARY?



HOW TO TREAT BOUNDARY?

- If two triangles share an edge, scan conversion should be consistent
  - No pixel drawn twice
  - No gaps

- Strategy ideas?

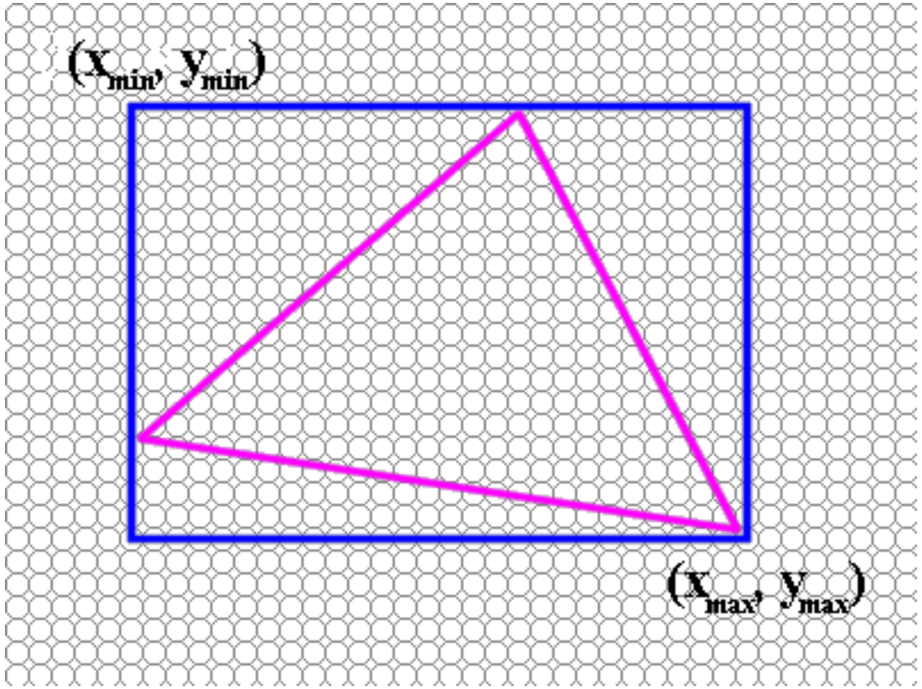


NAÏVE SCAN CONVERSION

- Testing every pixel is suboptimal
- Better ideas?

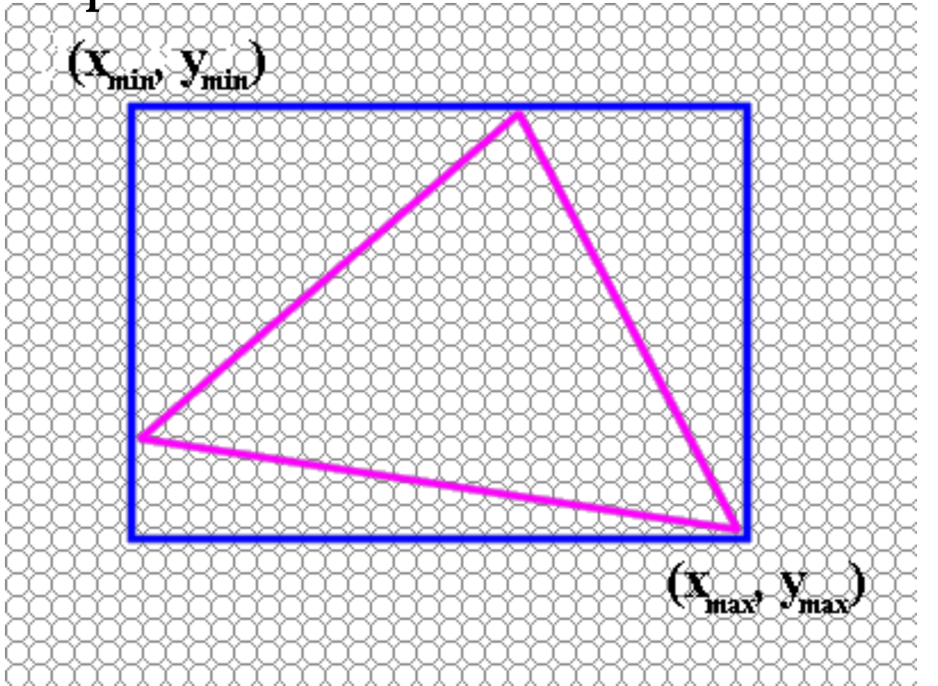
LESS NAÏVE SCAN CONVERSION

- Go over each pixel in bounding rectangle
- Check if pixel is inside/outside of triangle
  - Use sign of edge equations



SCANLINE IDEA (SIMPLIFIED)

- Basic structure of code:
  - Setup: compute edge equations, bounding box
  - (Outer loop) For each scanline in bounding box…
  - (Inner loop) …check each pixel on scanline, evaluating edge equations and drawing the pixel if all three are positive



SCANLINE: CODE

```
findBoundingBox(xmin, xmax, ymin, ymax);
setupEdges (a0,b0,c0,a1,b1,c1,a2,b2,c2);

for (int y = yMin; y <= yMax; y++) {
  for (int x = xMin; x <= xMax; x++) {
    float e0 = a0*x + b0*y + c0;
    float e1 = a1*x + b1*y + c1;
    float e2 = a2*x + b2*y + c2;
    if (e0 > 0 && e1 > 0 && e2 > 0)
        Image[x][y] = TriangleColor;
  }
}
```

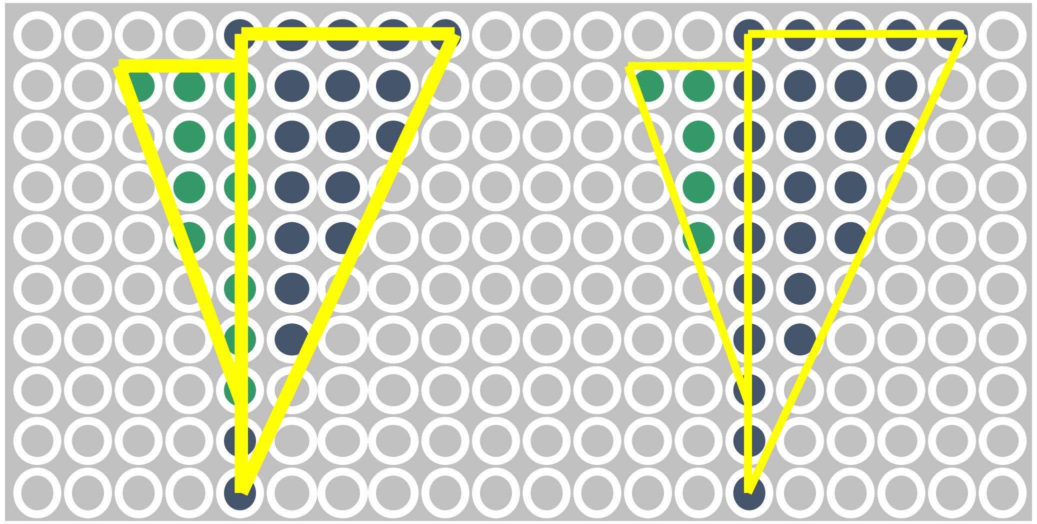
SCANLINE: OPTIMIZED CODE

```
// more efficient inner loop
for (int y = yMin; y <= yMax; y++) {
 float e0 = a0*xMin + b0*y + c0;
 float e1 = a1*xMin + b1*y + c1;
 float e2 = a2*xMin + b2*y + c2;
 for (int x = xMin; x <= xMax; x++) {
  if (e0 > 0 && e1 > 0 && e2 > 0)
    Image[x][y] = TriangleColor;

  e0 += a0;   e1+= a1;    e2 += a2;
 }
}
```

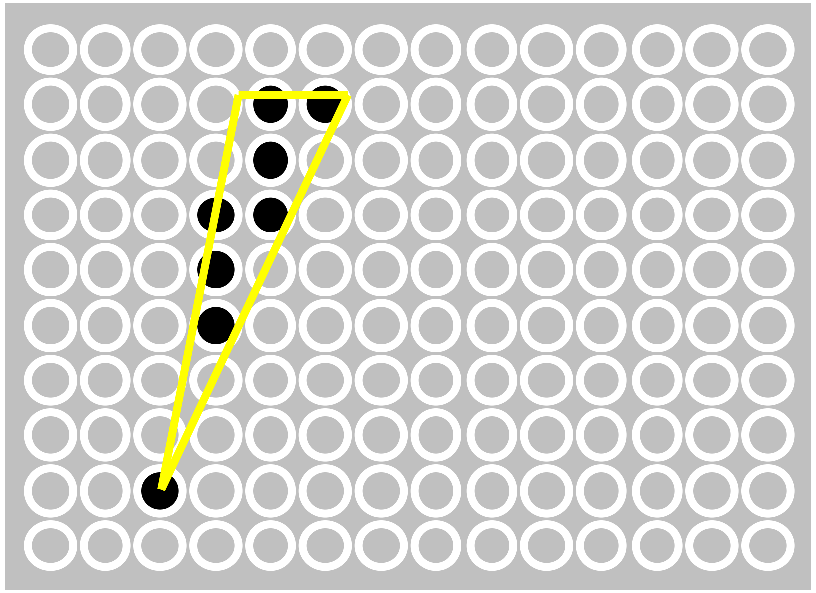
TRIANGLE RASTERIZATION ISSUES

- Exactly which pixels should be lit?
- A: Those pixels inside the triangle edges
- What about pixels exactly on the edge?

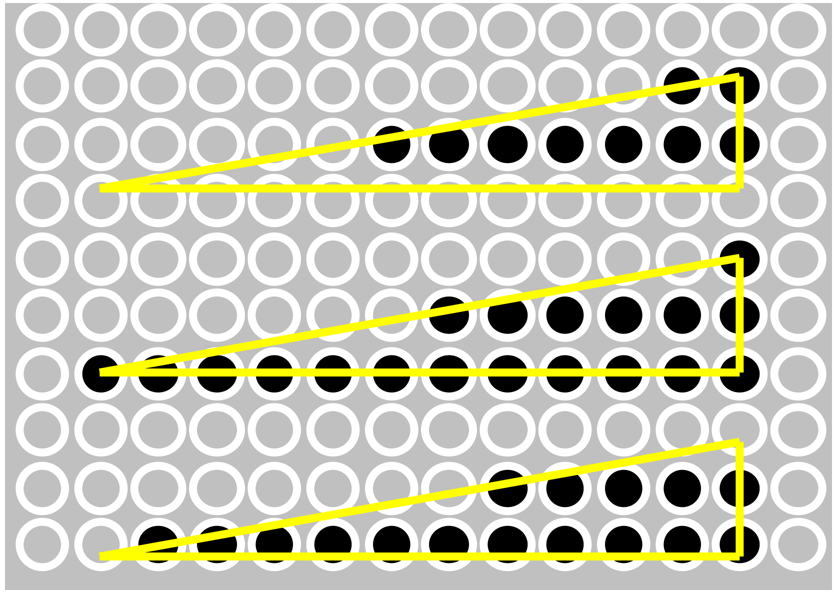


TRIANGLE RASTERIZATION ISSUES

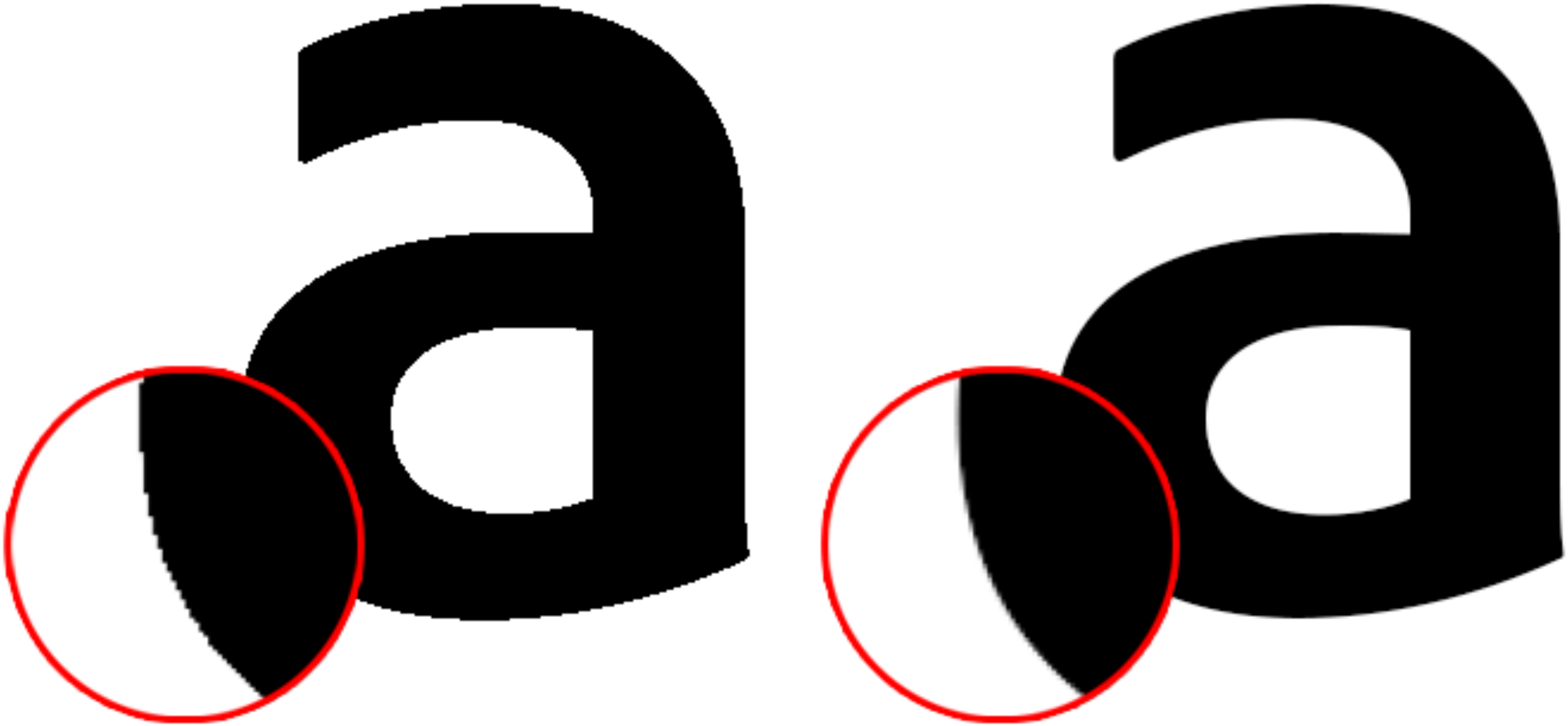
Sliver



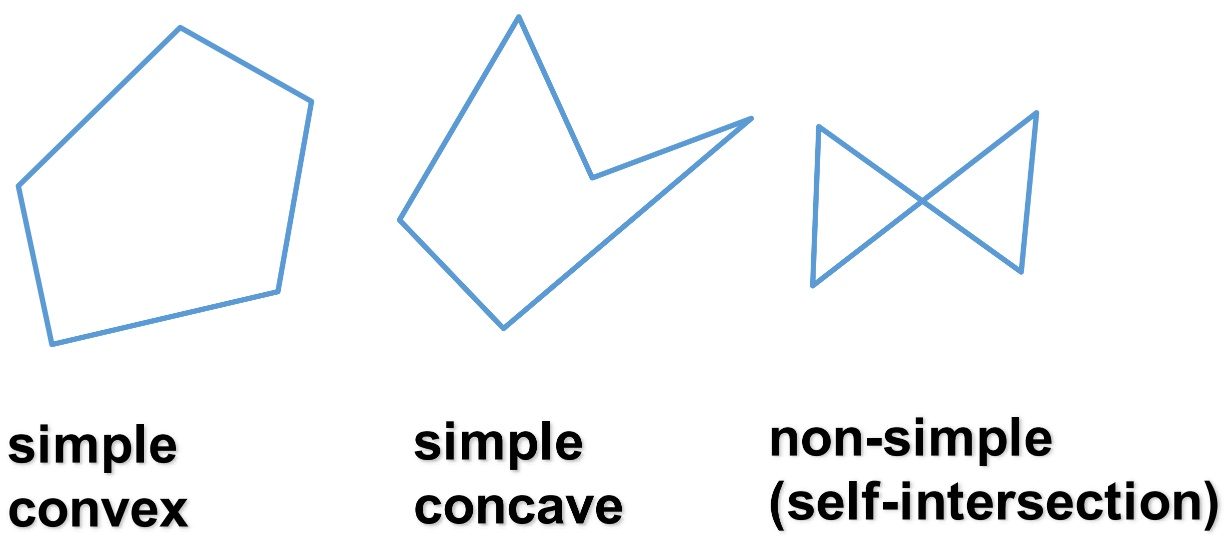
- Moving Slivers



ALIASING & ANTI-ALIASING



HOW TO TEST IF A POINT IS IN A POLYGON?

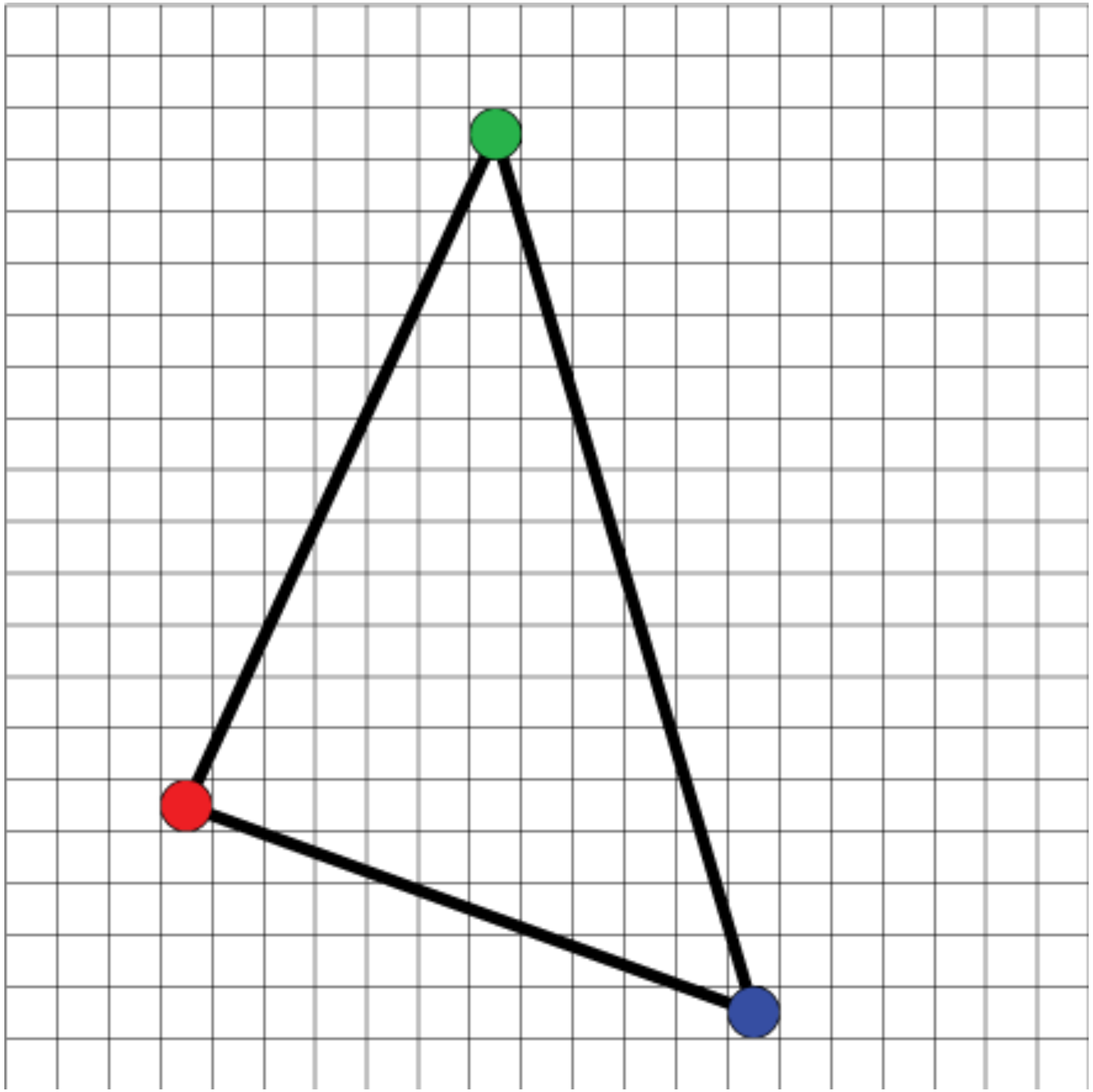


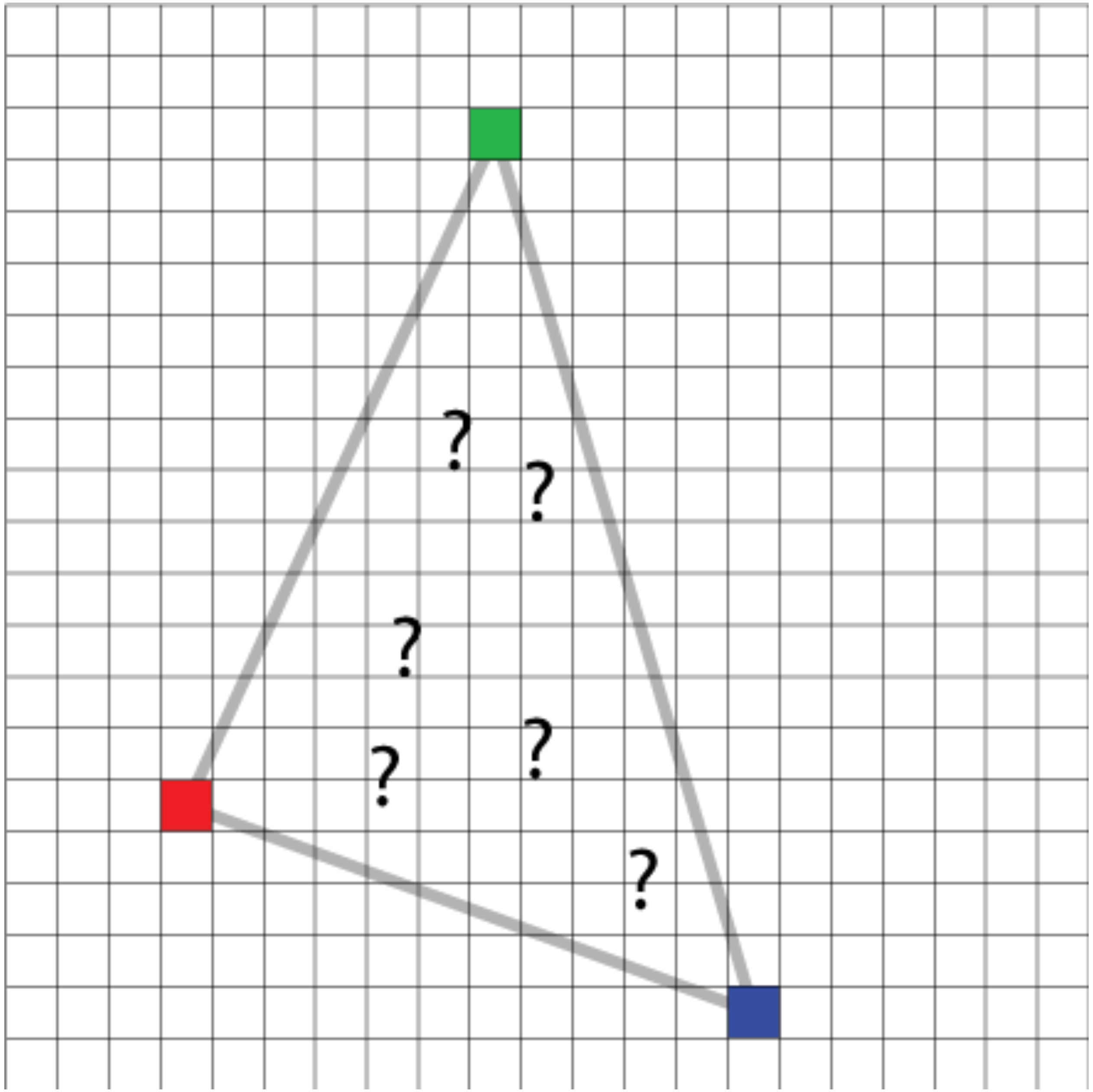
VALUES IN THE INTERIOR

Barycentric coordinates

INTERPOLATION – ACCESS TRIANGLE INTERIOR

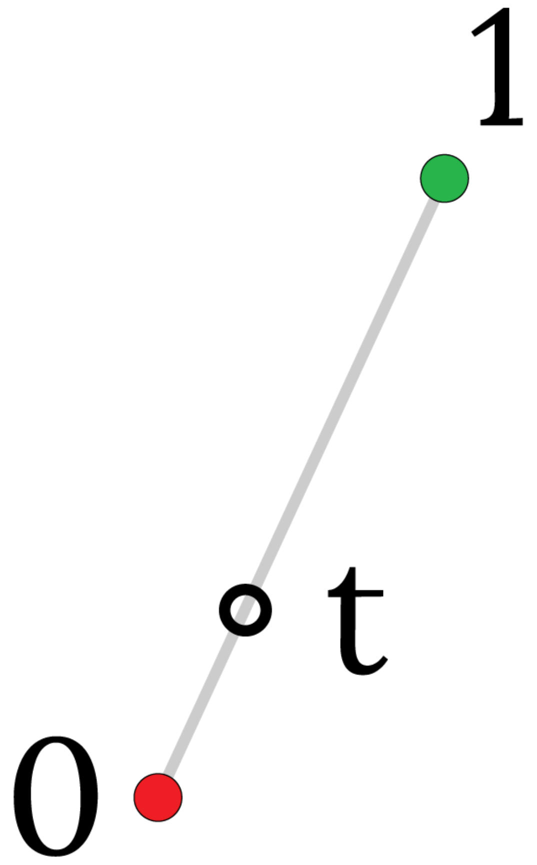
- Interpolate between vertices:
  - z
  - r,g,b - colour components
  - u,v - texture coordinates
  - N_x, N_y, N_z - surface normals
- Equivalent
  - Barycentric coordinates
  - Bilinear interpolation
  - Plane Interpolation





SIMPLER:

How to interpolate color between two points?

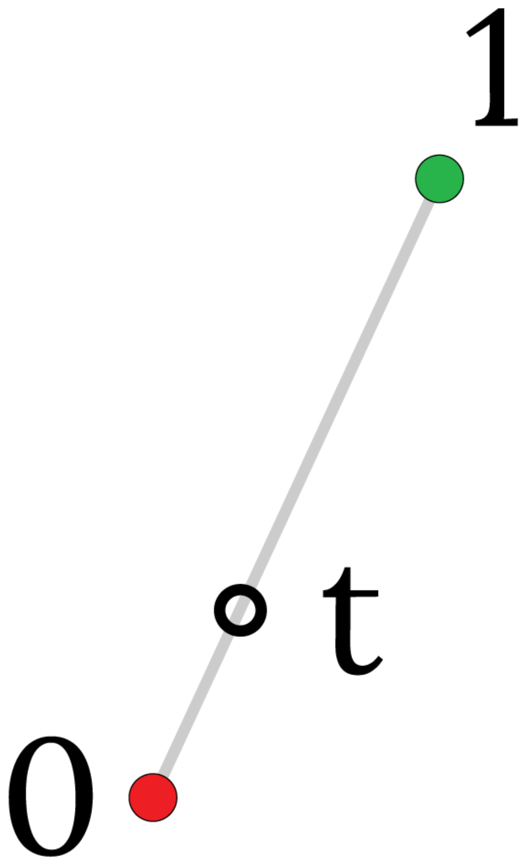


SIMPLER:

How to interpolate color between two points?

$$c(t) = c(0) \cdot (1 - t) + c(1) \cdot t$$

Linear interpolation

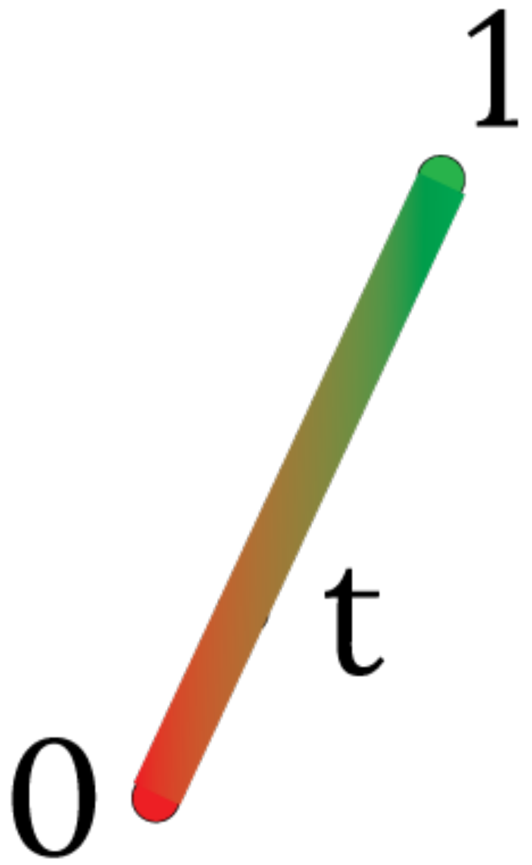


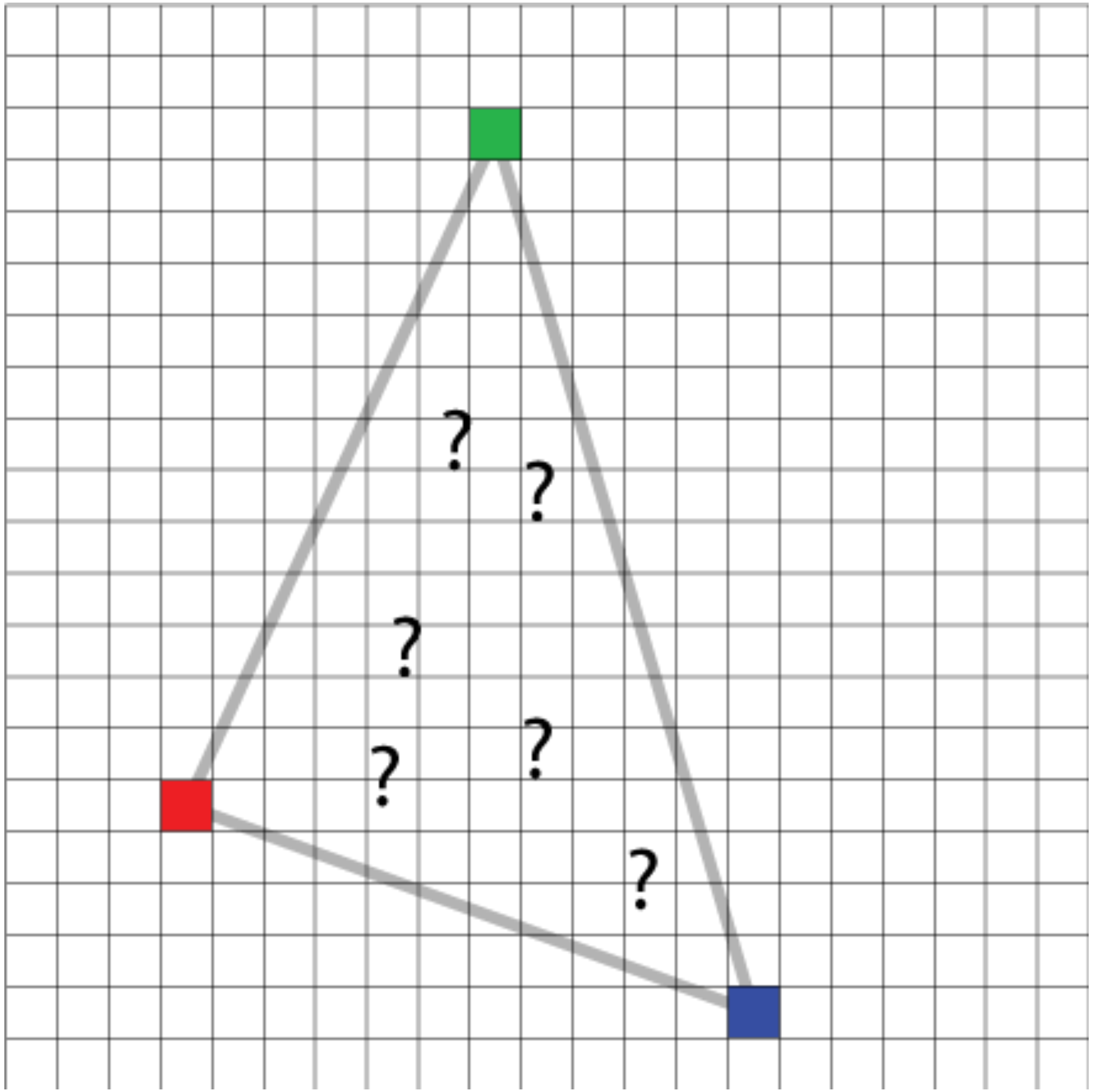
SIMPLER:

How to interpolate color between two points?

$$c(t) \approx c(0) \cdot (1 - t) + c(1) \cdot t$$

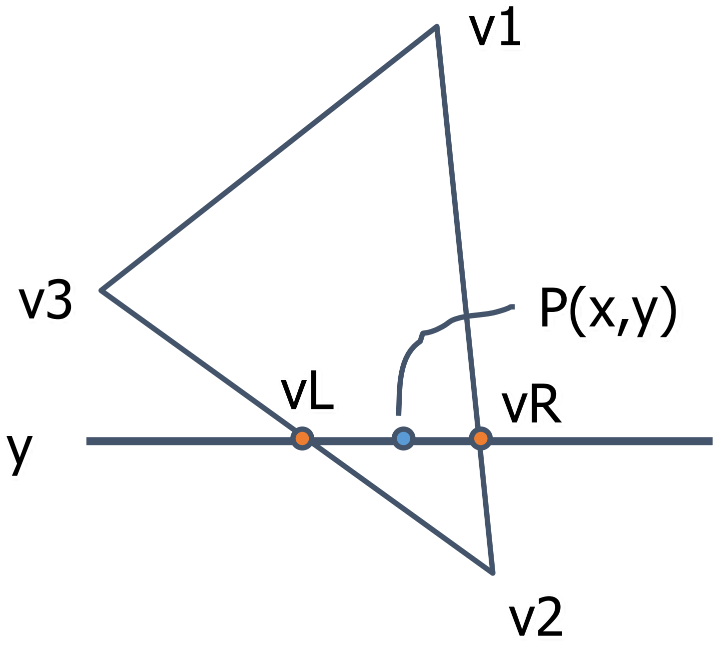
Linear interpolation



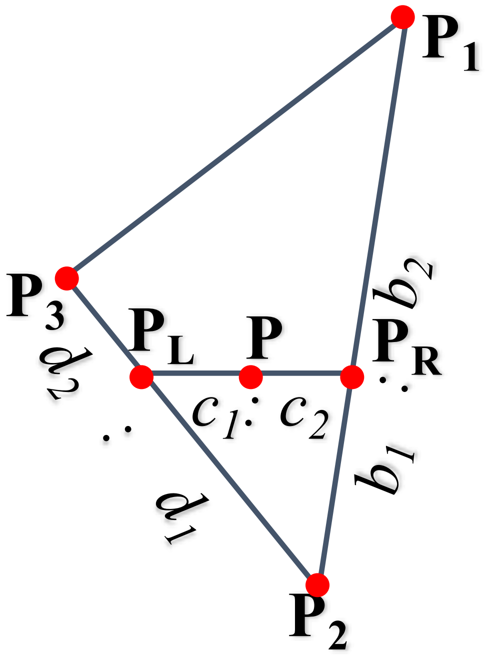


SIMPLE GENERALIZATION: BI-LINEAR INTERPOLATION

- Interpolate quantity along L and R edges
  - (as a function of y)
  - Then interpolate quantity as a function of x



BI-LINEAR INTERPOLATION



$$P = \frac{c_2}{c_1 + c_2} \cdot P_L + \frac{c_1}{c_1 + c_2} \cdot P_R$$

$$P_L = \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3$$

$$P_R = \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1$$

$$P = \frac{c_2}{c_1 + c_2} \left(\frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3 \right) + \frac{c_1}{c_1 + c_2} \left(\frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1 \right)$$

BARYCENTRIC COORDINATES

- Area

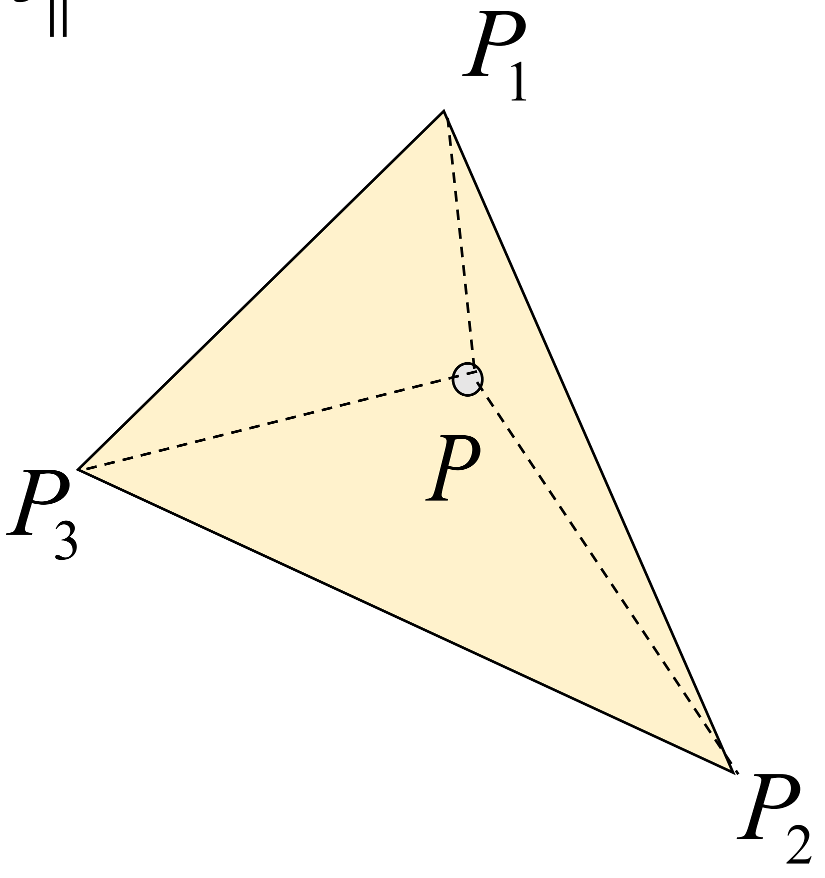
$$A = \frac{1}{2}\left\|\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}\right\|$$

- Barycentric coordinates

$$a_1 = A_{P_2P_3P} / A, a_2 = A_{P_3P_1P} / A,$$

$$a_3 = A_{P_1P_2P} / A,$$

$$P = a_1P_1 + a_2P_2 + a_3P_3$$

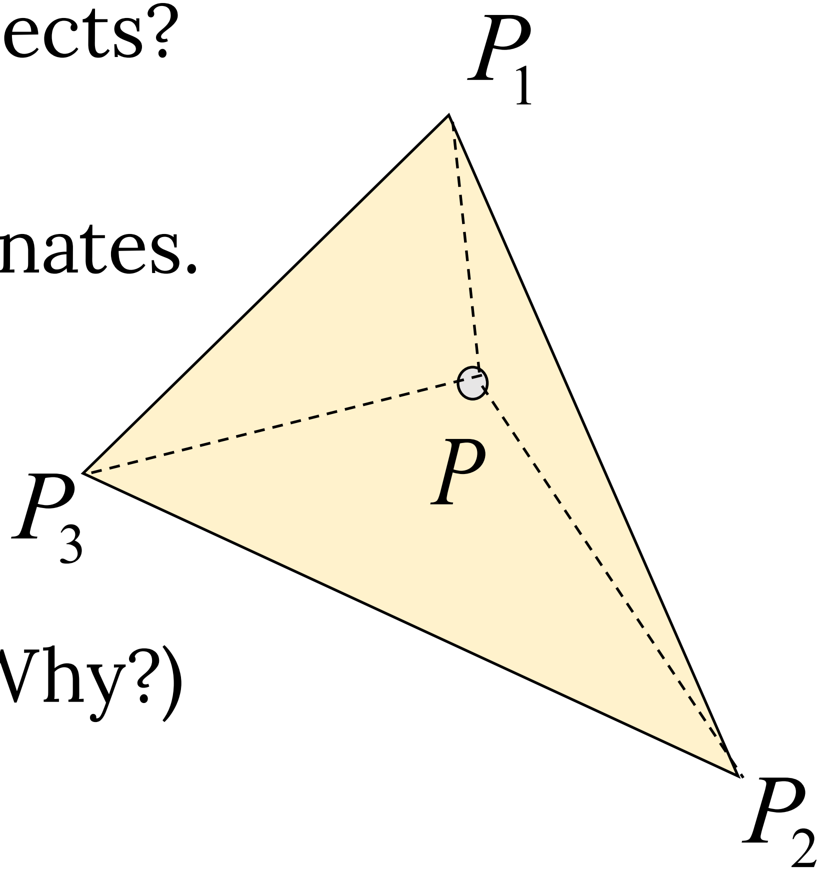


BARYCENTRIC COORDINATES

- Imagine there are little heavy objects at the vertices
- If P is the center of mass of such triangle,
- What are the masses of those objects?

- Those are the barycentric coordinates.

- (That's an equivalent definition. Why?)



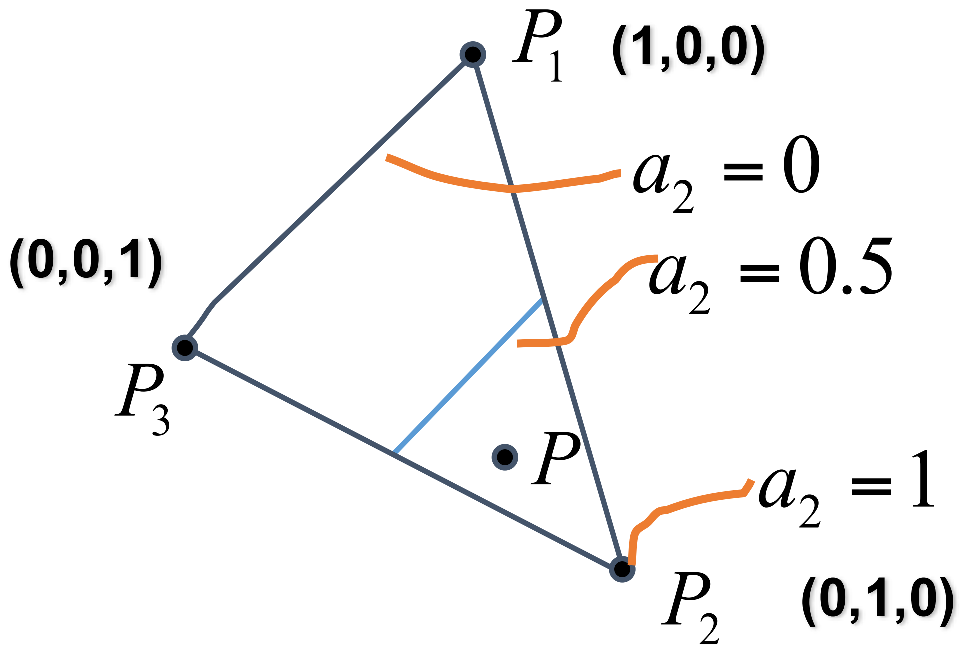
BARYCENTRIC COORDINATES

• weighted (affine) combination of vertices

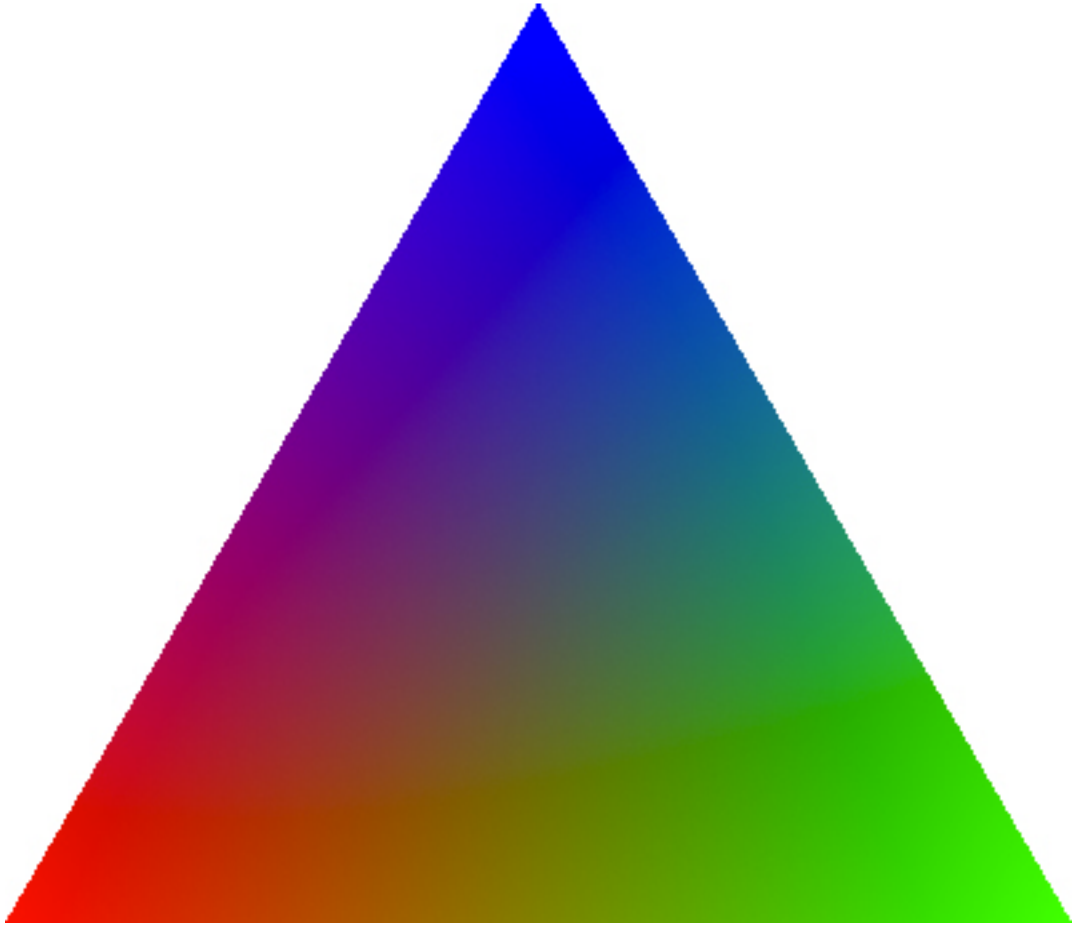
$$P = a_1 \cdot P_1 + a_2 \cdot P_2 + a_3 \cdot P_3$$

$$a_1 + a_2 + a_3 = 1$$

$$0 \leq a_1, a_2, a_3 \leq 1$$



BARYCENTRIC COORDINATES



NOTE:

- In reality, only two values are enough to encode a point in a triangle
- We added a 3rd one – a similar idea to homogeneous coordinates!

- Those are, however, unique because of this:

$$a_1 + a_2 + a_3 = 1$$

BARYCENTRIC COORDINATES

- Are used to interpolate
  - z
  - all varying variables
    - color
    - normals
  - Why do we interpolate z?
  - Problems when using perspective camera. We'll see later (in texture mapping)