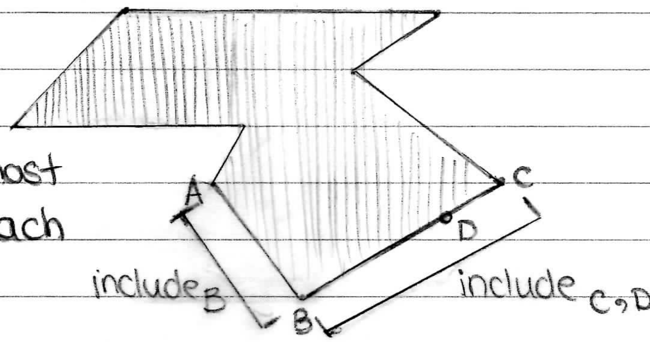
CPSC 314 HW Solutions

1. $N_b = \frac{N_f + N_g}{2} = \frac{(\langle -1,-1,0\rangle + \langle 1,-1,0\rangle)}{2}$

$= \frac{\langle 0,-2,0\rangle}{2}$ Already normalized!

$\boxed{= \langle 0,-1,0\rangle}$

2.



* use rightmost vertex on each face.

So, B = B
D = C
C = C, according to graph above

i) <u>POINT B TOTAL ILLUMINATION</u>. (Phong Lighting)

Total Illumination = Ambient + Diffuse + Specular.

$\text{Ambient}_B = k_a I_a = (0.5, 0.1, 0.5) \times (0.5, 0.2, 0.5)$
$\boxed{= (0.25, 0.02, 0.25)}$

$\text{Diffuse}_B = K_d I_{light} (\vec{n}_B \cdot \vec{l}_B)$

$\vec{l}_B =$ light position $- B = (-1, 0, 0) - (2, 3, 0)$
$= \langle -3, -3, 0\rangle$

* Remember to normalize directions in lighting calculations!

$= \frac{\vec{l}_B}{\|\vec{l}_B\|} = \frac{\langle -3,-3,0\rangle}{\sqrt{(-3)^2+(-3)^2}} = \frac{\langle -3,-3,0\rangle}{\sqrt{18}}$

$= \frac{\langle -3,-3,0\rangle}{3\sqrt{2}} = \boxed{\langle -1/\sqrt{2}, -1/\sqrt{2}, 0\rangle}$

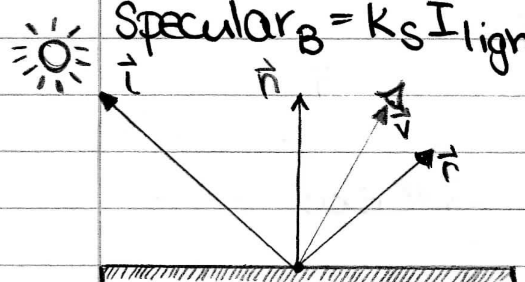
$\vec{n}_B = \langle 0, -1, 0 \rangle$, from Q1.

$$\text{Diffuse}_B = k_d I_{light} (\vec{n}_B \cdot \vec{l}_B)$$
$$= (0.3, 0.8, 0.2)(0.9, 1, 1)(\vec{n}_B \cdot \vec{l}_B)$$
$$= (0.27, 0.8, 0.2)(\langle 0, -1, 0 \rangle \cdot \langle -\tfrac{1}{\sqrt{2}}, -\tfrac{1}{\sqrt{2}}, 0 \rangle)$$
$$= (0.27, 0.8, 0.2)(\tfrac{1}{\sqrt{2}})$$
$$= \boxed{\left(\frac{0.27}{\sqrt{2}}, \frac{0.8}{\sqrt{2}}, \frac{0.2}{\sqrt{2}} \right)}$$

Dot product $\vec{n} \cdot \vec{l}$ is positive, so there must be diffuse component contribution to the total illumination at B!

$$\text{Specular}_B = k_s I_{light} (\vec{v}_B \cdot \vec{r}_B)^n$$

n: angular falloff
$\vec{v}$: view direction
$\vec{r}$: reflected ray direction



$$\vec{v}_B = \text{Eye point} - B = (1, -2, 0) - (2, 3, 0)$$
$$= \langle -1, -5, 0 \rangle$$

* normalize $\vec{v}$

$$= \frac{\vec{v}_B}{\|\vec{v}_B\|} = \frac{\langle -1, -5, 0 \rangle}{\sqrt{1 + 25}} = \frac{\langle -1, -5, 0 \rangle}{\sqrt{26}}$$

$$= \boxed{\left\langle -\tfrac{1}{\sqrt{26}}, -\tfrac{5}{\sqrt{26}}, 0 \right\rangle}$$

$\vec{r}_B = -\vec{l}_B + 2(\vec{n}_B \cdot \vec{l}_B)\vec{n}_B$ *to find reflected ray direction

$\vec{l}_B = \langle -1/\sqrt{2}, -1/\sqrt{2}, 0 \rangle$ *previously calculated for diffuse

$\vec{n}_B \cdot \vec{l}_B = 1/\sqrt{2}$

$$\vec{r}_B = -\langle -1/\sqrt{2}, -1/\sqrt{2}, 0 \rangle + 2(1/\sqrt{2})\langle 0, -1, 0 \rangle$$
$$= \langle 1/\sqrt{2}, 1/\sqrt{2}, 0 \rangle + \langle 0, -2/\sqrt{2}, 0 \rangle$$
$$= \boxed{\langle 1/\sqrt{2}, -1/\sqrt{2}, 0 \rangle}$$

$\|\langle 1/\sqrt{2}, -1/\sqrt{2}, 0 \rangle\| = 1$, $\therefore$ $\vec{r}_B$ is already normalized

$$\text{Specular}_B = K_s I_{light} (\vec{V}_B \cdot \vec{r}_B)^n$$

$$\vec{V}_B \cdot \vec{r}_B = \langle -1/\sqrt{26}, -5/\sqrt{26}, 0 \rangle \cdot \langle 1/\sqrt{2}, -1/\sqrt{2}, 0 \rangle$$
$$= \frac{-1}{\sqrt{2}\sqrt{26}} + \frac{5}{\sqrt{2}\sqrt{26}}$$
$$= \frac{4}{\sqrt{2}\sqrt{26}}$$

*dot product $\vec{V} \cdot \vec{r}$ is positive, so there must be specular contribution to total illumination.

$$K_s I_{light} = (0.5, 1, 1)(0.9, 1, 1) = (0.45, 1, 1)$$

$$\text{Specular}_B = (0.45, 1, 1)\left(\frac{4}{\sqrt{2}\sqrt{26}}\right)^{10}$$
$$= \boxed{\left(0.45\left(\frac{4}{\sqrt{2}\sqrt{26}}\right)^{10}, \left(\frac{4}{\sqrt{2}\sqrt{26}}\right)^{10}, \left(\frac{4}{\sqrt{2}\sqrt{26}}\right)^{10}\right)}$$

$$\text{Total Illumination}_B = I_a K_a + \sum_P I_p (K_d (\vec{n}_B \cdot \vec{l}_B) \ldots$$
$$\ldots + K_s (\vec{r}_B \cdot \vec{v}_B)^n)$$

$$= (0.25, 0.02, 0.25) + \left(\frac{0.27}{\sqrt{2}}, \frac{0.8}{\sqrt{2}}, \frac{0.2}{\sqrt{2}}\right) + \ldots$$

$$\ldots \left(0.45\left(\frac{4}{\sqrt{2}\sqrt{26}}\right)^{10}, \left(\frac{4}{\sqrt{2}\sqrt{26}}\right)^{10}, \left(\frac{4}{\sqrt{2}\sqrt{26}}\right)^{10}\right)$$

$$\approx (0.25, 0.02, 0.25) + (0.1909, 0.5657, 0.1414) \ldots$$
$$\ldots + (0.0012, 0.0028, 0.0028)$$
$$\boxed{\approx (0.4422, 0.5885, 0.3942)}$$

ii) <u>POINT C TOTAL ILLUMINATION (and D)</u> (Phong Lighting)

$$\text{Ambient}_C = K_a I_a = \boxed{(0.25, 0.02, 0.25)}$$

* previously calculated for B

$$\text{Diffuse}_C = K_d I_{light} (\vec{n}_C \cdot \vec{l}_C)$$

$$\vec{n}_C = \frac{\vec{N}_C}{\|\vec{N}_C\|} = \frac{\langle 0.9, 0.5, 0 \rangle}{\sqrt{0.9^2 + 0.5^2}} = \frac{\langle 0.9, 0.5, 0 \rangle}{\sqrt{1.06}}$$

$$= \boxed{\left\langle \frac{0.9}{\sqrt{1.06}}, \frac{0.5}{\sqrt{1.06}}, 0 \right\rangle}$$

$$\vec{l}_C = \text{light position} - C = (-1, 0, 0) - (8, 12, 0)$$
$$= \langle -9, -12, 0 \rangle$$

$$= \frac{\vec{l}_C}{\|\vec{l}_C\|} = \frac{\langle -9, -12, 0 \rangle}{\sqrt{(-9)^2 + (-12)^2}} = \frac{\langle -9, -12, 0 \rangle}{\sqrt{225}}$$

$$\vec{l}_c = \boxed{\left\langle \frac{-9}{15}, \frac{-12}{15}, 0 \right\rangle} = \left\langle \frac{-3}{5}, \frac{-4}{5}, 0 \right\rangle$$

$$\text{Diffuse}_c = K_d I_{light} (\vec{n}_c \cdot \vec{l}_c)$$

$$\vec{n}_c \cdot \vec{l}_c = \left\langle \frac{0.9}{\sqrt{1.06}}, \frac{0.5}{\sqrt{1.06}}, 0 \right\rangle \cdot \left\langle \frac{-9}{15}, \frac{-12}{15}, 0 \right\rangle$$

This will evaluate to a negative number due to $\vec{n}_c$ being positive and $\vec{l}_c$ being negative. So, $\boxed{\text{there is no diffuse contribution!}}$

* we clamp $\vec{n}_c \cdot \vec{l}_c$ to 0

$$= \max(0, \vec{n}_c \cdot \vec{l}_c)$$

$$\text{Specular}_c = K_s I_{light} (\vec{V}_c \cdot \vec{r}_c)^n$$

$$\vec{V}_c = \text{Eye point} - c = (1, -2, 0) - (8, 12, 0)$$

$$= \langle -7, -14, 0 \rangle$$

$$= \frac{\vec{V}_c}{\|\vec{V}_c\|} = \frac{\langle -7, -14, 0 \rangle}{\sqrt{49 + 196}} = \boxed{\left\langle \frac{-7}{\sqrt{245}}, \frac{-14}{\sqrt{245}}, 0 \right\rangle}$$

$$\vec{r}_c = -l_c + 2(\vec{n}_c \cdot \vec{l}_c)\vec{n}_c$$

$$= -\left\langle \frac{-9}{15}, \frac{-12}{15}, 0 \right\rangle + \ldots$$

* remember though, we clamped $\vec{n}_c \cdot \vec{l}_c$ to 0!!!

$$\ldots 2(0) \left\langle \frac{0.9}{\sqrt{1.06}}, \frac{0.5}{\sqrt{1.06}}, 0 \right\rangle$$

$$= \boxed{\left\langle \frac{9}{15}, \frac{12}{15}, 0 \right\rangle}$$

* this is already normalized!

$\text{Specular}_c = K_s I_{light} (\vec{V_c} \cdot \vec{r_c})^n$

$$\vec{V_c} \cdot \vec{r_c} = \left\langle \frac{-7}{\sqrt{245}}, \frac{-14}{\sqrt{245}}, 0 \right\rangle \cdot \left\langle \frac{9}{15}, \frac{12}{15}, 0 \right\rangle$$

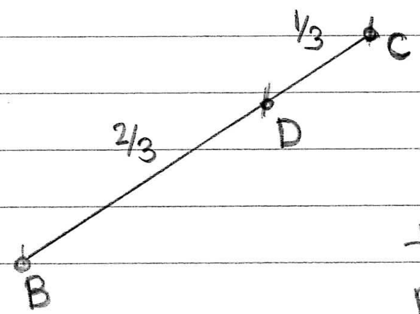
* this will evaluate to a negative number due to $\vec{V_c}$ being negative and $\vec{r_c}$ being positive. So, there is $\boxed{\text{no specular contribution!}}$

$\therefore$ Total Illumination$_c$ = Ambient
$= \boxed{(0.25, 0.02, 0.25)}$

$\therefore$ Total Illumination$_D$ = $\boxed{(0.25, 0.02, 0.25)}$

3. Calculate total illumination at points B, C, and D, using Gouraud shading model. This means we have to interpolate colors between vertices, which means the only affected point from Q2 is point D, since B and C are vertices and we have already calculated total illumination at these vertices. So,

B = equivalent to FLAT SHADING answer
C = equivalent to FLAT SHADING answer.
D = interpolate between B and C.



we can determine, by looking at the segment between B and C, the ratios of sub segments B→D and C→D.

Thus,

$$D = \tfrac{1}{3}B + \tfrac{2}{3}C$$

* B has a smaller contribution to D than C does, since D is closer to C than it is to B.

$$= \tfrac{1}{3}(0.4422, 0.5885, 0.3942) + \ldots$$
$$\tfrac{2}{3}(0.25, 0.02, 0.25)$$

$$= (0.1474, 0.1961, 0.1314) + \ldots$$
$$(0.1667, 0.0133, 0.1667)$$

$$= \boxed{(0.3141, 0.2094, 0.2981)}$$

$$B = \boxed{(0.4422, 0.5885, 0.3942)}$$
$$C = \boxed{(0.25, 0.02, 0.25)}$$

* you can also think about B and C as interpolating along the segment $B \to C$, where the ratio for B would be $1B + 0C$ and for C, $0B + 1C$.

4. Calculate total illumination at points B, C, and D, using the Phong Shading model. This means that we have to interpolate normals between vertices, which means the only affected point from Q2 is point D, since B and C are vertices so their normals do not change (their total illumination is the same as what we calculated in Q2).

By using the same ratios as in Q3, it can be determined that

$\vec{n}_D = \frac{1}{3}\vec{N}_B + \frac{2}{3}\vec{N}_C$ where

$\vec{N}_B = \langle 0, -1, 0 \rangle$

$\vec{N}_C = \left\langle \frac{0.9}{\sqrt{1.06}}, \frac{0.5}{\sqrt{1.06}}, 0 \right\rangle$ *previously calculated

$\frac{1}{3}N_B = \langle 0, -\frac{1}{3}, 0 \rangle$

$\frac{2}{3}N_C = \left\langle \frac{1.8}{3\sqrt{1.06}}, \frac{1}{3\sqrt{1.06}}, 0 \right\rangle \approx \langle 0.5828, 0.3238, 0 \rangle$

$\vec{n}_D = \langle 0.5828, -0.0096, 0 \rangle$

$$= \frac{\vec{n}_D}{\|\vec{n}_D\|} = \frac{\langle 0.5828, -0.0096, 0 \rangle}{\sqrt{0.3397}}$$

$$\approx \boxed{\langle 0.9999, -0.0165, 0 \rangle}$$

$\text{Ambient}_D = \boxed{(0.25, 0.02, 0.25)}$

$\text{Diffuse}_D = K_d I_a (\vec{n}_D \cdot \vec{l}_D)$

$\vec{l}_D = \text{light position} - D = (-1, 0, 0) - (6, 9, 0)$
$= \langle -7, -9, 0 \rangle$

$$= \frac{\vec{l}_D}{\|\vec{l}_D\|} = \frac{\langle -7, -9, 0 \rangle}{\sqrt{49+81}} = \frac{\langle -7, -9, 0 \rangle}{\sqrt{130}}$$

$$\vec{l}_D = \boxed{\left\langle \frac{-7}{\sqrt{130}}, \frac{-9}{\sqrt{130}}, 0 \right\rangle}$$

$$\vec{n}_D \cdot \vec{l}_D = \langle 0.9999, -0.0165, 0 \rangle \cdot \left\langle \frac{-7}{\sqrt{130}}, \frac{-9}{\sqrt{130}}, 0 \right\rangle$$

$$= (0.9999)\left(\frac{-7}{\sqrt{130}}\right) + (-0.0165)\left(\frac{-9}{\sqrt{130}}\right)$$

$\approx \boxed{-0.6009}$

* because $\vec{n}_D \cdot \vec{l}_D$ is negative (<0), there is no diffuse contribution! Also, we clamp $\vec{n}_D \cdot \vec{l}_D$ to 0
$= \max(0, \vec{n}_D \cdot \vec{l}_D)$

$\text{Specular}_D = K_s I_{light} (\vec{V}_D \cdot \vec{r}_D)^n$

$\vec{V}_D = \text{Eye position} - D = (1, -2, 0) - (6, 9, 0)$
$= \langle -7, -11, 0 \rangle$

$$\vec{V}_D = \frac{\vec{V}_D}{\|\vec{V}_D\|} = \frac{\langle -7, -11, 0 \rangle}{\sqrt{49+121}} = \frac{\langle -7, -11, 0 \rangle}{\sqrt{170}}$$

$$= \boxed{\left\langle \frac{-7}{\sqrt{170}}, \frac{-11}{\sqrt{170}}, 0 \right\rangle}$$

$\vec{r}_D = -l_D + 2(\vec{n}_D \cdot \vec{l}_D)\vec{n}_D$

* remember that because $\vec{n}_D \cdot \vec{l}_D < 0$, it is clamped to 0, just as if we are computing $\max(0, \vec{n}_D \cdot \vec{l}_D)$ for every instance.

$$= -\left\langle \frac{-7}{\sqrt{130}}, \frac{-9}{\sqrt{130}}, 0 \right\rangle + 2(0)\vec{n}_D$$

$$= \boxed{\left\langle \frac{7}{\sqrt{130}}, \frac{9}{\sqrt{130}}, 0 \right\rangle}$$ * Already normalized!

$$\text{Specular}_D = K_s I_{light} (\vec{V}_D \cdot \vec{r}_D)^n$$

$$\vec{V}_D \cdot \vec{r}_D = \left\langle \frac{-7}{\sqrt{170}}, \frac{-11}{\sqrt{170}}, 0 \right\rangle \cdot \left\langle \frac{7}{\sqrt{130}}, \frac{9}{\sqrt{130}}, 0 \right\rangle$$

* this will evaluate to negative because $\vec{V}_D$ is negative and $\vec{r}_D$ is positive. So, there will be no specular contribution!

$$\text{Total Illumination}_D = \text{Ambient}$$
$$= \boxed{(0.25, 0.02, 0.25)}$$

$B = \boxed{(0.4422, 0.5885, 0.3942)}$
$C = \boxed{(0.25, 0.02, 0.25)}$