University of British Columbia
CPSC 314 Computer Graphics
Jan-Apr 2016

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Final Review I

http://www.ugrad.cs.ubc.ca/~cs314/Vjan2016

Beyond 314: Other Graphics Courses

- 426: Computer Animation
  - will be offered next year (2016/2017)
- 424: Geometric Modelling
  - will be offered in two years (2017/2018)

- 526: Algorithmic Animation - van de Panne
- 530P: Sensorimotor Computation - Pai
- 533A: Digital Geometry – Sheffer
- 547: Information Visualization - Munzner

Final

- exam notes: noon Thu Apr 14 SWNG 122
  - exam will be timed for 2.5 hours, but reserve entire 3-hour block of time just in case
  - closed book, closed notes
  - except for 2-sided 8.5”x11” sheet of handwritten notes
    - ok to staple midterm sheet + new one back to back
  - calculator: a good idea, but not required
    - graphical OK, smartphones etc not ok
  - IDs out and face up

Final Emphasis

- covers entire course
- includes some material from before midterm
  - transformations, viewing
  - H1/H2, P1/P2
- but much heavier weighting for material after midterm
  - H3/H4, P3/P4
- post-midterm topics:
  - shaders
  - lighting/shading
  - raytracing
  - collision
  - rasterization / clipping
  - hidden surfaces / blending / picking
  - textures / procedural
  - color
- light coverage
  - animation, visualization

Sample Final

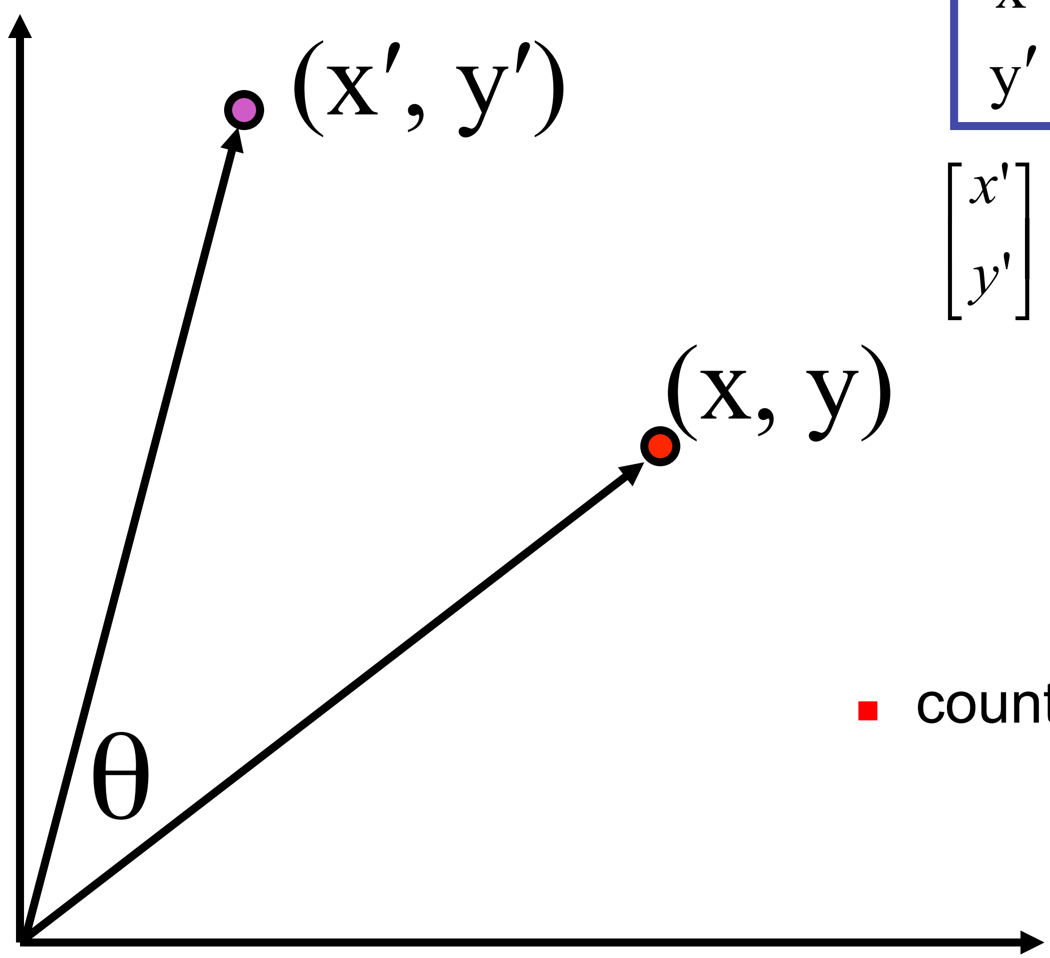
- final+solutions now posted
  - Jan 2007
- note some material not covered this time
  - projection types like cavalier/cabinet: Q1b, Q1c,
  - antialiasing/sampling: Q1d, Q1l, Q12
  - image-based rendering: Q1g
  - clipping algorithms: Q8, Q9
  - scientific visualization: Q14
  - curves/splines: Q18, Q19
- missing some new material
  - shaders

Studying Advice

- do problems!
  - work through old homeworks, exams
    - especially from years where I taught

Review – Fast!!

Review: 2D Rotation



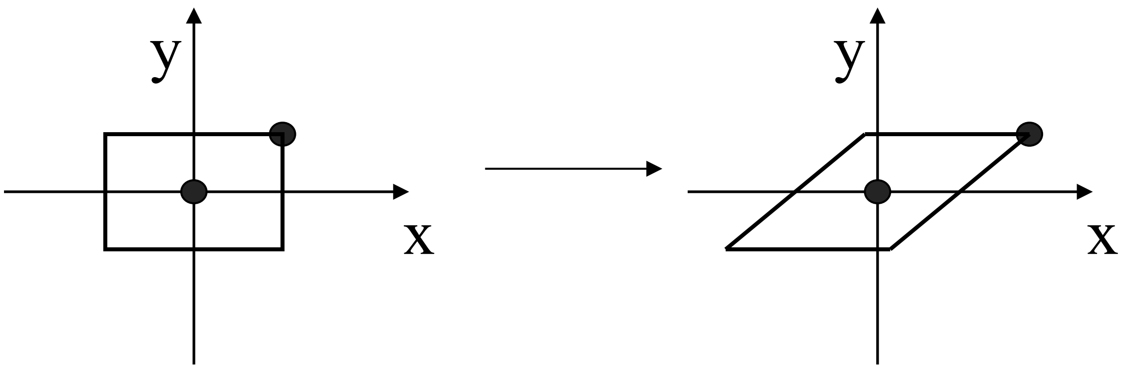
$$x' = x\ \mathbf{cos}(\theta) - y\ \mathbf{sin}(\theta)$$
$$y' = x\ \mathbf{sin}(\theta) + y\ \mathbf{cos}(\theta)$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- counterclockwise, RHS

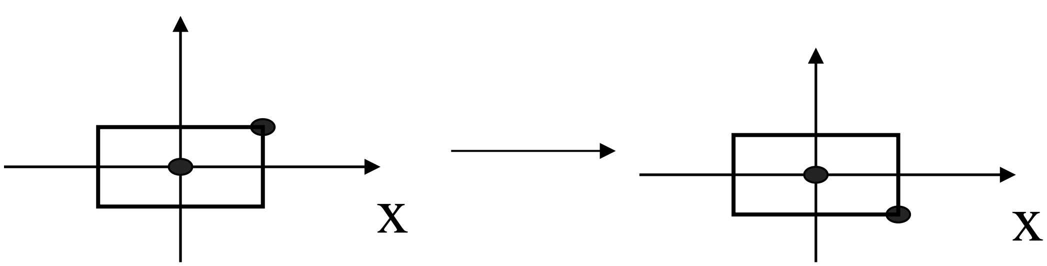
Review: Shear, Reflection

- shear along x axis
  - push points to right in proportion to height



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- reflect across x axis
  - mirror



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Review: 2D Transformations

matrix multiplication

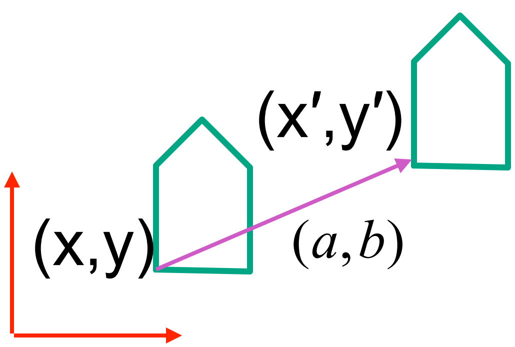
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}\begin{bmatrix} x \\ y \end{bmatrix}$$

scaling matrix

matrix multiplication

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}\begin{bmatrix} x \\ y \end{bmatrix}$$

rotation matrix



vector addition

$$\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

translation multiplication matrix??

Review: Linear Transformations

- linear transformations are combinations of
  - shear
  - scale
  - rotate
  - reflect

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x' = ax + by$$

$$y' = cx + dy$$

- properties of linear transformations
  - satisifes $T(s\mathbf{x}+t\mathbf{y}) = s\,T(\mathbf{x}) + t\,T(\mathbf{y})$
  - origin maps to origin
  - lines map to lines
  - parallel lines remain parallel
  - ratios are preserved
  - closed under composition

Review: Affine Transformations

- affine transforms are combinations of
  - linear transformations
  - translations

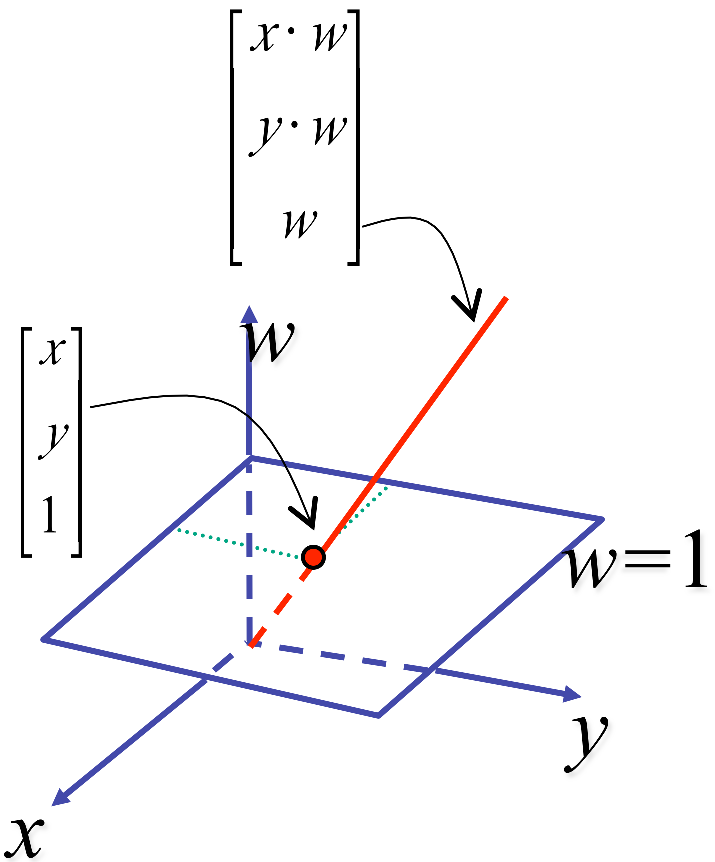
$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- properties of affine transformations
  - origin does not necessarily map to origin
  - lines map to lines
  - parallel lines remain parallel
  - ratios are preserved
  - closed under composition

Review: Homogeneous Coordinates

homogeneous → cartesian

$$(x, y, w) \xrightarrow{/\,w} \left(\frac{x}{w}, \frac{y}{w}\right)$$



- homogenize to convert homog. 3D point to cartesian 2D point:
  - divide by w to get (x/w, y/w, 1)
  - projects line to point onto w=1 plane
  - like normalizing, one dimension up
- when w=0, consider it as direction
  - points at infinity
  - these points cannot be homogenized
  - lies on x-y plane
- (0,0,0) is undefined

Review: 3D Homog Transformations

- use 4x4 matrices for 3D transformations

translate(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

scale(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & & & \\ & b & & \\ & & c & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\text{Rotate}(x,\theta)$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & \cos\theta & -\sin\theta & \\ & \sin\theta & \cos\theta & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\text{Rotate}(y,\theta)$

$$\begin{bmatrix} \cos\theta & & \sin\theta & \\ & 1 & & \\ -\sin\theta & & \cos\theta & \\ & & & 1 \end{bmatrix}$$

$\text{Rotate}(z,\theta)$

$$\begin{bmatrix} \cos\theta & -\sin\theta & & \\ \sin\theta & \cos\theta & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

Review: 3D Shear

- general shear $shear(hxy, hxz, hyx, hyz, hzx, hzy) = \begin{bmatrix} 1 & hyx & hzx & 0 \\ hxy & 1 & hzy & 0 \\ hxz & hyz & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- "x-shear" usually means shear along x in direction of some other axis
  - **correction: not** shear along some axis in direction of x
  - to avoid ambiguity, always say "shear along <axis> in direction of <axis>"

$$shearAlongXinDirectionOfY(h) = \begin{bmatrix} 1 & h & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shearAlongXinDirectionOfZ(h) = \begin{bmatrix} 1 & 0 & h & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

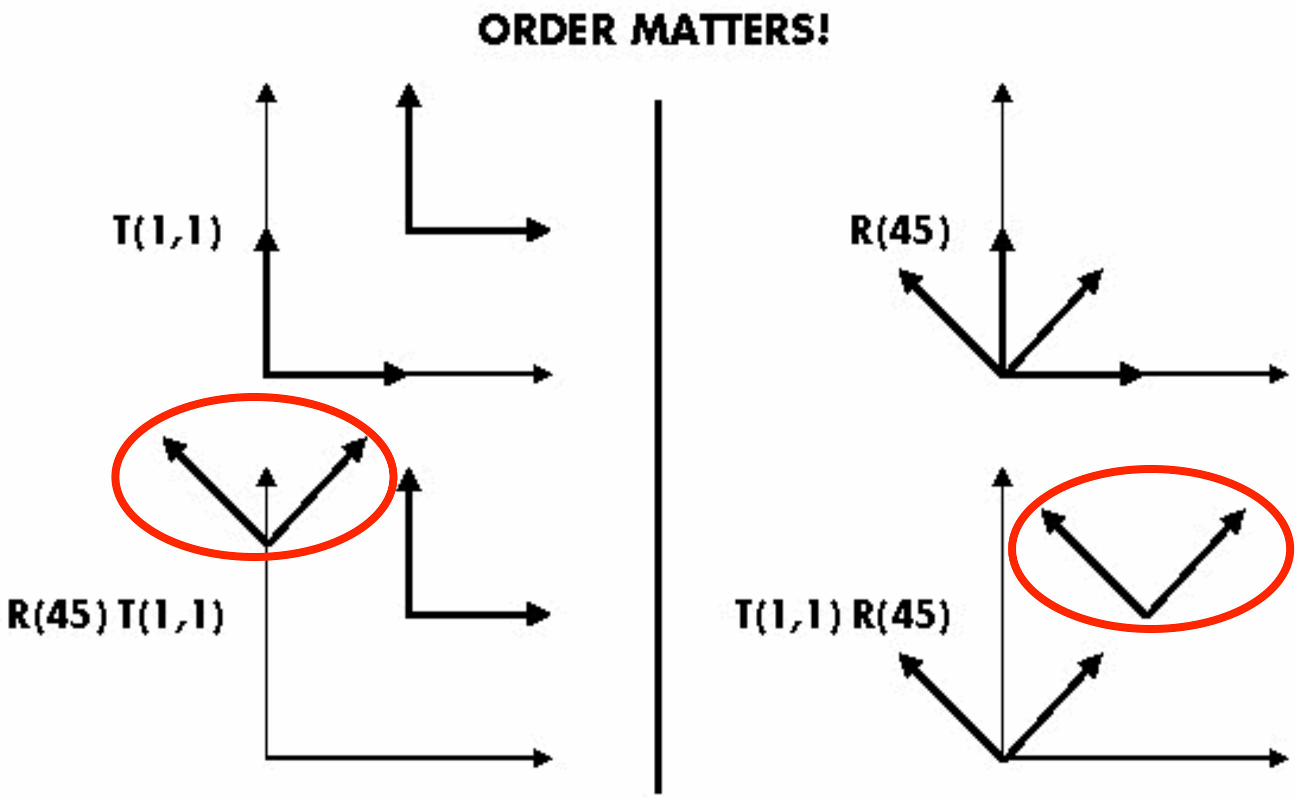
$$shearAlongYinDirectionOfX(h) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ h & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shearAlongYinDirectionOfZ(h) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shearAlongZinDirectionOfX(h) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ h & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shearAlongZinDirectionOfY(h) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & h & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Review: Composing Transformations



Ta Tb = Tb Ta, but Ra Rb != Rb Ra and Ta Rb != Rb Ta

- translations commute
- rotations around same axis commute
- rotations around different axes do not commute
- rotations and translations do not commute

Review: Composing Transformations

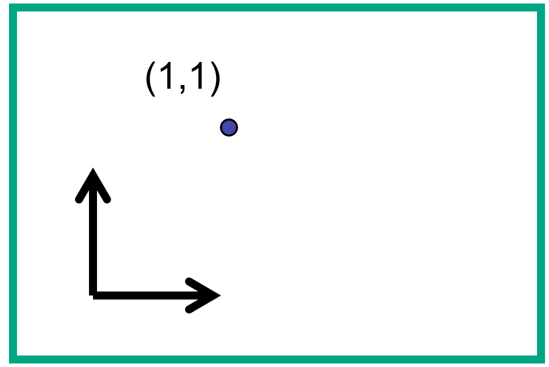
$$\mathbf{p'} = \mathbf{TRp}$$

- which direction to read?
  - right to left
    - interpret operations wrt fixed coordinates
    - moving object
  - left to right OpenGL pipeline ordering!
    - interpret operations wrt local coordinates
    - changing coordinate system
    - OpenGL updates current matrix with postmultiply
      - glTranslatef(2,3,0);
      - glRotatef(-90,0,0,1);
      - glVertexf(1,1,1);
    - specify vector last, in final coordinate system
    - first matrix to affect it is specified second-to-last

Review: Interpreting Transformations

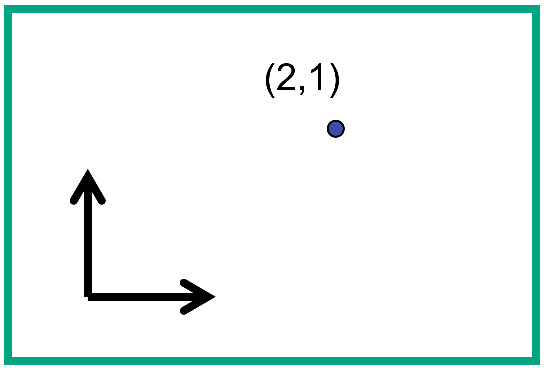
$$\mathbf{p}' = \mathbf{T}\mathbf{R}\mathbf{p}$$

right to left: moving object

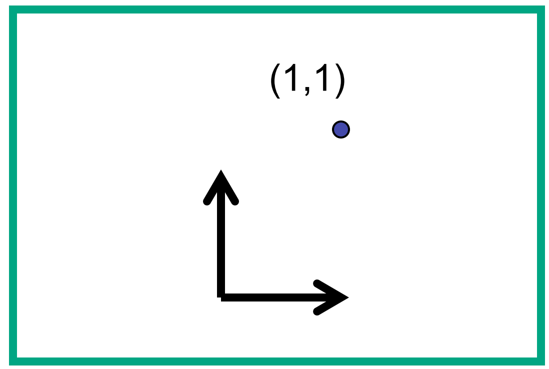


intuitive?

translate by (-1,0)



left to right: changing coordinate system



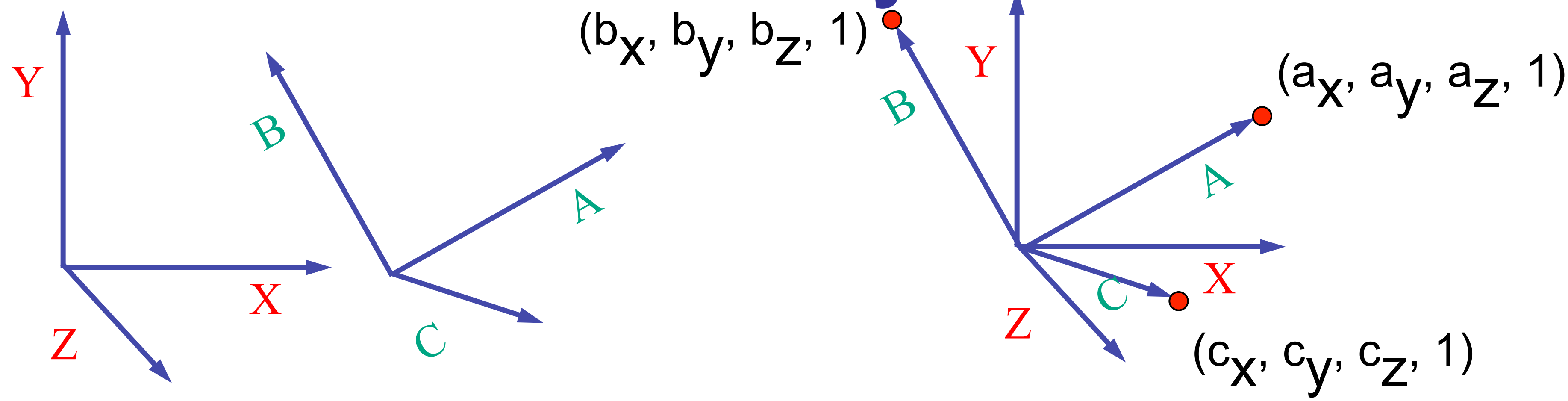
GL

- same relative position between object and basis vectors

Review: General Transform Composition

- transformation of geometry into coordinate system where operation becomes simpler
  - typically translate to origin
- perform operation
- transform geometry back to original coordinate system

Review: Arbitrary Rotation

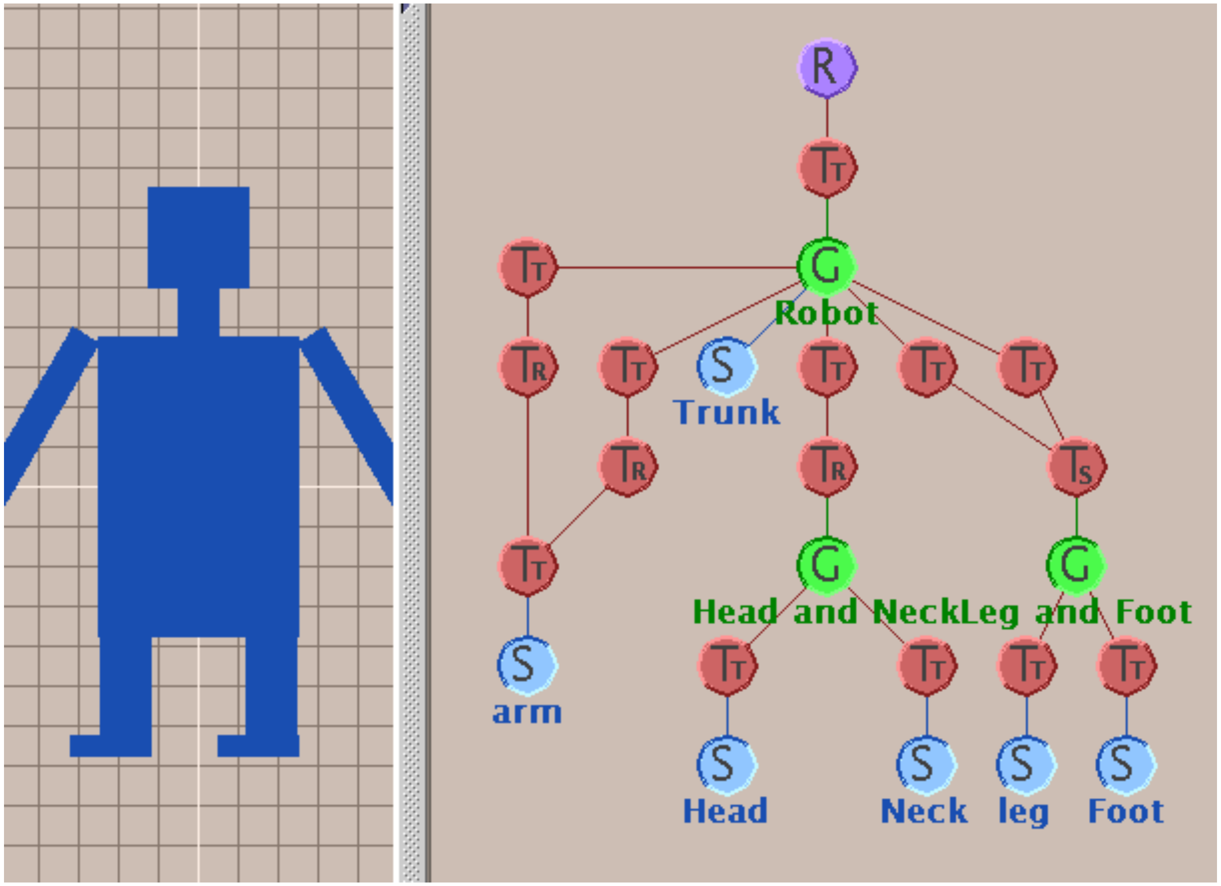


- arbitrary rotation: change of basis
  - given two orthonormal coordinate systems *XYZ* and *ABC*
    - *A*'s location in the XYZ coordinate system is $(a_x, a_y, a_z, 1)$, ...
- transformation from one to the other is matrix R whose columns are *A,B,C:*

$$R(X) = \begin{bmatrix} a_x & b_x & c_x & 0 \\ a_y & b_y & c_y & 0 \\ a_z & b_z & c_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = (a_x, a_y, a_z, 1) = A$$

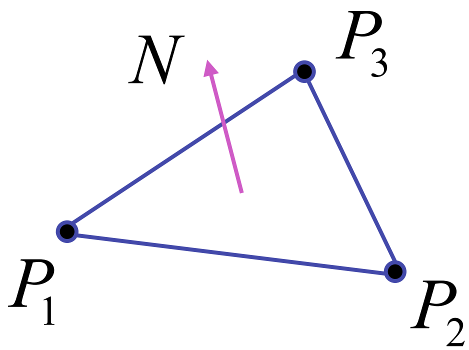
Review: Transformation Hierarchies

- transforms apply to graph nodes beneath them



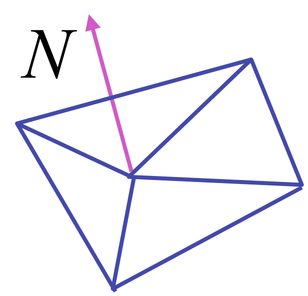
Review: Normals

- polygon:



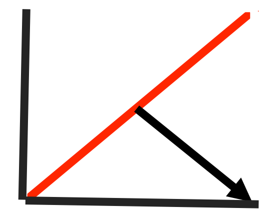
$$N = (P_2 - P_1) \times (P_3 - P_1)$$

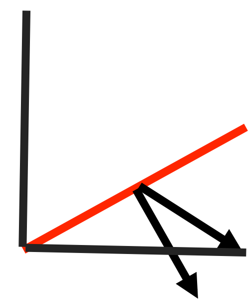
- assume vertices ordered CCW when viewed from visible side of polygon
- normal for a vertex
  - specify polygon orientation
  - used for lighting
  - supplied by model (i.e., sphere), or computed from neighboring polygons



Review: Transforming Normals

- cannot transform normals using same matrix as points
  - nonuniform scaling would cause to be not perpendicular to desired plane!





$$P \qquad\qquad P' = MP$$
$$N \longrightarrow N' = QN$$

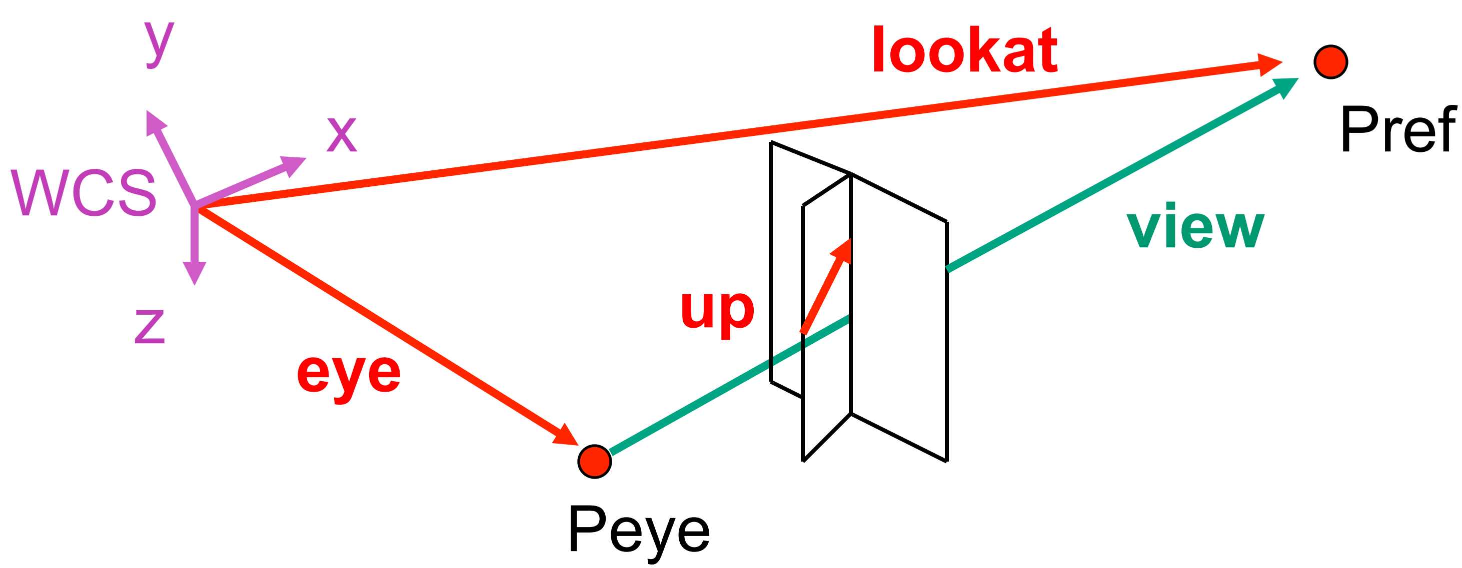
given M,
what should Q be?

$$\mathbf{Q} = \left(\mathbf{M}^{-1}\right)^{\mathrm{T}}$$

inverse transpose of the modelling transformation

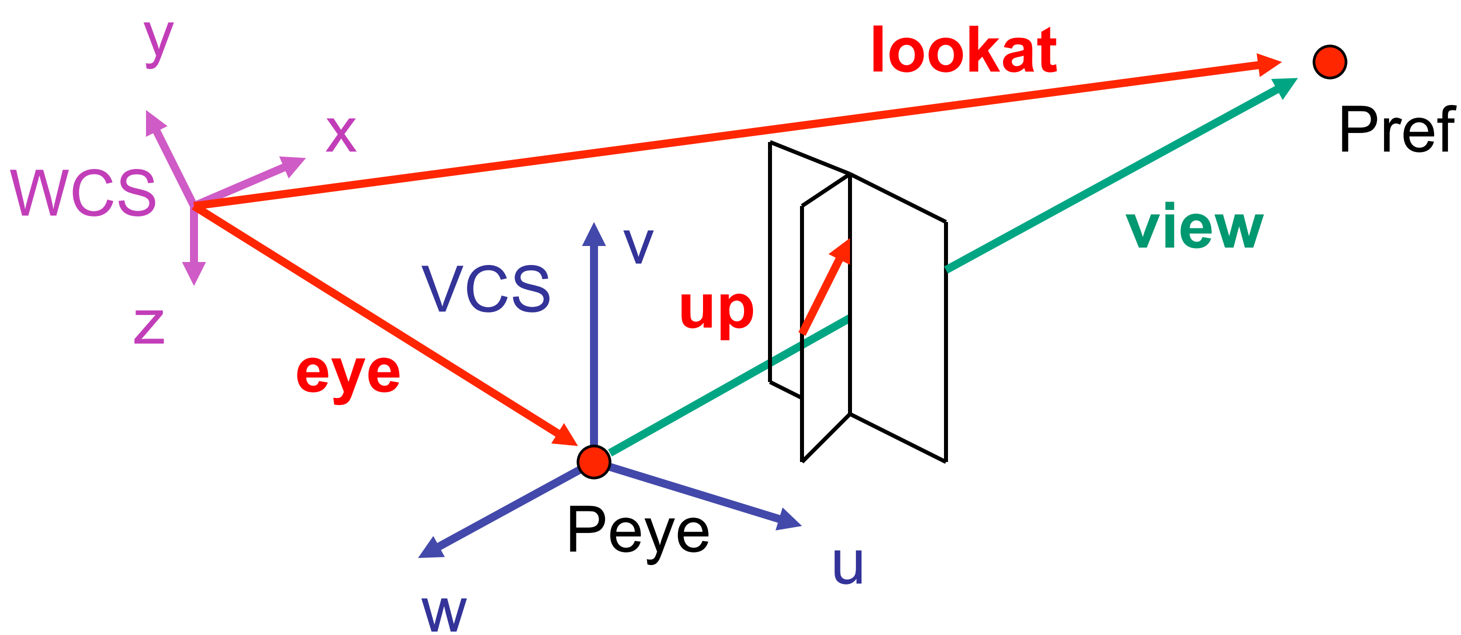
Review: Camera Motion

- rotate/translate/scale difficult to control
- arbitrary viewing position
  - eye point, gaze/lookat direction, up vector



Review: Constructing Lookat

- translate from origin to **eye**
- rotate **view** vector (**lookat** – **eye**) to **w** axis
- rotate around **w** to bring **up** into **vw**-plane



Review: V2W vs. W2V

- M_{V2W}=TR

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

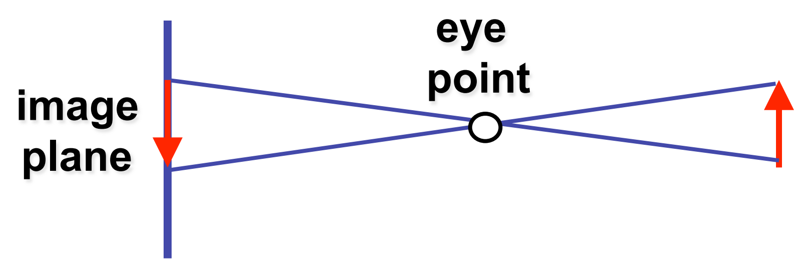
- we derived position of camera as object in world
  - invert for gluLookAt: go from world to camera!
- $M_{W2V}=(M_{V2W})^{-1}=R^{-1}T^{-1}$

$$\mathbf{R}^{-1} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

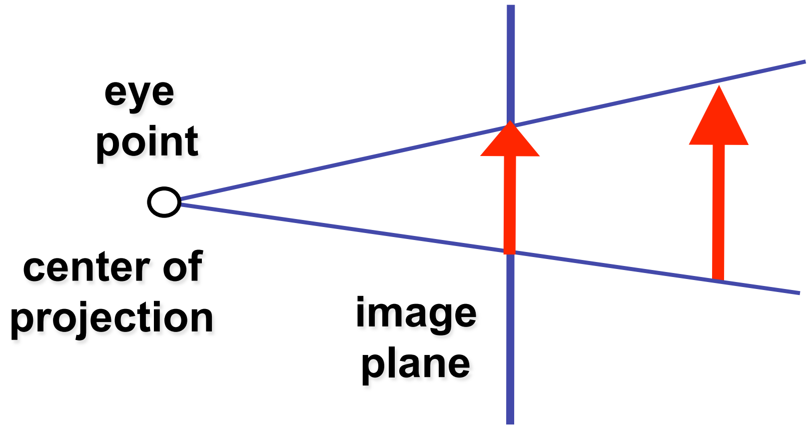
$$\mathbf{M}_{W2V} = \begin{bmatrix} u_x & u_y & u_z & -\mathbf{e}\bullet\mathbf{u} \\ v_x & v_y & v_z & -\mathbf{e}\bullet\mathbf{v} \\ w_x & w_y & w_z & -\mathbf{e}\bullet\mathbf{w} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z & -e_x*u_x+-e_y*u_y+-e_z*u_z \\ v_x & v_y & v_z & -e_x*v_x+-e_y*v_y+-e_z*v_z \\ w_x & w_y & w_z & -e_x*w_x+-e_y*w_y+-e_z*w_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Review: Graphics Cameras

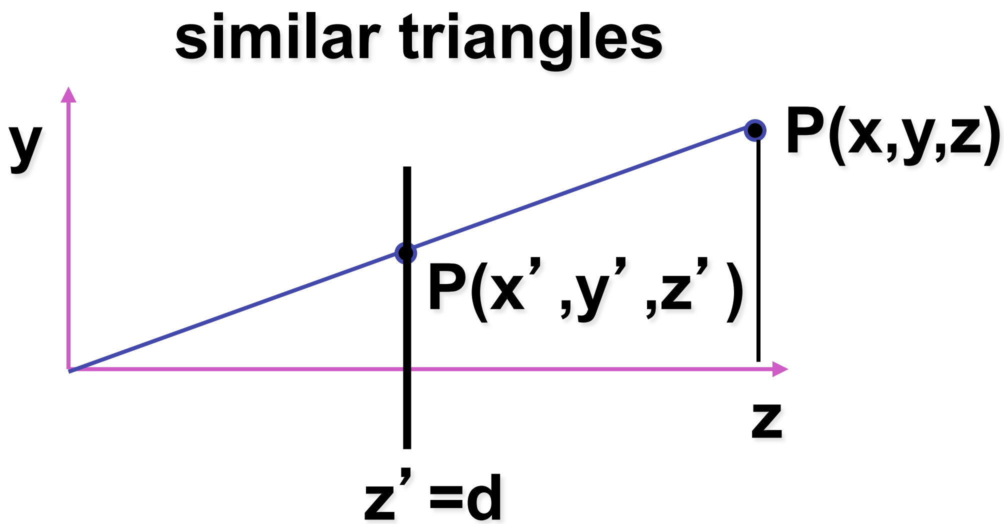
- real pinhole camera: image inverted



- computer graphics camera: convenient equivalent



Review: Basic Perspective Projection



$$\frac{y'}{d} = \frac{y}{z} \rightarrow y' = \frac{y \cdot d}{z}$$

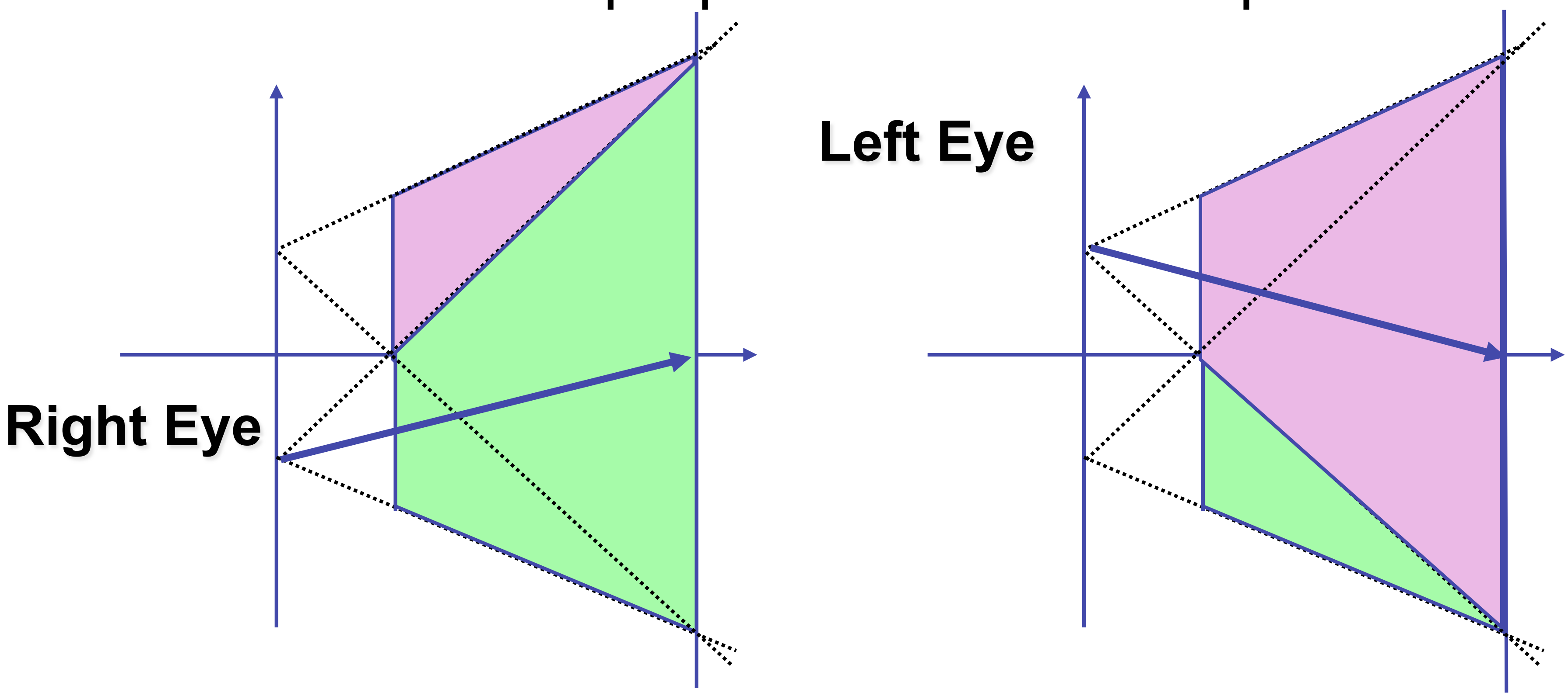
$$x' = \frac{x \cdot d}{z} \qquad z' = d$$

$$\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \end{bmatrix} \xrightarrow{\text{homogeneous coords}} \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

Review: Asymmetric Frusta

- our formulation allows asymmetry
  - why bother? binocular stereo
    - view vector not perpendicular to view plane



Review: Field-of-View Formulation

- FOV in one direction + aspect ratio (w/h)
  - determines FOV in other direction
  - also set near, far (reasonably intuitive)